Industrial Organization
Graduate-level Lecture Notes

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Contents

Topic 1
Monopoly

1.1 Swan's Durability Theorem

- Durability can be viewed as aspect of "quality" of a product.
- Suppose that firms control the durability for the products they produce.
- Of course, unit cost rises monotonically with durability.
- *"Loose" formulation of Swan's independence result*: Durability (or even quality) does not vary with the market structure.
- *More accurate formulation*: A monopoly will choose the same durability level as the social planner, which is the same as the one chosen by competitive firms.
- *Intuition*: It is sufficient for a monopoly to exercise its power using a price distortion, so quality distortion need not be utilized.
- *Therefore*: A monopoly (or any producer) distorts quality only if it cannot set the monopoly's profit-maximizing price.
- *Example*: Rent control in NYC: Landlords don't maintain their buildings.

A "light bulb" illustration of Swan's independence result

For a more general formulation see Tirole p.102.

- $\$V =$ consumers' maximum willingness to pay for lighting service per unit of time
- $c_1 =$ unit production cost of a light bulb which lasts for one unit of time.
- $c_2 =$ unit production cost of a light bulb which lasts for two unit of time.
- *Assumption*: $0 < c_1 < V$, $0 < c_2 < 2V$, and $c_1 < c_2$.
- *Remark*: At this stage we don't specify whether $c_2 < 2c_1$ (economies of durability production).

Monopoly's profit over 2 periods

- Produces nondurables: $p_1^m = \$V$, hence $\pi_1^m = 2(V - c_1)$.
- Produces durables: $p_2^m = 2\$V$, hence $\pi_2^m = 2V - c_2$.
- Hence, $\pi_2^m \geq \pi_1^m$ if and only if $c_2 \leq 2c_1$ (cost consideration only).

Competitive industry

- Many competing firms each offers long and short durability of light bulbs.
- Competitive prices: $p_1 = c_1$ and $p_2 = c_2$, both available in stores.
- Consumers buy durable if and only if $2V - p_2 \geq 2(V - p_1)$ implying that $c_2 \leq 2c_1$.

Result: Monopoly and competitive markets produce the same durability which would be also chosen by the social planner.

1.2 Durable Goods Monopoly

- Coase's Conjecture: A monopoly selling a durable good will charge below the price a monopoly charges for a nondurable (per period of usage).
- Two-period lived consumers, t, $t = 1, 2$.
- The good is per-period transportation services obtained from a car.
- A continuum of consumers having different valuations, $v \in [0, 100]$.
- Utility function of type v: $U \stackrel{\text{def}}{=} \max\{v - p_t, 0\}$.
  (*Instructor*: Explain the 2 interpretations of demand curves).
- Hence, inverse demand for one period of service: $p_t = 100 - Q_t$.
- Monopoly sells a durable product that lasts for two periods (zero costs)

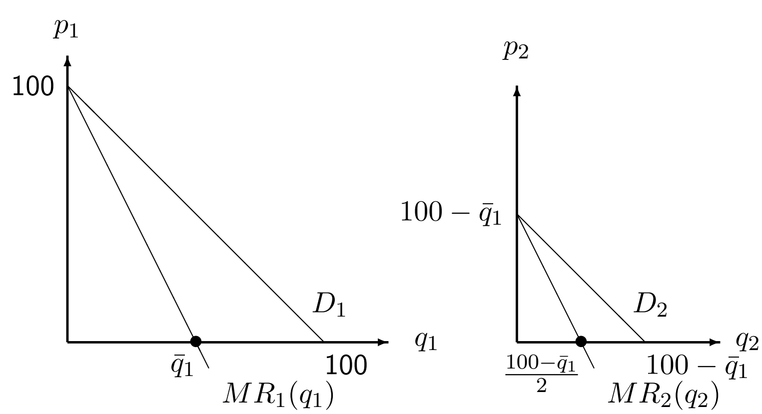


Figure 1.1: Durable-good monopoly: the case of downward sloping demand

The monopoly has two options:

Sell: for a price of p^S (transfer all ownership rights)

Rent (lease): For a price of p_t^R for period t (renter maintains ownership).

1.2.1 A renting (leasing) monopoly

The consumer *leases* Q_t each period $t = 1, 2$. The monopoly solves

$$MR(Q_t) = 100 - 2Q_t = 0 = MC(Q_t) \Longrightarrow Q_t^R = 50, \quad p_t^R = 50, \text{ and } \pi_t^R = 2,500 \text{ for } t = 1, 2.$$

Hence, the life-time sum of profits of the renting monopoly is given by $\pi^R = 5,000$.

1.2.2 A seller monopoly

- The seller knows that those consumers who purchase the durable good in $t = 1$ will not repurchase in period $t = 2$.
- Thus, in $t = 2$ the monopoly will face a lower demand.
- The reduction in $t = 2$ demand equals exactly the amount it sold in $t = 1$.
- Therefore, in $t = 2$ the monopoly will have to sell at a lower price than in $t = 1$.
- We compute a SPE for this two-period game.

The second period

- Suppose that the monopoly sells $\bar{q}_1$ units have been sold in $t = 1$.
- $t = 2$ residual demand is $q_2 = 100 - \bar{q}_1 - p_2$ or $p_2 = 100 - \bar{q}_1 - q_2$.
- In $t = 2$ the monopoly solves

$$MR_2(q_2) = 100 - \bar{q}_1 - 2q_2 = 0 \Longrightarrow q_2 = 50 - \frac{\bar{q}_1}{2}.$$

Hence, the second period price and profit levels are given by

$$p_2(\bar{q}_1) = 100 - \bar{q}_1 - \left(50 - \frac{\bar{q}_1}{2}\right) = 50 - \frac{\bar{q}_1}{2}, \quad \text{and} \quad \pi_2(\bar{q}_1) = p_2 q_2 = \left(50 - \frac{\bar{q}_1}{2}\right)^2.$$

The first period

- Given expected p_1 and p_2, find the consumer type $\tilde{v}$ who is indifferent between buying at $t = 1$ and postponing to $t = 2$.
- The "indifferent" consumer must satisfy $2\tilde{v} - p_1 = \tilde{v} - p_2$.
- Substitute $\tilde{v} = 100 - q_1$ (only high vs buy at $t = 1$) to obtain $2\overbrace{(100 - \bar{q}_1)}^{\tilde{v}} - p_1 = \overbrace{(100 - \bar{q}_1)}^{\tilde{v}} - q_2$.

Hence,

$$2(100 - \bar{q}_1) - p_1 = (100 - \bar{q}_1) - \underbrace{\left(50 - \frac{\bar{q}_1}{2}\right)}_{q_2}.$$

Solving for p_1 yields

$$p_1 = 150 - \frac{3\bar{q}_1}{2}.$$

In a SPE the selling monopoly chooses a first-period output level $\bar{q}_1$ that solves

$$\max_{q_1}(\pi_1+\pi_2) = \underbrace{\left(150-\frac{3q_1}{2}\right)q_1}_{\pi_1} + \underbrace{\left(50-\frac{q_1}{2}\right)^2}_{\pi_2} \tag{1.1}$$

yielding a first-order condition given by

$$0 = \frac{\partial(\pi_1+\pi_2)}{\partial q_1} = 150 - 3q_1 - \frac{100-q_1}{2} = 100 - \frac{5q_1}{2}.$$

Denoting the solution values by a superscript S, we have that $q_1^S = 40$, $q_2^S = 50 - 40/2 = 30$, $p_2^S = 50 - 40/2 = 30$ and $p_1^S = 100 - 40 + 30 = 90$. Hence,

$$\Pi^S = p_1^S q_1^S + p_2^S q_2^S = 4,500 < 5,000 = \Pi^{pm}.$$

These results manifest Coase's conjecture.

- Therefore, a monopoly selling a durable goods earns a lower profit than a renting monopoly.
- This result has led some economists to claim that monopolies have the incentives to produce less than an optimal level of durability (e.g., light bulbs that burn very fast).
- We discuss the (in)validity of this argument in Sections 1.1 and 1.3

1.3 Monopoly and Planned Obsolescence

- The literature on planned obsolescence may suggest that a monopoly my shorten durability in order to enhance future sales.
- But, look at your old computer, old printer, old TV, old music player. Don't you want to replace them with newer "faster" models? Aren't they "too" durable?
- Here we ask: Is short durability really bad?
- Answer: No, according to Fishman, Gandal, and Shy (1993) short durability may have some welfare enhancing effects such as the introduction of new technologies.
- Overlapping generations model, each $t = 1, 2, \ldots$ one two-period lived consumer enters the market.
- One good that can be improved via innovation, "many" firms.
- Each firm can produced a durable which lasts for 2 periods with unit cost c^D.
- Each firm can produced a nondurable which lasts 1 period with unit cost c^{ND}.
- *Assumption*: Production of a durable is less costly: $c^D < 2c^{ND}$.
- The utility from the initial technology at $t = 0$ is $v > 0$
- Utility from period t state-of-the-art technology under continuous innovation: $\lambda^t v$, where $\lambda > 1$.
- Continuous innovation means that technology $\lambda^{t-1}v$ prevailed at in $t-1$.
- Each t one firm is randomly endowed with ability to invest F and improve upon $t-1$ technology.
- *Instructor*: You must stress that this exposition compares only continuous innovation with continuous stagnation (simplification).

1.3.1 Welfare analysis of durability

Welfare analysis: Continuous stagnation (No innovation)

- Per-period welfare given that only *nondurables* are sold: $W = 2(v - c^{ND})$.
- Per-period welfare given that only *durables* are sold: $W = 2v - c^{D}$.
- Hence, welfare is higher when durables are produced since $c^{D} < 2c^{ND}$. (†)

Welfare analysis: Continuous innovation

- Per-period welfare given that only *nondurables* are sold: $W = 2(\lambda^t v - c^{ND}) - F$.
- Per-period welfare given that only *durables* are sold: $W = \lambda^t v + \lambda^{t-1} v - c^D - F$ (old guys don't switch to the new technology).
- Hence, welfare is higher when nondurables are produced if $2c^{ND} - c^D \leq \lambda^{t-1}(\lambda - 1)v$.
- In particular, it must hold in $t = 1$, hence, $2c^{ND} - c^D \leq (\lambda - 1)v$. $(*)$
- *Remark*: In the paper we assume that the above condition holds.

1.3.2 Profit-maximizing choice of durability and innovation

Instructor: Explain that this paper does not solve for a SPE. It only searches for an outcome which is more profitable to firms in the long run.

