file=Grad-IO-Fall07-midterm.tex

INSTRUCTIONS (please read!)

1. Please answer all the questions. You are not allowed to use any course material. Calculators are permitted.
2. Please PRINT your name and your UM-ID number on each notebook that you submit.
3. Maximum Time Allowed: 1 hour and 20 minutes (08:40–10:00).
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip it and go to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is 'visible' and that your sentences are properly organized. If you think that your handwriting is poor, please print each word!
10. Good Luck !

---

(1) Stadium Boulevard can be best described as the interval $[0,1]$. There are two stores, labeled A and B selling a homogenous product. Store A is located at the western corner of the street, $x=0$; whereas store B is located at the eastern corner of the street, $x=1$. All production costs are normalized to equal zero.

The wind on Stadium Boulevard blows from east to west, thereby making the transportation cost of traveling to the east twice as high as the transportation cost of traveling in the western direction. Formally, consumers are uniformly distributed on the unit interval with unit density. The utility of a consumer indexed by x, $x \in [0,1]$, is assumed to be given by

$$U_x \stackrel{\text{def}}{=} \begin{cases} \beta - \tau x - p_A & \text{if buys from } A \\ \beta - 2\tau(1-x) - p_B & \text{if buys from } B \\ 0 & \text{not buys at all,} \end{cases} \quad \text{where} \quad \beta > \frac{20\tau}{9}$$

measures the basic valuation (willingness to pay) for the product, and τ is the transportation cost parameter. Solve the following problems:

(1a) [15 points] Compute the pair of prices $\langle p_A, p_B \rangle$ which constitutes a Nash-Bertrand equilibrium. Which firm charges a higher price and why?

(1b) [5 points] Compute the equilibrium profit levels and market shares. Which firm earns a higher profit and which sells more units? Explain why.

(1c) [5 points] Solve problems (1a) and (1b) assuming that $\beta < 16\tau/9$.

---

(2) A company can produce a 20 page-per-minute (PPM) laser printer at a cost of $\$50$ per unit. In addition, your firm can replace a memory chip on each printer for an additional cost of $\$10$ per unit, which would slow the printer down to 10 PPM (thereby raising the unit cost of the damaged printer to $\$60$ per unit). The table below displays potential consumers' maximum willingness to pay for the two printer configurations.

i (Speed)	$\ell = 1$	$\ell = 2$	μ_i (Unit Cost)
F (Fast)	$V_1^F = \$80$	$V_2^F = \$180$	$\$50$
S (Slow)	$V_1^S = \$80$	$V_2^S = \$90$	$\$50 + \$10$
N_ℓ (# consumers)	$N_1 = 100$	$N_2 = 200$	

Solve the following problems:

(2a) [10 points] Compute the profit-maximizing price of the fast model assuming that the slow printer is <u>not</u> introduced to the market.

(2b) [15 points] Compute the profit-maximizing prices of the fast and slow printers assuming now that the slow (damaged) model is also sold on the market.

Conclude whether the introduction of the slow printer is profit enhancing or profit reducing.

---

(3) The price of milk in Cowanda is regulated by the government and is set at the level of $\$p$ per gallon. There are N milk drinkers in Cowanda and two producers of milk labeled $i = A, B$. Assume that $p < 4N$. Consumers are not aware of the existence of a specific firm unless they receive an ad from this particular firm. Each consumer buys at most one gallon (either A or B). Consumers who receive two ads equally split between the two stores.

Let ϕ_i, $0 \le \phi_i \le 1$ denote the fraction of the consumer population receiving an ad from firm i, $i = A, B$. Assume that firm i bears a cost of $(\phi_i N)^2$ for reaching $\phi_i N$ consumers.

(3a) [10 points] Formulate the profit functions of each firm, $\pi_A(\phi_A, \phi_B)$ and $\pi_B(\phi_A, \phi_B)$. Explain your formulation.

(3b) [10 points] Solve for the firms' best-response functions $\phi_A(\phi_B)$ and $\phi_B(\phi_A)$. Explain whether the advertising strategies should be considered as strategically substitutes or strategically complements.

(3c) [5 points] Conclude how the equilibrium advertising levels are affected by an increase in the consumer population N, and the regulated price p. Prove your results!

---

(4) A monopoly firm is promised to receive a grant of $G = \$9$ if it can prove that it is a low-cost firm. The grantor does not know whether this monopoly is a low-cost firm (unit cost equals to $c_L = \$2$), or a high cost firm (in which case $c_H = \$6$).

This single-period monopoly faces a downward-sloping inverse demand function $p = 12 - Q$. Solve the following problems:

(4a) [15 points] Compute the range of output levels that, if produced, provide a signal that this firm is a low-cost producer. Show your derivation.

(4b) [10 points] Prove that a low-cost firm earns a higher profit when it signals that it a low-cost producer by producing the quantity level you found in part (4a). Show your derivation.

THE END