(1) [20 points] In the stage II, the incumbent deters entry by setting $p^I = p^e$, thereby earning a profit of $\pi^I_D = p^e(12 - p^e)$. In this case, $\pi^e =$.

Also, in the stage II, if the incumbent accommodates entry by setting $p^I > p^e$, the incumbent solves

$$\max \pi^I_A = (12 - k - p^I)p^I \Longrightarrow q^I_A = \frac{12 - k}{2} \Longrightarrow \pi^I_A = \left(\frac{12 - k}{2}\right)^2.$$

Moving to stage I, suppose that the entrant sets $\langle k, p^e \rangle = \langle 2, \$3 \rangle$. Then,

$$\pi^I_D = 3(12 - 3) = \$27 > \$25 = \left(\frac{12 - 2}{2}\right)^2 = \pi^I_A.$$

Hence, entry is deterred.

Also, if the entrant sets $\langle k, p^e \rangle = \langle 4, \$1 \rangle$,

$$\pi^I_D = 1(12 - 1) = \$11 < \$16 = \left(\frac{12 - 4}{2}\right)^2 = \pi^I_A.$$

Hence, entry is accommodated, and the entrant earns $\pi^e = 4 \cdot 1 = \$4$.

Concluding from the above analysis, the subgame-perfect equilibrium strategies of player e and player I are $\langle k, p^e \rangle = \langle 4, \$1 \rangle$ and

$$p^I = \begin{cases} \$3 & \text{if } \langle k, p^e \rangle = \langle 2, \$3 \rangle \\ \$4 & \text{if } \langle k, p^e \rangle = \langle 4, \$1 \rangle. \end{cases}$$

(2a) [15 points] Since the seller is a monopoly, it extract full surplus from consumers. Let $\bar{t}$ be the most recent date at which a new technology has been adopted. Therefore, the next technology replacement will occur in period $\bar{t} + \Delta$. Hence, the seller will introduce a new technology in period $t + \Delta$ if

$$\lambda(\bar{t} + \Delta) + \frac{1}{3}120 + 120 \geq \lambda\bar{t} + 120 + 120.$$

Hence,

$$\Delta = \frac{80}{\lambda} = \begin{cases} 4 & \text{if } \lambda = 20 \\ 2 & \text{if } \lambda = 40. \end{cases}$$

Thus, in this model, technology is replaced more often when technology, as reflected by the parameter λ, advances faster.

(2b) [5 points] Using the above logic, technology is replaced when

$$\lambda(\bar{t} + \Delta) + N\left(\frac{1}{3} + 1\right) \geq \lambda\bar{t} + N + N \Longrightarrow \Delta = \frac{2N}{3\lambda}.$$

Therefore, in this model, technology is replaced less often in economies with a larger consumer population size.

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(3) [20 points] Innovation is classified as "major" if the innovator's monopoly price satisfies $p^m < c$. To find p^m solve the monopoly problem $MR(q) = a - 2q = c - x = MC(q)$ yielding

$$q^m = \frac{a-c+x}{2} \quad \text{and} \quad p^m = \frac{a+c-x}{2} < c \Longleftrightarrow x > a - c.$$

Thus, in this example, innovation if major if $x > a - c$.

For which values of λ minor innovation is profit-maximizing for the firm? Under minor innovation, the innovating firms solves

$$\max_x \pi = x(a-c) - \lambda \frac{x^2}{2} \Longrightarrow x = \frac{a-c}{\lambda} \leq a - c \Longleftrightarrow \lambda \geq 1.$$

Thus, innovation is minor if $\lambda \geq 1$. This makes sense since a high value of λ means that innovation is costly.

Now suppose that $0 < \lambda < 1$ which means that the firm engages in major innovation leading to monopoly pricing. This monopoly solves,

$$\max_x \pi^m = [p^m - (c-x)]q^m = \left[\frac{a+c-x}{2} - (c-x)\right]\frac{a-c+x}{2} - \lambda\frac{x^2}{2} = \left(\frac{a-c+x}{2}\right)^2 - \lambda\frac{x^2}{2}$$

yielding

$$x^m = \frac{a-c}{2\lambda - 1} > a - c \Longleftrightarrow 0 < \lambda < 1$$

which confirms that this innovation is major.

(4a) [10 points] Airline α maximizes the airfare p_α subject to:

$$\pi_\beta = \eta p_\beta \geq 2\eta(p_\alpha - 4 + f_\beta - f_\alpha) = 2\eta(p_\alpha - 4 + 3 - 6) = 2\eta(p_\alpha - 7).$$

Airline β maximizes the airfare p_β subject to:

$$\pi_\alpha = \eta p_\alpha \geq 2\eta(p_\beta - 4 + f_\alpha - f_\beta) = 2\eta(p_\beta - 4 + 6 - 3) = 2\eta(p_\beta - 1).$$

Solving 2 equations with 2 variables, the equilibrium airfares and airlines' profit levels are given by

$$p_\alpha = 10, \quad p_\beta = 6, \quad \text{and} \quad \pi_\alpha = 10\eta, \quad \pi_\beta = 6\eta.$$

(4b) [10 points] Under a code-sharing agreement, passengers of all airlines are exposed to the same frequency of flights given by $f = f_\alpha + f_\beta$.

Airline α maximizes the airfare p_α subject to:

$$\pi_\beta = \eta p_\beta \geq 2\eta(p_\alpha - 4 + f - f) = 2\eta(p_\alpha - 4).$$

Airline β maximizes the airfare p_β subject to:

$$\pi_\alpha = \eta p_\alpha \geq 2\eta(p_\beta - 4 + f - f) = 2\eta(p_\beta - 4).$$

Solving 2 equations with 2 variables, the equilibrium airfares and airlines' profit levels are given by

$$p_\alpha = 8, \quad \text{and} \quad \pi_\alpha = \pi_\beta = 8\eta.$$

Clearly airline β benefits from the code-sharing agreement since it provides a lower frequency of flights. Airline α loses from this agreement since it can no longer charge an extra premium for providing a higher frequency of flights.

(5a) [8 points] The trigger strategy of player $i = G, F$ is for every period $\tau = 1, 2, \ldots$ to set

$$p_i(\tau) = \begin{cases} p^H & \text{if } p_G(t) = p_F(t) = p^H \text{ for every } t = 1, 2, \ldots, \tau - 1 \\ p^L & \text{otherwise.} \end{cases}$$

(5b) [6 points] GM's discounted sum of profits when it does not deviate from the collusive high price, and when it deviates from the collusive price are given by

$$\pi_G = \frac{5}{1-\rho} \quad \text{and} \quad \pi'_G = 5 + \rho \frac{4}{1-\rho}.$$

Hence, $\pi_G \geq \pi'_G$ for every ρ satisfying $0 < \rho < 1$. Intuitively, it follows directly from the profit levels in the above table that GM cannot benefit even from one-period deviation since $\pi_G(p^L, p^H) = 5 = \pi_G(p^H, p^H)$.

(5c) [6 points] Ford's discounted sum of profits when it does not deviate from the collusive high price, and when it deviates from the collusive price are given by

$$\pi_F = \frac{4}{1-\rho} \quad \text{and} \quad \pi'_F = 6 + \rho \frac{3}{1-\rho}.$$

Hence, $\pi_F \geq \pi'_F$ if $\rho > 2/3$.

THE END