Profit under continuous stagnation (No innovation)

- Prices fall to marginal costs (no firm maintains any patent right): $p^D = c^D$ and $p^{ND} = c^{ND}$.
- Consumers buy only durables since $c^D < 2c^{ND}$ implies $U^D = 2v - c^D > 2(v - c^{ND}) = U^{ND}$. (†)

Profit under continuous innovation

- The prices and profits below are for 2 consumption periods.
- Maximum price that can be charged for a *nondurable* is solved from: $\lambda^t v - p^{ND} \geq \lambda^{t-1} v - c^{ND}$, because consumers can always buy an outdated nondurable for a price of c^{ND}.
- Hence, $p^{ND} \leq \lambda^{t-1}(\lambda - 1)v + c^{ND}$.
- Therefore, $\pi^{ND} = 2(p^{ND} - c^{ND}) - F = 2\lambda^{t-1}(\lambda - 1)v - F$. $(**)$
- Maximum price that can be charged for a *durable* is solved from: $2\lambda^t v - p^D \geq \lambda^{t-1} v - c^{ND} + \lambda^t v - c^{ND}$, because consumers can always buy an outdated nondurable for a price of c^{ND}.
- Hence, $p^D \leq \lambda^{t-1}(\lambda - 1)v + 2c^{ND}$.
- Therefore, $\pi^D = p^D - c^D - F = \lambda^{t-1}(\lambda - 1)v - c^D + 2c^{ND} - F$. (selling to young only) $(**)$
- *Remark*: Notice that outdated durables are also available at competitive prices. Hence, we should also verify that p^D also satisfies $2\lambda^t v - p^D \geq 2\lambda^{t-1} v - c^D$.

- Comparing the two profit levels marked by $(**)$, we conclude that, under continuous innovation, production of nondurables is more profitable than durables, $\pi^{ND} \geq \pi^{D} \iff 2c^{ND} - c^{D} \leq \lambda^{t-1}(\lambda - 1)v$.
- In particular, it must hold in $t = 1$, hence, $2c^{ND} - c^{D} \leq (\lambda - 1)v$. $(*)$
- Hence, if the production of *ND* is profitable, it is also socially optimal.

Summary of results: A welfare evaluation of profit decisions

Continuous stagnation: Comparing the two outcomes marked by $(\dagger)$, production of nondurables is both unprofitable and socially undesirable.

Continuous innovation: Comparing the two outcomes marked by $(*)$, production of durables is both unprofitable and socially undesirable $\iff 2c^{ND} - c^{D} \leq (\lambda - 1)v$.

Conclusion: Planned obsolescence (short durability) is "essential" for technology growth.

Topic 2
A Taxonomy of Business Strategies

2.1 Major Issues

(1) Reinterprets Stackelberg (sequential-moves) equilibrium as a sequence of *commitments*, rather than as a sequential-move output (production game).

(2) Commitments (other than output and price, not really commitments) include:

   (a) investment in capital
   (b) advertising cost
   (c) choice of standard
   (d) contracts
   (e) brand diversity (brand prolification)
   (f) coupons and price commitment

(3) Major question: should the first mover engage in *over* or *under* investment in the relevant strategic variable?

(4) To provide a classification of different optimal behavior of the first mover.

2.2 Is there Any Advantage to the First Mover?

Not necessarily! In sequential-move *price* games, or *auction* games, the last mover earns higher profit than the first mover.

$$q_1 = 168 - 2p_1 + p_2 \quad \text{and} \quad q_2 = 168 + p_1 - 2p_2. \tag{2.1}$$

The single-period game Bertrand prices and profit levels are $p_i^b = 56$ and $\pi_i^b = 6272$ (assuming costless production).

We look for a SPE in prices where firm 1 sets its price before firm 2.
In the first period, firm 1 takes firm 2's best-response function as given, and chooses p_1 that solves

$$\max_{p_1} \pi_1(p_1, R_2(p_1)) = \left(168 - 2p_1 + \frac{168 + p_1}{4}\right) p_1. \tag{2.2}$$

The first-order condition is

$$0 = \frac{\partial \pi_1}{\partial p_1} = 210 - \frac{7}{2} p_1.$$

Therefore, $p_1^s = 60$, hence, $p_2^s = 57$. Substituting into (2.1) yields that $q_1^s = 105$ and $q_2 = 114$. Hence, $\pi_1^s = 60 \times 105 = 6300 > \pi_1^b$, and $\pi_2^s = 57 \times 114 = 6498 > \pi_2^b$.

Result 2.1
Under a sequential-moves price game (or more generally, under any game where actions are strategically complements):
(a) Both firms collect a higher profit under a sequential-moves game than under the single-period Bertrand game. Formally, $\pi_i^s > \pi_i^b$ for $i = 1, 2$.
(b) The firm that sets its price first (the leader) makes a lower profit than the firm that sets its price second (the follower).
(c) Compared to the Bertrand profit levels, the increase in profit to the first mover (the leader) is smaller than the increase in profit to the second mover (the follower). Formally, $\pi_1^s - \pi_1^b < \pi_2^s - \pi_2^b$.

Reason: Firm 1 is slightly undercut in the 2nd period. Therefore, it keeps the price above the Bertrand level.

2.3 Classification of Best-Response Functions

Consider a static two-firm Nash game.

Action/Strategy space: x_i strategic variable of firm i, $i = 1, 2$. $x_i \in [0, \infty]$. If $x_i = q_i$, we have Cournot-quantity game. If $x_i = p_i$, we have Bertrand-price game.

Payoff Functions:

$$\pi_i(x_i, x_j) = (\alpha - \beta x_i + \gamma x_j)x_i, \quad \alpha > 0.$$

Note: the sign of γ is not specified!

Assumption 2.1
Own-price effect: $\beta > 0$, $(-\beta < 0)$, *(also, need for concavity w/r/t x_i)*

Cross effect: $\beta^2 > \gamma^2$ may need to be assumed (meaning that own effect is "stronger" than the cross effect). This assumption also implies the following:

Stability:

$$\left(\frac{\partial^2 \pi_1}{\partial (x_1)^2}\right)\left(\frac{\partial^2 \pi_2}{\partial (x_2)^2}\right) = 4\beta^2 > \gamma^2 = \left(\frac{\partial^2 \pi_1}{\partial (x_1)\partial (x_2)}\right)\left(\frac{\partial^2 \pi_2}{\partial (x_1)\partial (x_2)}\right),$$

meaning that the own-price *coefficient dominates the rival's price coefficient.*

The first-order conditions yield the best-response functions

$$x_i = R_i(x_j) = \frac{\alpha}{2\beta} + \frac{\gamma}{2\beta}x_j, \quad i, j = 1, 2; \quad i \neq j. \tag{2.3}$$

Definition 2.1
(a) Players' strategies are said to be **strategic substitutes** *if the best-response functions are downward sloping. That is, if $R_i'(p_j) < 0$ ($\gamma < 0$ in our example).*
(b) Players' strategies are said to be **strategic complements** *if the best-response functions are upward sloping. That is, if $R_i'(p_j) > 0$ ($\gamma > 0$ in our example).*

Note: Strategic substitutes and complements are defined by whether a more "aggressive" strategy by 1 lowers or raises 2's *marginal* profit.

Solving the two best-response function yield

$$x_1^N = x_2^N = \frac{\alpha}{2\beta - \gamma}, \quad \text{and} \quad \pi_1^N = \pi_2^N = \frac{\alpha^2 \beta}{(2\beta - \gamma)^2}. \tag{2.4}$$

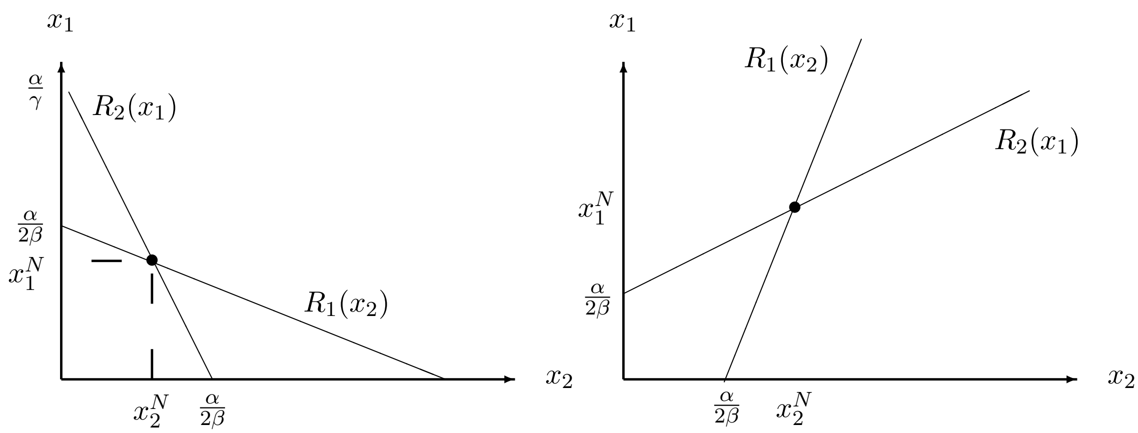


Figure 2.1: *Left:* Strategic substitutes. *Right:* Strategic complements

2.4 The Two-stage Game

Two-stage game.

Stage 1: Incumbent chooses to invest k_1.

Stage 2: $k_1 = \bar{k}_1$ is given. Incumbent and entrant play Nash in $x_1(\bar{k}_1)$ and $x_2(\bar{k}_1)$, respectively. Profits, $\pi_1^N(x_1(\bar{k}_1), x_2(\bar{k}_1))$ and $\pi_2^N(x_1(\bar{k}_1), x_2(\bar{k}_1))$, are collected.

Remarks:

(1) the post-entry market structure is given.

(2) if $\pi_2^N = 0$, we say that entry is *deterred*.

(3) assume that entry is accommodated.

What is the effect of increasing k_1 on π_1? The *total* effect is defined by

$$\underbrace{\frac{\mathsf{d}\pi_1}{\mathsf{d}k_1}}_{\text{Total Effect}} = \underbrace{\frac{\partial\pi_1}{\partial k_1}}_{\text{Direct Effect}} + \underbrace{\frac{\partial\pi_1}{\partial x_2}\frac{\mathsf{d}x_2}{\mathsf{d}k_1}}_{\text{Strategic Effect}} .$$

Now,

$$\frac{\mathsf{d}x_2}{\mathsf{d}k_1} = \frac{\partial x_2}{\partial x_1} \times \frac{\mathsf{d}x_1}{\mathsf{d}k_1} = R_2'(x_1) \times \frac{\mathsf{d}x_1}{\mathsf{d}k_1}.$$

Assumption 2.2

(a) There are no direct effects. Formally,

$$\frac{\partial\pi_1}{\partial k_1} = 0.$$

(b)

$$\textit{sign}\left(\frac{\partial\pi_1}{\partial x_2}\right) = \textit{sign}\left(\frac{\partial\pi_2}{\partial x_1}\right).$$

Hence,

$$\text{sign}\left(\frac{\partial\pi_1}{\partial x_2}\frac{\mathsf{d}x_2}{\mathsf{d}k_1}\right) = \text{sign}\left(\frac{\partial\pi_2}{\partial x_1}\frac{\mathsf{d}x_1}{\mathsf{d}k_1}\right) \times \text{sign}\left(R_2'\right) \tag{2.5}$$

DEFINITION 2.2
Investment makes firm 1 **tough** *(***soft***) if*

$$\frac{\partial \pi_2}{\partial x_1}\frac{dx_1}{dk_1} < 0 \ \ (>0).$$

	Investment makes firm 1 Tough	Soft
$R' > 0$ (complements)	Puppy dog (underinvest)	Fat cat (overinvest)
$R' < 0$ (substitutes)	Top dog (overinvest)	Lean & hungry (underinvest)

Table 2.1: Classification of optimal business strategies.

