(1) Stadium Boulevard can be best described as the interval $[0,1]$. There are two stores, labeled A and B selling a homogenous product. Store A is located at the western corner of the street, $x=0$; whereas store B is located at the eastern corner of the street, $x=1$. All production costs are normalized to equal zero.

The wind on Stadium Boulevard blows from east to west, thereby making the transportation cost of traveling to the east twice as high as the transportation cost of traveling in the western direction. Formally, consumers are uniformly distributed on the unit interval with unit density. The utility of a consumer indexed by x, $x \in [0,1]$, is assumed to be given by

$$U_x \stackrel{\text{def}}{=} \begin{cases} \beta - \tau x - p_A & \text{if buys from } A \\ \beta - 2\tau(1-x) - p_B & \text{if buys from } B \\ 0 & \text{not buys at all,} \end{cases} \quad \text{where} \quad \beta > \frac{20\tau}{9}$$

measures the basic valuation (willingness to pay) for the product, and τ is the transportation cost parameter. Solve the following problems:

(1a) [15 points] Compute the pair of prices $\langle p_A, p_B \rangle$ which constitutes a Nash-Bertrand equilibrium. Which firm charges a higher price and why?

Answer: The "indifferent" consumer is indexed by

$$\hat{x}(p_A, p_B) = \frac{p_B - p_A + 2\tau}{3\tau}.$$

Store A chooses p_A to maximize $\pi_A(p_A, p_B) = p_A \hat{x}(p_A, p_B)$. Store B chooses p_B to maximize $\pi_B(p_A, p_B) = p_B[1 - \hat{x}(p_A, p_B)]$. The Nash-Bertrand equilibrium prices are $p_A^b = 5\tau/3 > 4\tau/3 = p_B^b$. Clearly, store A charges a higher price since it exercise more monopoly power over consumers seeking to reduce their transportation costs.

For the above to be the solution, we must verify that consumer $\hat{x} = 5/9$ is better off buying than not buying. Formally, using the above utility function, we must check that

$$\beta - \tau\frac{5}{9} - \frac{5\tau}{3} \geq 0, \quad \text{hence } \beta \geq \tau\frac{20}{9},$$

which is assumed.

(1b) [5 points] Compute the equilibrium profit levels and market shares. Which firm earns a higher profit and which sells more units? Explain why.

Answer:

$$\hat{x}^b = \frac{5}{9}, \quad \pi_A^b = \frac{25\tau}{27} > \frac{16\tau}{27} = \pi_B^b.$$

Store A has a higher market share and also earns a higher profit since consumers pay lower transportation cost for traveling to A compared with traveling to store B.

(1c) [5 points] Solve problems (1a) and (1b) assuming that $\beta < 16\tau/9$.

Answer: A low basic valuation for the product will make each store act as a local monopoly since consumers located sufficiently far from both stores will choose not to buy at all.

If store A is a local monopoly, the utility function implies that the consumer indifferent between buying from A and not buying at all is $\hat{x}_A = (\beta - p_A)/\tau$. Store A chooses p_A to maximize $\pi_A = p_A\hat{x}_A$, yielding

$$p_A = \frac{\beta}{2}, \quad \hat{x}_A = \frac{\beta}{4\tau}, \quad \text{and} \quad \pi_A = \frac{\beta^2}{4\tau}.$$

If store B is a local monopoly, the utility function implies that the consumer indifferent between buying from A and not buying at all is $\hat{x}_B = (p_B - \beta + 2\tau)/\tau$. Store B chooses p_B to maximize $\pi_B = p_B(1 - \hat{x}_B)$, yielding

$$p_B = \frac{\beta}{2}, \quad \hat{x}_B = \frac{4\tau - \beta}{4\tau}, \quad \text{and} \quad \pi_B = \frac{\beta^2}{8\tau}.$$

We must verify that some consumers are not served under the above restriction on the parameter β. Formally,

$$\hat{x}_B - \hat{x}_A = \frac{4\tau - \beta}{4\tau} - \frac{\beta}{4\tau} > 0 \quad \text{if} \quad \beta < \frac{16\tau}{9},$$

which is assumed.

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(2) A company can produce a 20 page-per-minute (PPM) laser printer at a cost of $\$50$ per unit. In addition, your firm can replace a memory chip on each printer for an additional cost of $\$10$ per unit, which would slow the printer down to 10 PPM (thereby raising the unit cost of the damaged printer to $\$60$ per unit). The table below displays potential consumers' maximum willingness to pay for the two printer configurations.

i (Speed)	$\ell = 1$	$\ell = 2$	μ_i (Unit Cost)
F (Fast)	$V_1^F = \$80$	$V_2^F = \$180$	$\$50$
S (Slow)	$V_1^S = \$80$	$V_2^S = \$90$	$\$50 + \$10$
N_ℓ (# consumers)	$N_1 = 100$	$N_2 = 200$	

Solve the following problems:

(2a) [10 points] Compute the profit-maximizing price of the fast model assuming that the slow printer is <u>not</u> introduced to the market.

Answer: There are two pricing options for selling the fast original model only. First, selling at a high price, $p_F^H = \$180$, so only type $\ell = 2$ consumers buy it. At this price, type $\ell = 1$ consumers do not buy it because $V_1^F = 80 < 180 = p_F^H$ (the price exceeds their valuation). The resulting profit is therefore $\pi_F^H = 200(180 - 50) = \$26,000$.

Second, the firm can lower the price to $p_F^L = \$80$, thereby serving both consumer types. The resulting profit is therefore $\pi_F^L = (100 + 200)(80 - 50) = \$9000$.

