INSTRUCTIONS (please read!)

1. Please answer all the questions. You are not allowed to use any course material. Calculators are permitted.
2. Please PRINT your name and your UM-ID number on each notebook that you submit.
3. Maximum Time Allowed: 2 hours (10:30–12:30).
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading to a numerical solution.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is 'visible' and that your sentences are properly organized. If you think that your handwriting is poor, please print each word!
10. Good Luck !

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(1) [20 points] The market inverse demand function for flights between two cities is $Q = 12 - p$, where p denotes the airfare, and Q denotes the number of passengers in a given period. There are no production costs. Consider the "Judo Economics" entry deterrence/accommodation two stage game.

In stage I, an entering airlines, denoted by e, invests in capacity k (number of passengers that can be flown) and sets the airfare p^e, both cannot be changed in stage II of the game.

In stage II, the incumbent airlines, denoted by I, sets its airfare, p^I. Therefore, the demand facing each airlines in stage II is:

$$q^I = \begin{cases} 12 - p^I & \text{if } p^I \le p^e \\ 12 - k - p^I & \text{if } p^I > p^e \end{cases} \quad \text{and} \quad q^e = \begin{cases} k & \text{if } p^e < p^I \\ 0 & \text{if } p^e \ge p^I. \end{cases}$$

Assuming that the entrant's choice of capacity and price $\langle k, p^e \rangle$ is limited to two pairs *only* given by $\langle 2, \$3 \rangle$ and $\langle 4, \$1 \rangle$, compute the subgame-perfect equilibrium of this game.

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(2) Consider a technology replacement adoption model with network externalities. In each period t, N_t two-period lived consumers enter the market and choose whether to adopt a new technology or an old technology product. The new (state-of-the-art) technology is only partially backward compatible in the sense that it is compatible with only $\frac{1}{3}$ of the "old" consumers at t. In contrast, the old technology is 100% compatible

New technologies "arrive" exogenously according to $T_t = \lambda t$, for $t = 1, 2, \ldots$. New technologies are not always adopted, so the path of actually-adopted technologies is given by

$$V_t = \begin{cases} T_t & \text{if the young at } t \text{ adopt the new technology} \\ V_{t-1} & \text{otherwise.} \end{cases}$$

Consumers buy the product only when they are young and use it for two periods and dispose it. The utility function of a young consumer at τ is:

$$U^\tau = \begin{cases} T_\tau + \frac{1}{3}N_{\tau-1} + N_\tau - p_\tau(T_\tau) & \text{if generation } \tau \text{ buys period } \tau\text{'s newest technology} \\ \\ V_{\tau-1} + N_{\tau-1} + N_\tau - p_\tau(V_{\tau-1}) & \text{if generation } \tau \text{ buys the same technology used by the old at } \tau, \end{cases}$$

where $p_\tau(V)$ is the period τ price of technology of quality V, set by a single firm which acts as a monopoly over all technologies.

(2a) [15 points] Let Δ denote the duration of each technology (number of periods each technology lasts before it is replaced by a more advanced technology). Compute Δ assuming constant population, $N_\tau = 120$ for all τ, and $\lambda = 20$.

How your result would change if we assume $\lambda' = 40$? Conclude from your answer about the relationship between technology advance and the frequency of technology replacement.

(2b) [5 points] Suppose now that $N_t = N$ for all $t = 1, 2, \ldots$. Would an increase in the parameter N (size of a each generation of consumers) increase or decrease the frequency of technology replacement? Prove your answer.

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(3) [20 points] The inverse market demand function for 8mb memory chips is given by $p = a - q$. Initially, there are "many" firms that have identical technologies for producing these chips, where each chip costs $\$c$ $(0 < c < a)$ to produce.

Suppose now that one and only one firm engages in cost-reducing R&D, in which a reduction in unit cost from c to $c - x$ requires an investment of $\lambda x^2/2$, where $\lambda > 0$.

Assume that the new technology is patentable for one period only. Compute the profit-maximizing level of cost reduction x. Also, indicate for which values of λ the cost-reducing innovation will be considered to be major (or drastic), and for which values the innovation will be minor (nondrastic).

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(4) Consider two airline companies: airline α and airline β, who are the only airlines providing a service connecting city A with city B. Suppose that the frequency of flights provided by airline α and airline β are $f_\alpha = 6$ and $f_\beta = 3$, respectively. That is, airline α provides 6 flights per day, whereas airline β provides only 3 flights per day. There are η consumers who are oriented towards airline α, and η who are oriented towards airline β. Suppose now that passengers' utility functions are given by:

$$U_\alpha \stackrel{\text{def}}{=} \begin{cases} f_\alpha - p_\alpha & \text{flies } \alpha \\ f_\beta - 4 - p_\beta & \text{flies } \beta, \end{cases} \quad \text{and} \quad U_\beta \stackrel{\text{def}}{=} \begin{cases} f_\alpha - 4 - p_\alpha & \text{flies } \alpha \\ f_\beta - p_\beta & \text{flies } \beta. \end{cases}$$

Assume that the airline firms do not bear any type of cost. Answer the following questions.

(4a) [10 points] Calculate the UPE airfare charged by each airline and the associated profit levels assuming that there are no agreements between the two airline firms.

(4b) [10 points] For our purposes, a code-share agreement means that passengers buying a ticket from any airline can board any flight including those operated by the other airline. This means that a code-sharing agreement increases passengers' frequency of flights. Calculate the Undercut-proof Equilibrium (UPE) airfares and the associated profit levels assuming that the two airline firms are engaged in a code-sharing agreement. Therefore, the effective frequency of flights becomes the sum of the frequencies. Assume that airline α continues to maintain $f_\alpha = 6$ flights per day and airline β continues to maintain $f_\beta = 3$ flights per day even after the agreement is signed.

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(5) [20 points] Consider an infinitely-repeated price competition game between GM and FORD. Each firm can set a high price or a low price in each period $t = 0, 1, 2, \ldots$. The profit of each outcome are given in the following matrix:

		FORD: Low Price (p^L)		FORD: High Price (p^H)	
GM	Low (p^L)	4	3	5	1
	High (p^H)	1	6	5	4

Suppose that each firm adopts a trigger-price strategy under which the firms may be able to implicitly collude on setting the high price. Let ρ $(0 < \rho < 1)$ denote the time discount factor.

(5a) [8 pts.] Formulate the trigger price strategy for each player under which the players collude on setting high prices.

(5b) [6 pts.] Compute the minimum threshold value of ρ which would ensure that GM sets p^H in every period t. Show and explain your derivations.

(5c) [6 pts.] Compute the minimum threshold value of ρ which would ensure that FORD sets p^H in every period t. Show and explain your derivations.

THE END