2.5 Cost reduction investment: makes firm 1 tough

Stage 1: firm 1 chooses k_1 which lowers its *marginal cost*
Stage 2: firms compete in quantities or prices

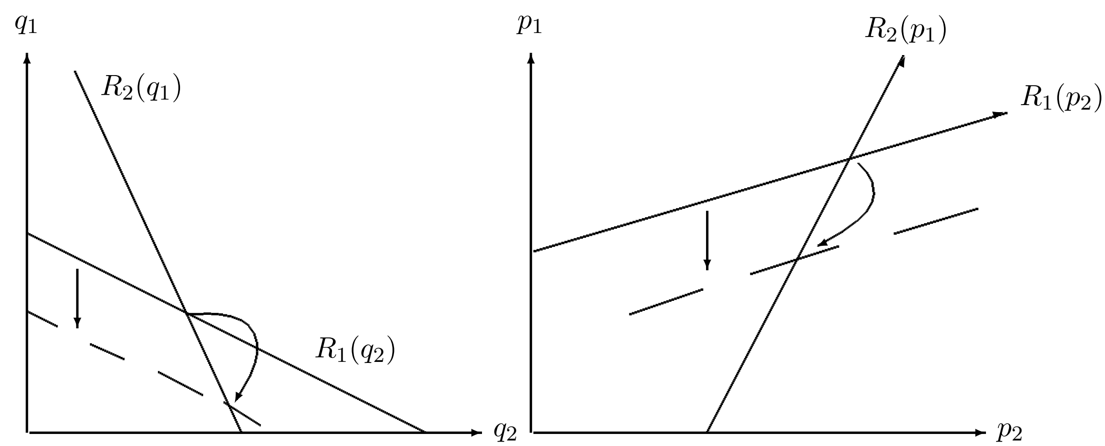


Figure 2.2: Investment makes 1 tough. *Left:* quantity game. *Right:* price game.

Quantity game: $k_1\uparrow \Longrightarrow q_1\uparrow \Longrightarrow \pi_2\downarrow \Longrightarrow \frac{\partial \pi_2}{\partial q_1}\frac{\mathrm{d}q_1}{\mathrm{d}k_1} < 0 \Longrightarrow$ Tough!
Hence, should *overinvest (Top dog)*.

Price game: $k_1\uparrow \Longrightarrow p_1\downarrow \Longrightarrow \pi_2\downarrow \Longrightarrow \frac{\partial \pi_2}{\partial p_1}\frac{\mathrm{d}p_1}{\mathrm{d}k_1} < 0 \Longrightarrow$ Tough!
Hence, should *underinvest (Puppy dog)*.

2.6 Advertising investment: makes firm 1 soft

Stage 1: firm 1 chooses k_1 which boosts its demand
Stage 2: firms compete in quantities or prices

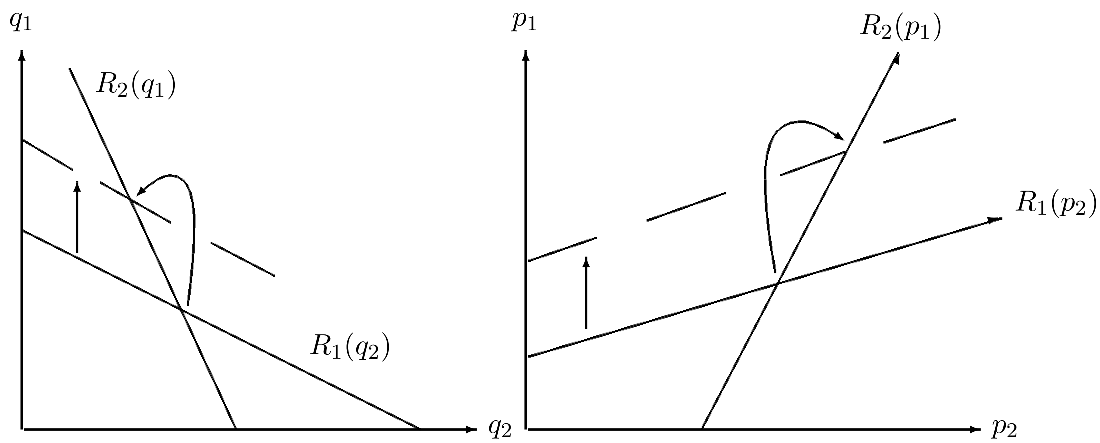


Figure 2.3: Investment makes 1 soft. *Left:* quantity game. *Right:* price game.

Quantity game: $k_1 \uparrow \Longrightarrow q_1 \downarrow \Longrightarrow \pi_2 \uparrow \Longrightarrow \frac{\partial \pi_2}{\partial q_1} \frac{\mathsf{d} q_1}{\mathsf{d} k_1} > 0 \Longrightarrow$ Soft!
Hence, should *underinvest (Lean & hungry look)*.

Price game: $k_1 \uparrow \Longrightarrow p_1 \uparrow \Longrightarrow \pi_2 \uparrow \Longrightarrow \frac{\partial \pi_2}{\partial p_1} \frac{\mathsf{d} p_1}{\mathsf{d} k_1} > 0 \Longrightarrow$ Soft!
Hence, should *overinvest (Fat cat)*.

Topic 3
Product Differentiation

3.1 Major Issues

(1) Firms choose the specification of the product in addition to price. (Specification may be quality in general and durability in particular).

(2) Firms choose to differentiate their brand to reduce competition (maintain higher monopoly power).

(3) Policy question: Too much differentiation? Or, too little?

3.2 Horizontal Versus Vertical Differentiation

We demonstrate the difference using Hotelling's model.

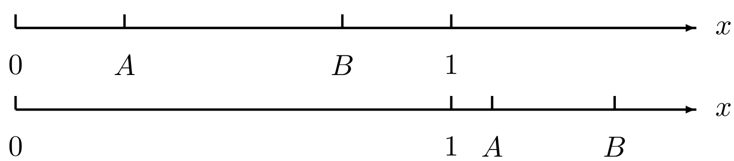


Figure 3.1: Horizontal versus vertical differentiation. *Up:* horizontal differentiation; *Down:* vertical differentiation

$$U_x \equiv \begin{cases} -p_A - \tau|x-A| & \text{if he buys from } A \\ -p_B - \tau|x-B| & \text{if she buys from } B \end{cases} \qquad \tau > 0. \tag{3.1}$$

Definition 3.1
Let brand prices be given.
(a) Differentiation is said to be **horizontal** *if, when the level of the product's characteristic is augmented in the product's space, there exists a consumer whose utility rises and there exists another consumer whose utility falls.*
(b) Differentiation is said to be **vertical** *if all consumers benefit when the level of the product's characteristic is augmented in the product space.*

- In Figure 3.1, brands are horizontally differentiated if $A, B < 1$, and vertically differentiated when $A, B > 1$.

- As τ increases, differentiation increases. When $\tau \to 0$ the brands become *perfect substitutes*, which means that the industry becomes more competitive.

- Alternative definition of vertical differentiations is that if all brands are equally priced, *all* consumers prefer one brand over all others.

3.3 Horizontal Differentiation: Hotelling's Linear City Model

- Firm B is located to the right of firm A, b units of distance from point L.
- Each consumer buys one unit of the product.
- Production is costless (not critical)

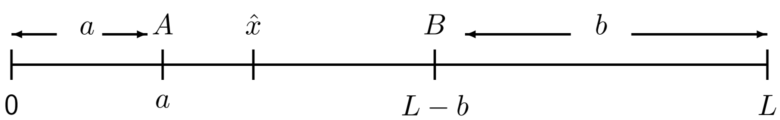


Figure 3.2: Hotelling's linear city with two firms

The utility function of a consumer located at point x by

$$U_x \equiv \begin{cases} -p_A - \tau|x-a| & \text{if he buys from } A \\ -p_B - \tau|x-(L-b)| & \text{if she buys from } B. \end{cases} \tag{3.2}$$

Here there is no reservation utility. Adding a reservation utility may result in partial market coverage in the sense that consumers around the center will prefer not to buy any brand.

Formally, if $a < \hat{x} < L - b$, then

$$-p_A - \tau(\hat{x} - a) = -p_B - \tau(L - b - \hat{x}).$$

Hence,

$$\hat{x} = \frac{p_B - p_A}{2\tau} + \frac{(L - b + a)}{2},$$

which is the demand function faced by firm A. The demand function faced by firm B is

$$L - \hat{x} = \frac{p_A - p_B}{2\tau} + \frac{(L + b - a)}{2}.$$

We now look for a Bertrand-Nash equilibrium in price strategies. That is, Firm A takes p_B as given and chooses p_A to

$$\max_{p_A} \pi_A = \frac{p_B p_A - (p_A)^2}{2\tau} + \frac{(L - b + a)p_A}{2}. \tag{3.3}$$

The first-order condition is given by

$$0 = \frac{\partial \pi_A}{\partial p_A} = \frac{p_B - 2p_A}{2\tau} + \frac{(L - b + a)}{2}. \tag{3.4}$$

Firm B takes p_A as given and chooses p_B to

$$\max_{p_B} \pi_B = \frac{p_B p_A - (p_B)^2}{2\tau} + \frac{(L + b - a)p_B}{2}. \tag{3.5}$$

The first-order condition is given by

$$0 = \frac{\partial \pi_B}{\partial p_B} = \frac{p_A - 2p_B}{2\tau} + \frac{L + b - a}{2}.$$

Hence, the equilibrium prices are given by

$$p_A^h = \frac{\tau(3L - b + a)}{3} \text{ and } p_B^h = \frac{\tau(3L + b - a)}{3}. \tag{3.6}$$

The equilibrium market share of firm A is given by

$$\hat{x}^h = \frac{3L - b + a}{6}. \tag{3.7}$$

Note that if $a = b$, then the market is equally divided between the two firms. The profit of firm A is given by

$$\pi_A^h = \hat{x}^h p_A^h = \frac{\tau(3L - b + a)^2}{18}, \tag{3.8}$$

Shows the *Principle of Minimum Differentiation*.