Clearly, $p_F^H = \$180$ is the profit-maximizing price when only the original fast model is sold.

(2b) [15 points] Compute the profit-maximizing prices of the fast and slow printers assuming now that the slow (damaged) model is also sold on the market.

Conclude whether the introduction of the slow printer is profit enhancing or profit reducing.

Answer: Suppose now that the slow (damaged) model is introduced at an extra per-unit cost of $\$10$. Type $\ell = 1$ consumers would buy the slow model if $V_1^S - p_S \geq V_1^F - p_F$, hence, if $p_S \leq p_F$. Similarly, type $\ell = 2$ consumers would buy the fast model if $V_2^S - p_S \leq V_2^F - p_F$, hence if $p_S \geq p_F - 90$ or $p_F \leq p_S + 90$. These two equations determine the range of prices that segment the market between the two consumer types.

The table implies that the highest price type 1 consumers are willing to pay for the slow model is $p_S = \$80$. Therefore, $p_F = 80 + 90 = \$170$. The resulting total profit is therefore

$$\pi_{F,S} = 200(170 - 50) + 100(80 - 60) = \$26,000.$$

Because $\pi_F^H = \pi_{F,S} = \$26,000$, the seller earns the same profit whether or not the slow model (damaged good) is introduced into the market.

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(3) The price of milk in Cowanda is regulated by the government and is set at the level of $\$p$ per gallon. There are N milk drinkers in Cowanda and two producers of milk labeled $i = A, B$. Assume that $p < 4N$. Consumers are not aware of the existence of a specific firm unless they receive an ad from this particular firm. Each consumer buys at most one gallon (either A or B). Consumers who receive two ads equally split between the two stores.

Let ϕ_i, $0 \le \phi_i \le 1$ denote the fraction of the consumer population receiving an ad from firm i, $i = A, B$. Assume that firm i bears a cost of $(\phi_i N)^2$ for reaching $\phi_i N$ consumers.

(3a) [10 points] Formulate the profit functions of each firm, $\pi_A(\phi_A, \phi_B)$ and $\pi_B(\phi_A, \phi_A)$. Explain your formulation.

Answer:

$$\pi_A = p\left[\phi_A(1-\phi_B)N + \phi_A\phi_B\frac{N}{2}\right] - (\phi_A N)^2 \quad \text{and} \quad \pi_B = p\left[\phi_B(1-\phi_A)N + \phi_A\phi_B\frac{N}{2}\right] - (\phi_B N)^2.$$

Explanation: $\phi_A(1-\phi_B)N$ consumers receive A's ad only. $\phi_A\phi_B N$ consumers receive both ads (hence equally split between the stores).

(3b) [10 points] Solve for the firms' best-response functions $\phi_A(\phi_B)$ and $\phi_B(\phi_A)$. Explain whether the advertising strategies should be considered as strategically substitutes or strategically complements.

Answer: Maximizing π_A with respect to ϕ_A, and π_B with respect to ϕ_B yields

$$\phi_A(\phi_B) = \frac{p(2-\phi_B)}{4N} \quad \text{and} \quad \phi_B(\phi_A) = \frac{p(2-\phi_A)}{4N}.$$

The advertising best-response functions are strategically substitutes (downward sloping).

(3c) [5 points] Conclude how the equilibrium advertising levels are affected by an increase in the consumer population N, and the regulated price p. Prove your results!

Answer: Solving the two advertising best-response functions yields

$$\phi = \phi_A = \phi_B = \frac{2p}{4N+p}.$$

Notice that $\phi < 1$ by the assumption that $p < 4N$. Next, it follows that $\mathrm{d}\phi/\mathrm{d}N = -8p/(4N+p)^2 < 0$ and $\mathrm{d}\phi/\mathrm{d}p = 8N/(4N+p)^2 > 0$.

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(4) A monopoly firm is promised to receive a grant of $G = \$9$ if it can prove that it is a low-cost firm. The grantor does not know whether this monopoly is a low-cost firm (unit cost equals to $c_L = \$2$), or a high cost firm (in which case $c_H = \$6$).

This single-period monopoly faces a downward-sloping inverse demand function $p = 12 - Q$. Solve the following problems:

(4a) [15 points] Compute the range of output levels that, if produced, provide a signal that this firm is a low-cost producer. Show your derivation.

Answer: A high-cost firm which does not receive a grant will act as a monopoly, by solving $MR = 12 - 2Q = 6 = c_H$ yielding $Q^m = 3$, $p^m = \$9$ and a profit of $\pi^m = (9-6)3 = \$9$.

If a high-cost firm attempts to provide a signal that it is a low-cost firm, it should expand its output level to a level that yields a profit lower than $\$9$. Formally, the firm must choose q^s to that solves

$$(12 - q^s - c_H)q^s + G = (12 - q^s - 6)q^s + 9 \leq \$9 \Longrightarrow q^s \geq 6.$$

(4b) [10 points] Prove that a low-cost firm earns a higher profit when it signals that it a low-cost producer by producing the quantity level you found in part (4a). Show your derivation.

Answer: If a low-cost producer does not signal, it does not receive the grant, and therefore acts like a monopoly by solving $MR = 12 - 2Q = 2 = c_L$ yielding $Q^m = 5$, $p^m = \$7$ and a profit of $\pi^m = (7-2)5 = \$25$.

By signalling via $q^s = 6$, a low-cost producer receives the grant and earns

$$\pi = (12 - q^s - c_L)q^s + G = (12 - 6 - 2)6 + 9 = 24 + 9 = \$33 > \$25.$$

Thus, signaling is profitable to a low-cost firm.

THE END