Result 3.1

(a) If both firms are located at the same point ($a+b = L$, meaning that the products are homogeneous), then $p_A = p_B = 0$ is a unique equilibrium.

(b) A unique equilibrium exists and is described by (3.6) and (3.7) if and only if the two firms are not too close to each other; formally if and only if

$$\left(L + \frac{a - b}{3}\right)^2 \geq \frac{4L(a + 2b)}{3} \quad \textit{and} \quad \left(L + \frac{b - a}{3}\right)^2 \geq \frac{4L(b + 2a)}{3}$$

the unique equilibrium is given by (3.6), (3.7), and (3.8).

Proof. (1) When $a + b = 1$...undercutting (Bertrand).
To demonstrate assume $a = b$, $a < L/2$. Then, we are left to show that the equilibrium exists if and only if $L^2 \geq 4La$, or if and only if $a \leq L/4$.

When $a = b$, the distance between the two firms is $L - 2a$.
Also, if equilibrium exists, $p_A = p_B = \tau L$.

Figure 3.3 has three regions:
Region I: A'a maximal profit is given by $\pi_A = p_A L$.
Region II: Substituting the equilibrium $p_B = \tau L$ into (3.3) yields

$$\pi_A = \frac{L}{2} + \frac{L}{2} p_A - \frac{(p_A)^2}{2\tau}, \tag{3.9}$$

which is drawn in Region II of Figure 3.3. Maximizing (3.9) with respect to p_A yields $\pi_A = \tau L^2/2$.
Region III: High price, no market share.

In equilibrium

$$\pi_A^{II} = \frac{\tau L^2}{2} \geq \pi_A^I = [\tau L - \tau(L - 2a)]L = 2\tau a L,$$

implying that $a \leq L/4$. □

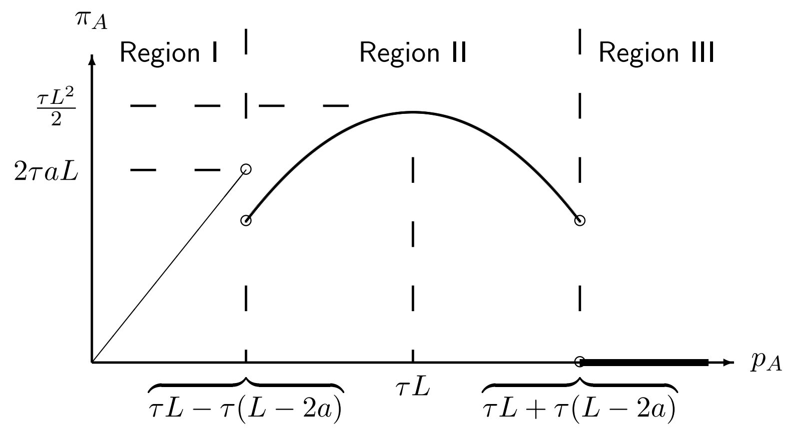


Figure 3.3: Existence of equilibrium in the linear city: The profit of firm A for a given $\bar{p}_B = \tau L$

3.4 Horizontal Differentiation: Behavior-based Pricing

- Suppose that firms can identify consumers who have purchased their brands before.
- How? For example, by product registration, frequent mileage, and trade-in.
- Therefore, they can set different prices for *loyal* consumers and consumers *switching* from competing brands.
- Two firms, A located at $x = 0$, and B located at $x = 1$.
- Two periods: $t = 0$ is history. Price competition takes place at $t = 1$.
- History of consumer $x \in [0, 1]$ is the function $h(x) : [0, 1] \to \{A, B\}$ describing whether x has purchased A or B in $t = 0$.
- Example: $h(x) = A$ means that consumer x has purchased brand A in $t = 0$ (public information).
- Firm A sets p_A for its loyal consumers, and q_A for consumers switching from brand B.
- Firm B sets p_B for its loyal consumers, and q_B for consumers switching from brand A.
- σ_{AB} and σ_{BA} exogenously-given switching costs A to B, and B to A.

Utility of a consumer indexed by x with a purchase history of brand $h(x) \in A, B$ is defined by

$$U(x) \stackrel{\text{def}}{=} \begin{cases} \beta - p_A - \tau x & \text{if } h(x) = A \text{ and continues to purchase brand } A \\ \beta - q_B - \tau(1-x) - \sigma_{AB} & \text{if } h(x) = A \text{ and now switches to brand } B \\ \beta - p_B - \tau(1-x) & \text{if } h(x) = B \text{ and continues to purchase brand } B \\ \beta - q_A - \tau x - \sigma_{BA} & \text{if } h(x) = B \text{ and now switches to brand } A. \end{cases}$$

- For our purposes, we now set $\sigma_{AB} = \sigma_{BA} = 0$.[1]
- *Assumption*: A's inherited market share constitutes of consumers indexed by $x \leq x_0$.

[1] Gehrig et. al. 2007 demonstrate why $\sigma_{AB} > 0$ and $\sigma_{BA} > 0$ are needed for generating persistent dominance.

- Hence, consumers indexed by $x > x_0$ have a history of buying brand B.
- *Assumption*: With no loss of generality $x_0 \geq 0.5$ (A was dominant in $t = 0$).

Figure 3.4 illustrates how the history of purchases relates to current brand preferences.

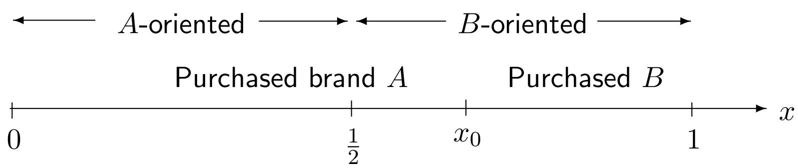


Figure 3.4: Purchase history relative to current preferences

The utility function implies that consumers indifferent between switching brands and not switching are given by

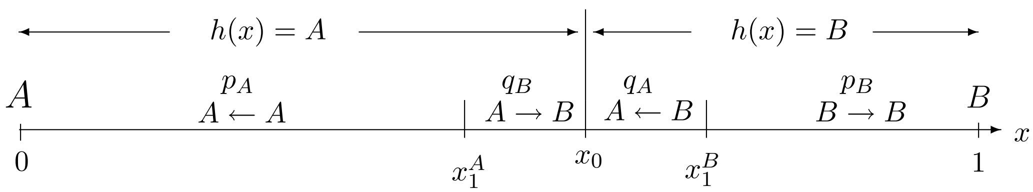


Figure 3.5: Consumer allocation between the brands *Note*: Arrows indicate direction of switching (if any). Prices indicated the prices paid by the relevant range of consumers.

$$x_1^A = \frac{1}{2} + \frac{q_B - p_A}{2\tau} \quad \text{and} \quad x_1^B = \frac{1}{2} + \frac{p_B - q_A}{2\tau}.$$

Firms' profit maximization problems are:

$$\max_{p_A, q_A} \pi_A(p_A, q_A) \stackrel{\text{def}}{=} p_A x_1^A + q_A(x_1^B - x_0) \tag{3.10}$$

$$\max_{p_B, q_B} \pi_B(p_B, q_B) \stackrel{\text{def}}{=} p_B(1 - x_1^B) + q_B(x_0 - x_1^A).$$

yielding the Nash equilibrium prices

$$p_A = \frac{\tau(2x_0 + 1)}{3}, \quad q_A = \frac{\tau(3 - 4x_0)}{3}, \quad p_B = \frac{\tau(3 - 2x_0)}{3}, \quad \text{and} \quad q_B = \frac{\tau(4x_0 - 1)}{3}.$$

and therefore

$$x_1^A = \frac{2x_0 + 1}{6}, \quad x_1^B = \frac{2x_0 + 3}{6}, \quad \text{and} \quad \pi_A = \pi_B = \frac{5\tau(2x_0^2 - 2x_0 + 1)}{9}.$$

Define

$$m_1^A = x_1^A + (x_1^B - x_0) = \frac{2 - x_0}{3} \quad \text{and} \quad m_1^B = (x_0 - x_1^A) + (1 - x_1^B) = \frac{1 + x_0}{3}.$$

Results

(1) The loyalty price of each firm increases with the firm's inherited market share. Formally, p_A increases and p_B decreases with an increase in x_0.

(2) Each firm's poaching price decreases with the firm's inherited market share. Formally, q_A decreases and q_B increases with an increase in x_0.

(3) The dominant firm charges a loyalty premium. Formally, $p_A \geq q_A$.

(4) The small firm offers a loyalty discount $p_B < q_B$ if and only if its inherited market share exceeds $1/3$ (i.e., $x_0 > 2/3$).

(5) With behavior-based price discrimination, the firm with inherited dominance is bound to lose its dominance.[2]

[2] Gehrig et. al. 2007 reverses this result by assuming strictly positive switching costs: $\sigma_{AB} > 0$ and $\sigma_{BA} > 0$.

3.5 Horizontal Differentiation: Salop's Circular City

- Model advantages: (1) Number of firms (brands) is endogenously determined. (2) Can have service time differentiation applications when the circle is interpreted as a clock.
- Notation: (1) N firms, endogenously determined. (2) $F=$ fixed cost, $c=$ marginal cost. (3) q_i and $\pi_i(q_i)$ the output and profit levels of the firm-producing brand i,

$$\pi_i(q_i) = \begin{cases} (p_i - c)q_i - F & \text{if } q_i > 0 \\ 0 & \text{if } q_i = 0. \end{cases} \tag{3.11}$$

Consumers:
Then, assuming that firms 2 and N charge p,

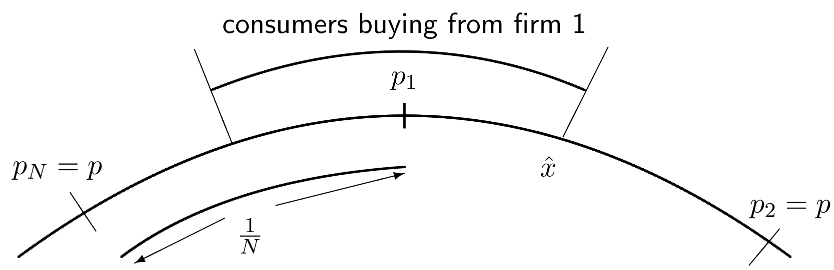


Figure 3.6: The position of firms on the unit circle

$$p_1 + \tau\hat{x} = p + \tau(1/N - \hat{x})$$

Hence,

$$\hat{x} = \frac{p - p_1}{2\tau} + \frac{1}{2N}. \tag{3.12}$$

$$q_1(p_1, p) = 2\hat{x} = \frac{p - p_1}{\tau} + \frac{1}{N}. \tag{3.13}$$

Definition 3.2
The triplet $\{N^\circ, p^\circ, q^\circ\}$ is an equilibrium if
(a) Firms: Each firm behaves as a monopoly on its brand; that is, given the demand for brand i (3.13) and given that all other firms charge $p_j = p^\circ, j \neq i$, each firm i chooses p° to

$$\max_{p_i} \pi_i(p_i, p^\circ) = p_i q_i(p_i) - (F + cq_i) = (p_i - c)\left(\frac{p^\circ - p_i}{\tau} + \frac{1}{N}\right) - F.$$

(b) Free entry: Free entry of firms (brands) will result in zero profits; $\pi_i(q^\circ) = 0$ for all $i = 1, 2, \ldots, N^\circ$.

The first-order condition for firm i's maximization problem is

$$0 = \frac{\partial \pi_i(p_i, p^\circ)}{\partial p_i} = \frac{p^\circ - 2p_i + c}{\tau} + \frac{1}{N}.$$

Therefore, in a symmetric equilibrium, $p_i = p^\circ = c + \tau/N$.

To find the equilibrium number of brands N, we set

$$0 = \pi_i(p^\circ, p^\circ) = (p^\circ - c)\frac{1}{N} - F = \frac{\tau}{N^2} - F.$$

Hence

$$N^\circ = \sqrt{\frac{\tau}{F}}, \quad p^\circ = c + \frac{\tau}{N} = c + \sqrt{\tau F}, \quad q^\circ = \frac{1}{N}. \tag{3.14}$$

The cost of the average consumer who is located half way between $\hat{x} = 1/(2N)$ and a firm. The average consumer has to travel $1/(4N)$, which yields

$$T(N) = \frac{\tau}{4N}. \tag{3.15}$$

$$\min_N L(F, \tau, N) \equiv NF + T(N) + Ncq = NF + \frac{\tau}{4N} + c. \tag{3.16}$$

The first-order condition is $0 = \frac{\partial L}{\partial N} = F - \tau/(4N^2)$. Hence,

$$N^* = \frac{1}{2}\sqrt{\frac{\tau}{F}} < N^\circ. \tag{3.17}$$

3.6 Vertical Differentiation: A Modified Hotelling Model

- Continuum of consumers uniformly distributed on $[0, 1]$.
- Two firms, denoted by A and B and located at points a and b $(0 \le a \le b \le 1)$ from the origin, respectively.[3]
- p_A and p_B are the price charged by firm A and B.

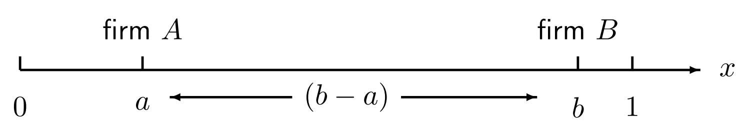


Figure 3.7: Vertical differentiation in a modified Hotelling model

$$U_x(i) \equiv \begin{cases} ax - p_A & i = A \\ bx - p_B & i = B \end{cases} \tag{3.18}$$

The "indifferent" consumer is determined by

$$U_{\hat{x}}(A) = a\hat{x} - p_A = b\hat{x} - p_B = U_{\hat{x}}(B). \tag{3.19}$$

Solving for $\hat{x}$ from (3.19) yields

$$\hat{x} = \frac{p_B - p_A}{b - a} \quad \text{and} \quad 1 - \hat{x} = 1 - \frac{p_B - p_A}{b - a}. \tag{3.20}$$

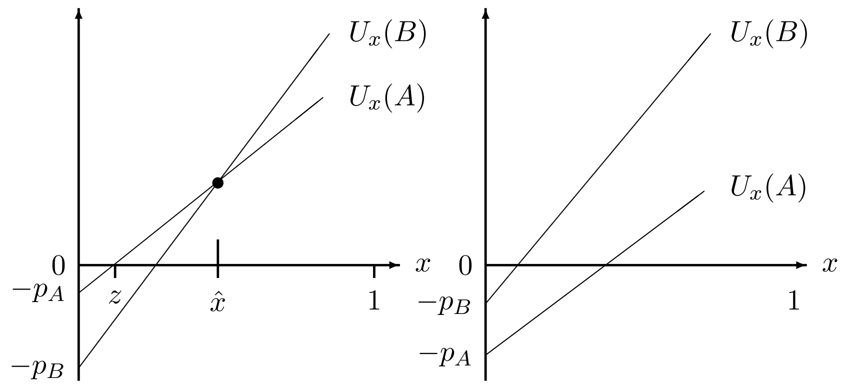


Figure 3.8: Determination of the indifferent consumer among brands vertically differentiated on the basis of quality. *Left:* $p_A < p_B$, *Right:* $p_A > p_B$

Formally, firm A and B solve

$$\max_{p_A} \pi_A(a,b,p_A,p_B) = p_A\hat{x} = p_A\left[\frac{p_B - p_A}{b-a}\right] \tag{3.21}$$
$$\max_{p_B} \pi_B(a,b,p_A,p_B) = p_B\,(1-\hat{x}) = p_B\left[1 - \frac{p_B - p_A}{b-a}\right].$$

Definition 3.3
The quadruple $< a^e, b^e, p^e_A(a,b), p^e_B(a,b) >$ *is said to be a vertically differentiated industry equilibrium if*

Second period: For (any) given locations of firms (a and b), $p^e_1(a,b)$ and $p^e_2(a,b)$ constitute a Nash equilibrium.

First period: Given the second period-price functions of locations $p^e_A(a,b)$, $p^e_B(a,b)$, and $\hat{x}(p^e_A(a,b), p^e_B(a,b))$, (a^e, b^e) is a Nash equilibrium in location.

3.6.1 The Second Stage: Choice of Prices Given Location

The first-order conditions to (3.21) are given by

$$0 = \frac{\partial \pi_A}{\partial p_A} = \frac{p_B - 2p_A}{b-a} \quad \text{and} \quad 0 = \frac{\partial \pi_B}{\partial p_B} = 1 - \frac{2p_B - p_A}{b-a}. \tag{3.22}$$

Hence,

$$p^e_A(a,b) = \frac{b-a}{3} \quad p^e_B(a,b) = \frac{2(b-a)}{3}, \quad \text{and} \quad \hat{x} = \frac{1}{3}. \tag{3.23}$$

Result 3.2
The firm producing the higher-quality brand charges a higher price even if the production cost for low-quality products is the same as the production cost of high-quality products.

[3] The assumption that $a, b \le 1$ is needed only for the two-stage game where in stage I firms choose their qualities, a and b, respectively. In general, vertical differentiation can be defined also for $a, b > 1$.

Substituting (3.23) into (3.21) yields that

$$\begin{aligned} \pi_A(a,b) &= \frac{1}{b-a}\left[\frac{2(b-a)^2}{9} - \frac{(b-a)^2}{9}\right] = \frac{b-a}{9} \\ \pi_B(a,b) &= \frac{1}{b-a}\left[\frac{4(b-a)^2}{9} - \frac{2(b-a)^2}{9}\right] = \frac{4(b-a)}{9}. \end{aligned} \quad (3.24)$$

3.6.2 The first Stage: Choice of Location (Quality)

The following result is known as the *principle of maximum differentiation*.

Result 3.3
In a vertically differentiated quality model each firm chooses maximum differentiation from its rival firm.

Remark: Here the assumption that $a, b \in [0,1]$ (or any compact interval) is crucial. However, the model can be extended to $a, b \in [0, \infty)$ provided that we assume and subtract the (convex) cost of developing quality levels, $c(a)$ and $c(b)$.

3.7 Non-address Approach: Monopolistic Competition

Model advantage: General equilibrium which is therefore suited to analyze welfare and international trade.

Consumers

Representative consumer

$$u(q_1, q_2, \ldots) \equiv \sum_{i=1}^{\infty} \sqrt{q_i}. \quad (3.25)$$

$$\lim_{q_i \to 0} \frac{\partial u}{\partial q_i} = \lim_{q_i \to 0} \frac{1}{2\sqrt{q_i}} = +\infty.$$

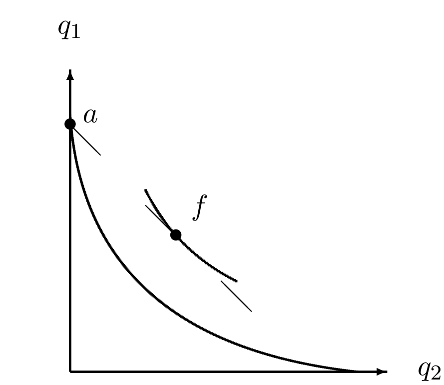


Figure 3.9: CES indifference curves for $N = 2$

$$\sum_{i=1}^{N} p_i q_i \le I \equiv L + \sum_{i=1}^{N} p_i q_i. \quad (3.26)$$

We form the Lagrangian

$$L(q_i, p_i, \lambda) \equiv \sum_{i=1}^{N} \sqrt{q_i} - \lambda \left[I - \sum_{i=1}^{N} p_i q_i \right].$$

The first-order condition for every brand i is

$$0 = \frac{\partial L}{\partial q_i} = \frac{1}{2\sqrt{q_i}} - \lambda p_i \quad i = 1, 2, \ldots, N.$$

Thus, the demand and the price elasticity (η_i) for each brand i are given by

$$q_i(p_i) = \frac{1}{4\lambda^2 (p_i)^2}, \quad \text{or} \quad p_i(q_i) = \frac{1}{4\lambda\sqrt{q_i}} \quad \eta \equiv \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} = -2. \tag{3.27}$$

Brand-producing firms

Each brand is produced by a single firm.

$$TC_i(q_i) = \begin{cases} F + cq_i & \text{if } q_i > 0 \\ 0 & \text{if } q_i = 0. \end{cases} \tag{3.28}$$

Defining a monopolistic-competition market structure

DEFINITION 3.4
The triplet $\{N^{mc}, p_i^{mc}, q_i^{mc}, i = 1, \ldots, N^{mc}\}$ *is called a Chamberlinian* **monopolistic-competition equilibrium** *if*

(1) Firms: Each firm behaves as a monopoly over its brand; that is, given the demand for brand i *(3.27), each firm* i *chooses* q_i^{mc} *to* $\max_{q_i} \pi_i = p_i(q_i) q_i - (F - cq_i)$.

(2) Consumers: Each consumer takes his income and prices as given and maximizes (3.25) subject to (3.26), yielding a system of demand functions (3.27).

(3) Free entry: Free entry of firms (brands) will result in each firm making zero profits; $\pi_i(q_i^{mc}) = 0$ *for all* $i = 1, 2, \ldots, N$.

(4) Resource constraint: Labor demanded for production equals the total labor supply; $\sum_{i=1}^{N}(F + cq_i) = L$.

Solving for a monopolistic-competition equilibrium

$$MR_i(q_i) = p_i \left(1 + \frac{1}{\eta}\right) = p_i \left(1 + \frac{1}{-2}\right) = \frac{p_i}{2} = c = MC(q_i).$$

Hence, the equilibrium price of each brand is given by $p_i^{mc} = 2c$ (twice the marginal cost).

The zero-profit condition implies

$$0 = \pi_i(q_i^{mc}) = (p_i^{mc} - c) q_i^{mc} - F = c q_i^{mc} - F.$$

Hence, $q_i^{mc} = F/c$.

The resource-constraint condition: that $N[F + c(F/c)] = L$. Hence, $N = L/(2F)$. Altogether, we have it that

$$p_i^{mc} = 2c; \quad q_i^{mc} = \frac{F}{c}; \quad N^{mc} = \frac{L}{2F}.$$

3.7.1 Monopolistic competition in international markets

$N^f = L/F = 2N^a$, where f and a denote equilibrium values under free trade and under autarky. Also, $(q_i^f = q_i^a = F/c)$, but the entire population has doubled, under free trade each consumer (country) consumes one-half of the world production $(F/(2c))$.

$$\begin{aligned} u^f &= N^f\sqrt{q_i^f} = \frac{L}{F}\sqrt{\frac{F}{2c}} = \frac{L}{\sqrt{2}\sqrt{cF}} \\ &> \frac{L}{2\sqrt{cF}} = \frac{L}{2F}\sqrt{\frac{F}{c}} = N^a\sqrt{q_i^a} = u^a. \end{aligned} \tag{3.29}$$

3.8 Damaged Goods

- Manufacturers intentionally damage some features of a good or a service in order to be able to price discriminate among the consumer groups.
- A proper implementation of this technique may even generate a Pareto improvement.
- The most paradoxical consequence of this technique is that the more costly to produce good (the damaged good) is sold for a lower price as it has a lower quality.
- Deneckere and McAfee (1996), Shapiro and Varian (1999), and McAfee (2007) list a wide variety of industries where this technique is commonly observed. For example:

Costly delay: Overnight mail carriers, such as Federal Express and UPS, offer deliveries at 8:30 A.M. or 10 A.M., and a standard service promising an afternoon delivery. Carriers make two trips to the same location instead of delivering the standard packages during the morning.

Reduced performance: Intel has removed the math coprocessor from its 486DX chip and renamed it as 486SX in order to be able to sell it for a low price of \$333 to low-cost consumers, as compared with \$588 that it charged for the undamaged version (in 1991 prices).

Delay in Internet services: Real-time information on stock prices is sold for a premium, whereas twenty-minute delayed information is often provided for free.

- A good (service) is produced (delivered) at a high quality level, H, with a unit cost of $\mu_H = \$2$.
- The seller posses a technology of damaging the good so it becomes a low quality product denoted by L. The cost of damaging is $\mu_D = \$1$.
- The cost of damaging is $\mu_D = \$1$. Therefore, the total unit cost of producing good L is $\mu_L = \mu_H + \mu_D$.

The seller has to to consider the following options:

Selling H to type 2 consumers only: This is accomplished by not introducing a damaged version and by setting a sufficiently high price, $p_H = \$20$, under which consumer $\ell = 1$ will not buy. The resulting profit is $\pi = 100(20-2) = \$1,800$.

Selling H to both consumer types: Again, selling only the original high-quality good but at a much lower price, $p_H = \$10$, in order to induce consumer $\ell = 1$ to buy. The resulting profit is $\pi = (100+100)(10-2) = \$1,600$.

i (Quality)	$\ell = 1$	$\ell = 2$	μ_i (Unit Cost)
H (Original)	$V_1^H = \$10$	$V_2^H = \$20$	\$2
L (Damaged)	$V_1^L = \$8$	$V_2^L = \$9$	\$2+\$1
N_ℓ (# consumers)	$N_1 = 100$	$N_2 = 100$	

Table 3.1: Maximum willingness for original and quality-damaged product/service.

Selling H to type 2, and L to type 1 consumers: Introducing the damaged good into the market. Consumer 2 will choose H over L if $V_2^H - p_H \geq V_2^L - p_L$. Thus, the seller must set $p_H \leq V_2^H - V_2^L + p_L = 11 + p_L$. To induce type 1 consumers to buy the damaged good L, the seller should set $p_L = \$8$ which implies that $p_H = 11 + 8 = \$19$. Total profit is therefore $\pi = 100(19 - 2) + 100(8 - 2 - 1) = \$22,000$. Most profitable !!!

- *Note*: Selling H to type 2 and L to type 1 makes no one worse off compared with selling only H to type 2 only.
- Introducing the damaged good L lowers the price of the H good to $p_H = 19$ thereby increasing the welfare of type 2 consumers.
- Type 1 consumers remain indifferent.
- Seller's profit is enhanced to $\pi = \$22,000$ from $\pi = \$18,000$.

Figure 3.10 below illustrates buyers' decisions on which quality to purchase in the p_L–p_H space.

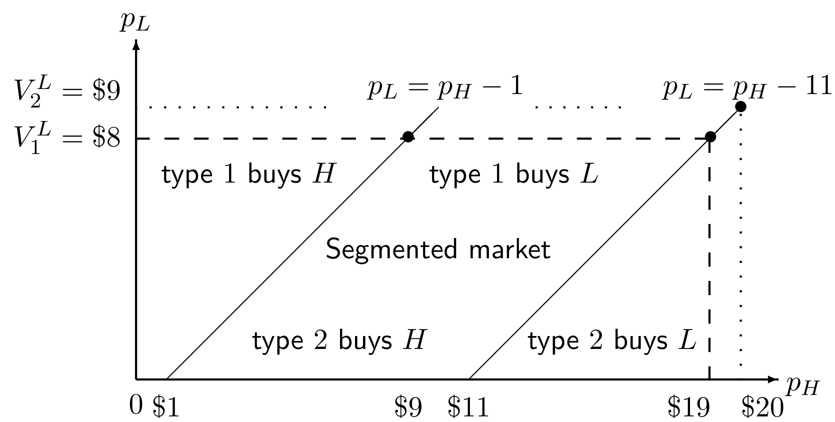


Figure 3.10: Segmenting the market with a "damaged" good. *Note*: The three bullet marks represent candidate profit-maximizing price pairs.

Topic 4
Advertising

4.1 Major Issues

- Integral part of our life, in many forms: TV, radio, printed media, billboards, buses, trains, junk mail, junk e-mail, Internet.
- Very little is understood about the effects of advertising.
- In developed economies: about 2% of GNP is spent, 3% of personal expenditure
- Ratio of advertising/sales vary across industry (low for vegetables, 20% to 80% in cosmetics and detergents).
- Is this ration correlated with size? The Big-3 are among the largest advertisers. In 1986, GM (largest producer) spends \$63/car, Ford \$130/car, Chrysler \$113/car (although over all less).
- Kaldor (1950): manipulative, hence welfare reducing due to reduced competition (prices of advertised brands rise above MC).
- More recently, Tesler (1964), Nelson (1970, 1974), Demetz (1979): tool for information transmission, thereby reducing consumers' search cost.
- Nelson distinguishes: *search* goods, and *experience* goods.
- Economics literature: *persuasive* v. *informative* advertising.

4.2 Persuasive Advertising: Dorfman-Steiner Condition

- Monopoly, $TC(q,s) = C(q) + s$ (s advertising expenditure).
- Market demand: $Q = D(p,s)$, $D_1 < 0$, $D_2 > 0$.
- What is the monopoly's profit maximizing advertising level?

Define

$$\epsilon_p \stackrel{\text{def}}{=} -\frac{\partial D(p,s)}{\partial p}\frac{p}{D(p,s)}, \quad \text{and} \quad \epsilon_s \stackrel{\text{def}}{=} \frac{\partial D(p,s)}{\partial s}\frac{s}{D(p,s)}.$$

The monopoly solves

$$\max_{p,s} \pi^M = pD(p,s) - C(D(p,s)) - s.$$

$$\begin{aligned} 0 &= \pi_p^M = Q + pD_p - C'(\cdot)D_p \\ 0 &= \pi_s^M = pD_s - C'(\cdot)D_s - 1 \end{aligned}$$

Rearranging,

$$\frac{p^M - c'}{p^M} = \frac{-Q^M}{D_p} = \frac{1}{D_s} \Longleftrightarrow \left(\frac{p^M Q^M}{s}\right)\left(\frac{sD_s}{Q^M}\right) = -\left(\frac{D_p p^M}{Q^M}\right) \Longleftrightarrow \frac{s^M}{p^M Q^M} = \frac{\epsilon_s}{\epsilon_p}.$$

4.3 Informative Advertising

- Do sellers provide optimal amount of advertising?
- Butters (1977): All firms sell identical brands; advertising is only for price
- Grossman & Shapiro (1984): Advertising also conveys information about products' attributes.
- Benham (1972): finds that state laws prohibiting eyeglass advertising had higher-than-average prices.

The (unit circle) Grossman & Shapiro (1984) modified to the linear city in Tirole p.292:

- 2 firms $i = 1, 2$, locate on the edges of $[0, 1]$
- Continuum, uniform density of consumers, τ=transportation cost parameter.
- utility of consumer located at x from firm i is

$$U_x = \begin{cases} \beta - \tau x - p_A & \text{buy from } A \\ \beta - \tau(1-x) - p_B & \text{buy from } B \\ \beta - \tau x - p_A & \text{does not buy from any store.} \end{cases}$$

- ϕ_i= fraction of consumers receiving an ad from firm i
- Later on assume that $\phi_i \in \{\frac{1}{2}, 1\}$
- Cost of $\phi_i = \frac{1}{2} = a_L$. Cost of $\phi_i = 1 = a_H \geq a_L$. (Grossman & Shapiro $A(1) = \infty$)
- Two assumptions: (1) $c + 4\tau \leq \beta \leq \frac{11\tau}{2} - 4a_L + c$. (2) $a_L \leq \frac{3\tau}{8}$. To be explained below.
- Consumers do not know the existence of a store unless they received an ad from the specific store.

$\Longrightarrow$ $(1-\phi_2)\phi_1$= the fraction that receives ads only from store 1 $\Longrightarrow$ buy from store 1

$\Longrightarrow$ $(1-\phi_1)\phi_2$= the fraction that receives ads only from store 2 $\Longrightarrow$ buy from store 2

$\Longrightarrow$ $\phi_1\phi_2$ = fraction that receives both firms' ads. These consumers obtain information on location and prices, thus will follow Hotelling's basic model

$$\hat{x} = \frac{p_2 - p_1 + \tau}{2\tau}.$$

$\Longrightarrow$ Aggregate demand facing store

$$\hat{x} = \phi_1 \left[1 - \phi_2 + \phi_2\left(\frac{p_2 - p_1 + \tau}{2\tau}\right)\right]$$

$\Longrightarrow$ Checking the effect of advertising on price elasicity:

$$\epsilon_1 \stackrel{\text{def}}{=} \frac{\partial \hat{x}}{\partial p_1}\frac{p_1}{\hat{x}} = \frac{-\phi_1\phi_2}{2\tau}\frac{p_1}{\hat{x}} = \Big|_{\substack{\phi_1=\phi_2\\ p_1=p_2}} \frac{-\phi\phi}{2\tau}\frac{p}{\phi(1-\phi/2)} = \frac{-\phi p}{(2-\phi)\tau} \quad \text{[decreases (more elastic) with } \phi]$$

- Two-stage game: *Stage I*: Stores invest in advertising, ϕ_1 and ϕ_2. *Stage II*: Price game, p_1 and p_2.

Stage II: Equilibrium in prices for given advertising levels

We look for a Nash equilibrium in (p_1, p_2). Firm 1 takes ϕ_1, ϕ_2, and p_2 as given and solves

$$\max_{p_1} \pi_1 = \phi_1 \left[1 - \phi_2 + \phi_2\left(\frac{p_2 - p_1 + \tau}{2\tau}\right)(p_1 - c)\right] - a \quad \text{where} \quad a \in \{a_L, a_H\}$$

$$0 = \frac{\partial \pi_1}{\partial p_1} = \frac{\phi_1[\phi_2(c - 2p_1 + p_2) + \tau(2-\phi_2)]}{2\tau} \Longrightarrow p_1(p_2) = \frac{\tau(2-\phi_2)}{2\phi_2} + \frac{p_2 + c}{2}.$$

Firm 2 takes ϕ_1, ϕ_2, and p_1 as given and solves

$$\max_{p_2} \pi_2 = \phi_2 \left[1 - \phi_1 + \phi_1\left(1 - \frac{p_2 - p_1 + \tau}{2\tau}\right)(p_2 - c)\right] - a \quad \text{where} \quad a \in \{a_L, a_H\}$$

$$0 = \frac{\partial \pi_2}{\partial p_2} = \frac{\phi_2[\phi_1(c - 2p_2 + p_1) + \tau(2-\phi_1)]}{2\tau} \Longrightarrow p_2(p_1) = \frac{\tau(2-\phi_1)}{2\phi_1} + \frac{p_1 + c}{2}.$$

Solving the two price best-response functions yield the equilibrium prices as functions of the advertising levels

$$p_1 = c + \frac{\tau[2\phi_2 - \phi_1(3\phi_2 - 4)]}{3\phi_1\phi_2} \quad \text{and} \quad p_2 = c + \frac{\tau[4\phi_2 - \phi_1(3\phi_2 - 2)]}{3\phi_1\phi_2}.$$

Hence,

$$p_1 - p_2 = \frac{2\tau(\phi_1 - \phi_2)}{3\phi_1\phi_2} \ge 0 \Longleftrightarrow \phi_1 \ge \phi_2,$$

which means that the firm that places more ads charges a higher price.

Substituting the prices into the profit functions yields

$$\pi_1(\phi_1, \phi_2) = \frac{\tau[\phi_1(3\phi_2 - 4) - 2\phi_2]^2}{18\phi_1\phi_2} - a_1 \quad \text{and} \quad \pi_2 = \frac{\tau[\phi_1(3\phi_2 - 2) - 4\phi_2]^2}{18\phi_1\phi_2} - a_2.$$

Stage I: Equilibrium in advertising levels

- Each store $i = 1, 2$ chooses its advertising level $\phi_i \in \{\frac{1}{2}, 1\}$.
- Cost of advertising $a(\frac{1}{2}) = a_L$, $a(1) = a_H$, where $a_H \ge a_L \ge 0$.

Result 4.1

(a) Prices and profit are higher under $\phi_1 = \phi_2 = \frac{1}{2}$ compared with $\phi_1 = \phi_2 = 1$ (less adverting generates more profits).

(b) $p_1 \ge p_2 \Longleftarrow \phi_1 \ge \phi_2$ (more advertising leads to a higher price)

(c) $\phi_1 = \phi_2 = 1$ (maximum advertising) is NOT a Nash equilibrium.

(d) $\phi_1 = \phi_2 = \frac{1}{2}$ is a Nash equilibrium if $a_H - a_L > \frac{17\tau}{72}$

(e) Otherwise, there are two equilibria: $\langle \phi_1, \phi_2 \rangle = \langle \frac{1}{2}, 1 \rangle$ and $\langle \phi_1, \phi_2 \rangle = \langle 1, \frac{1}{2} \rangle$

1	Store 2: $\phi_2 = \frac{1}{2}$		$\phi_2 = 1$	
$\phi_1 = \frac{1}{2}$	$c + 3\tau$	$c + 3\tau$	$c + \frac{5}{3}\tau$	$c + \frac{7}{3}\tau$
$\phi_1 = 1$	$c + \frac{7}{3}\tau$	$c + \frac{5}{3}\tau$	$c + \tau$	$c + \tau$

1	Store 2: $\phi_2 = \frac{1}{2}$		$\phi_2 = 1$	
$\phi_1 = \frac{1}{2}$	$\frac{9}{8}\tau - a_L$	$\frac{9}{8}\tau - a_L$	$\frac{25}{36}\tau - a_L$	$\frac{49}{36}\tau - a_H$
$\phi_1 = 1$	$\frac{49}{36}\tau - a_H$	$\frac{25}{36}\tau - a_L$	$\frac{\tau}{2} - a_H$	$\frac{\tau}{2} - a_H$

Table 4.1: Equilibrium prices (p_1, p_2) (left) and profits (π_1, π_2) (right) under varying advertising levels

1	Store 2: $\phi_2 = \frac{1}{2}$		$\phi_2 = 1$	
$\phi_1 = \frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{5}{12}$	$\frac{7}{12}$
$\phi_1 = 1$	$\frac{7}{12}$	$\frac{5}{12}$	$\frac{1}{2}$	$\frac{1}{2}$

1	Store 2: $\phi_2 = \frac{1}{2}$		$\phi_2 = 1$	
$\phi_1 = \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{1}{6}$
$\phi_1 = 1$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{2}$

Table 4.2: Equilibrium sales (q_1, q_2) (left) and $\hat{x}$ (right) under varying advertising levels

Result 4.2

(a) The store that advertises more serves more consumers than the store that advertises less. Formally, $\phi_1 \geq \phi_2$ implies that $q_1 \geq q_2$. However,

(b) it serves less consumers that receive both adds, formally, $\hat{x} = \frac{1}{6}$.

The role of our 2 assumptions

The first assumption was $c + 4\tau \leq \beta \leq \frac{11\tau}{2} - 4a_L + c$. The left part, $c + 4\tau \leq \beta$, is needed so that $p_i = c + 3\tau - \tau \geq 0$ for the case where $\phi_A = \phi_B = \frac{1}{2}$ in Table 4.1.

The right part, $\beta \leq \frac{11\tau}{2} - 4a_L + c$ is needed to prevent a firm from raising the price to unbounded levels, lose all the market with shared information, and monopolize the market for consumers who receive only one ad. To monopolize, this firm can raise to price to a maximum of $\beta - \tau$. This is not profitable if

$$\frac{1}{4}(\beta - \tau - c) < \frac{9}{8}\tau - a_L \Longrightarrow \beta \leq \frac{11\tau}{2} - 4a_L + c.$$

The above assumption implies that the second assumption, $a_L \leq \frac{3\tau}{8}$, is needed to have a nonempty interval for β. Formally,

$$c + 4\tau \leq \beta \leq \frac{11\tau}{2} - 4a_L + c \Longrightarrow a_L \leq \frac{3\tau}{8}.$$

Socially optimal advertising level

- Computing social welfare for the outcomes $\langle \phi_1, \phi_2 \rangle = \langle \frac{1}{2}, 1 \rangle$ and $\langle \phi_1, \phi_2 \rangle = \langle 1, \frac{1}{2} \rangle$ would require the computation of transportation costs (distorted because $\hat{x} = \frac{1}{6}$ or $\hat{x} = \frac{5}{6}$. We omit this analysis.
- Table 4.2 shows that $\phi_1 = \phi_2 = \frac{1}{2}$ results in an exclusion of $\frac{1}{2}$ consumers. Therefore
- for sufficiently-high value of β (basic valuation), $\phi_1 = \phi_2 = 1$ should yield higher social welfare than $\phi_1 = \phi_2 = \frac{1}{2}$.
- For example, take β that satisfies $\frac{\beta}{4} > 2(a_H - a_L)$.

Results from Grossman & Shapiro "circular" city model

(1) Under fixed # brands: advertising is excessive (excessive competition over market shares

(2) Under free entry: the equilibrium # of brands exceeds the socially optimal, in this case, too little advertising.

(3) In general, advertising increases efficiency if it leads to a reduction of over-priced brands.

Topic 5

R&D and Patent Law

5.1 Classifications of Process Innovation

- classifies process (cost-reducing) innovation according to the magnitude of the cost reduction generated by the R&D process.
- industry producing a homogeneous product
- firms compete in prices.
- initially, all firms possess identical technologies: with a unit production cost $c_0 > 0$.

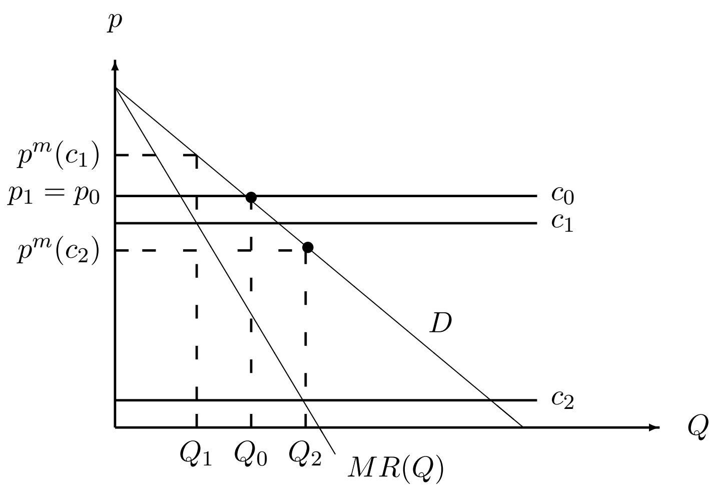


Figure 5.1: Classification of process innovation

Definition 5.1
Let $p^m(c)$ denote the price that would be charged by a monopoly firm whose unit production cost is given by c. Then,
(a) Innovation is said to be **large** *(or drastic, or major) if*
$p^m(c) < c_0$. *That is, if innovation reduces the cost to a level where the associated pure monopoly price is lower than the unit production costs of the competing firms.*
(b) Innovation is said to be **small** *(or nondrastic, or minor) if*
$p^m(c) > c_0$.

Example: Consider the linear inverse demand function $p = a - bQ$. Then, innovation is major if $c_1 < 2c_0 - a$ and minor otherwise.

5.2 Innovation Race

Two types of models:

Memoryless: Probability of discovery is independent of experience (thus, depends only on *current* R&D expenditure).

Cumulating Experience: Probability of discovery increases with cumulative R&D experience (like capital stock).

5.2.1 Memoryless model

Lee and Wilde (1980), Loury (1979), and Reinganum. The model below is Exercise 10.5, page 396 in Tirole.

- n firms race for a prize V (present value of discounted benefits from getting the patent)
- each firm is indexed by i, $i = 1, 2, \ldots, n$
- $x_i \geq 0$ is a commitment to a stream of R&D investment (at any t, $t \in [0, \infty)$).
- $h(x_i)$ is probability that firm i discovers the at Δt when it invests x_i in this time interval, where $h' > 0$, $h'' < 0$, $h(0) = 0$, $h'(0) = \infty$, $h'(+\infty) = 0$
- τ_i date firm i discovers (random variable)
- $\hat{\tau}_i \stackrel{\text{def}}{=} \min_{j \neq i}\{\tau_j(x_j)\}$ date in which first rival firm discovers (random variable).

Probability that firm i discovers before or at t is

$$\Pr(\tau_i(x_i) \leq t) = 1 - e^{-h(x_i)t}, \quad \text{density is:} \quad h(x_i)e^{-h(x_i)t}$$

Probability that firm i does not discover by t

$$\Pr(\tau_i(x_i) > t) = e^{-h(x_i)t}$$

Probability that all n firms do NOT discover before t

$$\Pr(\hat{\tau}_i \leq t) = 1 - \Pr\{\tau_j > t \ \ \forall i\} = 1 - \left(1 - e^{-\sum_{i=1}^{n} h(x_i)t}\right) = e^{-\sum_{i=1}^{n} h(x_i)t}$$

Define $a_i \stackrel{\text{def}}{=} \sum_{j \neq i} h_j(x_j)$.
Remark: Probability at least one rival discovers before t

$$\Pr(\hat{\tau}_i \leq t) = 1 - \Pr\{\tau_j > t \ \ \forall j \neq i\} = 1 - e^{-a_i t}$$

The expected value of firm i is

$$V_i = \int_0^{\infty} e^{-rt} e^{-t\sum_{i=1}^{n} h(x_i)} \left[h(x_i)V - x_i\right] dt = \frac{h(x_i)V - x_i}{r + a_i + h(x_i)}$$

Result 5.1
R&D investment actions are strategically complements

Proof. Let $x_j = x$ for all $j \neq i$. Then, $a_i = (n-1)h(x)$. First order condition is,

$$0 = \frac{\partial V_i}{\partial x_i} = \frac{1}{()^2}\left\{[h'(x_i)V - 1][(n-1)h(x) + h(x_i) + r] - h'(x_i)[h(x_i)V - x_i]\right\}.$$

Using the implicit function theorem,

$$\frac{\partial x_i}{\partial x} = \frac{-[h'(x_i)V - 1](n-1)h'(x)}{\phi} \quad \text{where}$$

$$\phi = Vh''(x_i)[(n-1)h(x)+h(x_i)+r]+h'(x_i)[h'(x_i)V-1]-[h'(x_i)V-1]h'(x_i)-h''(x_i)[h(x_i)V-x_i].$$

The second and third terms cancel out so,

$$\phi = Vh''(x_i)[(n-1)h(x) + h(x_i) + r] - h''(x_i)[h(x_i)V - x_i] < 0$$

Therefore, $\partial x_i/\partial x > 0$. □

Lee and Wilde show a series of propositions:

(1) give condition under which $\partial \hat{x}/\partial n > 0$.

(2) $\partial \tau_i/\partial n < 0$

(3) $\partial V_i/\partial n < 0$

(4) $\hat{x} > x^*$ (excessive R&D, where social optimal is calculated by maximizing nV)

5.2.2 Cumulating experience model

Fudenberg, Gilbert, Stiglitz, and Tirole (1983) provide a model with cumulative experience.

- V is prize (only to winner), c per-unit of time R&D cost
- t_i innovation starting date of firm i, $i = 1, 2$
- Assumption: $t_2 > t_1 = 0$ (firm 1 has a head start)
- $\omega_i(t)$ = experience (length of time) firm i is engaged in R&D
- hence, $\omega_1(t_2) > \omega_2(t_2) = 0$. Note that $\omega_2(t) = 0$ for $t \leq t_2$.
- $\mu_i(t) \stackrel{\text{def}}{=} \mu(\omega_i(t))$ = probability of discovering at $t + dt$
- Hence, $\mu_1(t) > \mu_2(t)$ for all $t > 0$. Also $\mu_2(t) = 0$ for $t \leq t_2$.
- $c =$ (flow) cost of engaging in R&D.

Expected instantaneous profit conditional that no firm having yet made the discovery is

$$\mu_i(t)V - c$$

Probability that *neither* firm makes the discovery before t

$$e^{-\int_0^t [\mu_1(\omega_1(\tau))+\mu_2(\omega_2(\tau))]d\tau}$$

When both firms undertake R&D, from t_1 and t_2, respectively,

$$V_i = \int_{t_i}^{\infty} e^{-\left[rt+\int_0^t [\mu_1(\omega_1(\tau))+\mu_2(\omega_2(\tau))]d\tau\right]} \left[\mu_i(\omega_i(t))V - c\right] dt$$

Assumptions:

(1) R&D is potentially profitable for the monopolist. Formally, there exists $\bar{\omega} > 0$ such that $\mu(\omega)V - c > 0$ for all $\omega > \bar{\omega}$; and $\mu(0)V - c < 0$.

(2) R&D is profitable for a monopolist. Formally,

$$\int_0^{\infty} e^{-\left[rt+\int_0^t [\mu(\tau)]d\tau\right]} \left[\mu(t)V - c\right] dt > 0.$$

(3) R&D is unprofitable for a firm in a duopoly. Formally,

$$\int_0^{\infty} e^{-\left[rt+\int_0^t [2\mu(\tau)]d\tau\right]} \left[\mu(t)V - c\right] dt < 0.$$

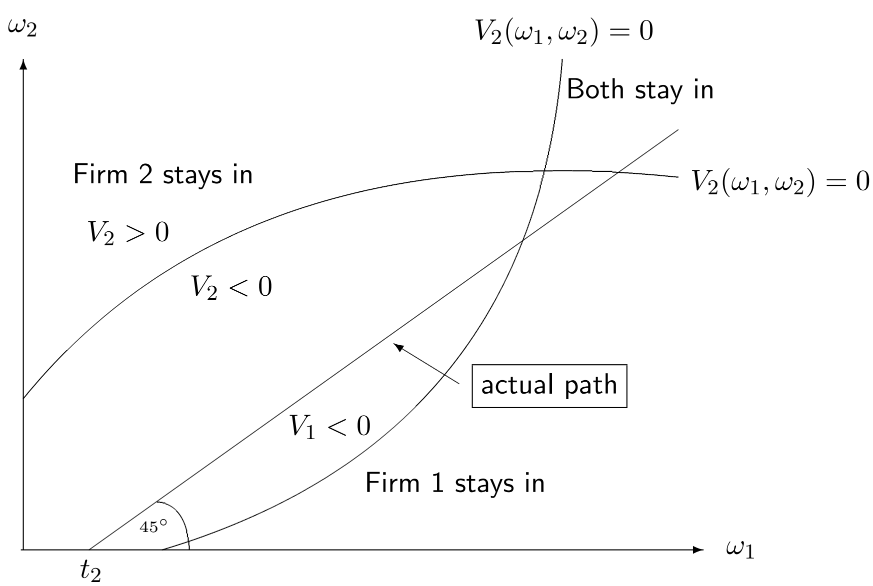


Figure 5.2: Locuses of $V_i(\omega_1(\tau),\omega_2(\tau)) = 0$, $i = 1,2$ (note: $t_2 > t_1 = 0$)

Results:

(1) ϵ-preemption. That is, firm 2 does not enter the race.

(2) if we add another stage of uncertainty (associated with developing the innovation), leapfrogging is possible and there is *no* ϵ-preemption

5.3 R&D Joint Ventures

Many papers (see Choi 1993; d'Aspremont and Jacquemin 1988; Kamien, Muller, and Zang 1992; Katz 1986, and Katz and Ordover 1990).

- two-stage game: at $t = 1$, firms determine (first noncooperatively and then cooperatively) how much to invest in cost-reducing R&D and, at $t = 2$, the firms are engaged in a Cournot quantity game
- market for a homogeneous product, aggregate demand $p = 100 - Q$.
- x_i the amount of R&D undertaken by firm i,
- $c_i(x_1, x_2)$ the unit production cost of firm i

$$c_i(x_1, x_2) \stackrel{\text{def}}{=} 50 - x_i - \beta x_j \quad i \neq j, \; i = 1, 2, \quad \beta \geq 0.$$

Definition 5.2
We say that R&D technologies exhibit (positive) **spillover effects** *if* $\beta > 0$.

Assumption 5.1
Research labs operate under decreasing returns to scale. Formally,

$$TC_i(x_i) = \frac{(x_i)^2}{2}.$$

We analyze 2 (out of 3) market structures:

(1) Noncoordination: Look for a Nash equilibrium in R&D efforts: x_1 and x_2.

(2) Coordination (semicollusion): Determine each firm's R&D level, x_1 and x_2 as to maximize joint profit, while still maintaining separate labs.

(3) R&D Joint Venture (RJV semicollusion): Setting a single lap. The present model does not fit this market structure.

5.3.1 Noncooperative R&D

The second period

$$\pi_i(c_1, c_2)|_{t=2} = \frac{(100 - 2c_i + c_j)^2}{9} \quad \text{for } i = 1, 2, \; i \neq j. \tag{5.1}$$

The first period

$$\begin{aligned}\max_{x_i} \pi_i &= \frac{1}{9}[100 - 2(50 - x_i - \beta x_j) + 50 - x_j - \beta x_i]^2 - \frac{(x_i)^2}{2} \\ &= \frac{1}{9}[50 + (2-\beta)x_i + (2\beta - 1)x_j]^2 - \frac{(x_i)^2}{2}.\end{aligned} \tag{5.2}$$

The first-order condition yields

$$0 = \frac{\partial \pi_i}{\partial x_i} = \frac{2}{9}[50 + (2-\beta)x_i + (2\beta-1)x_j](2-\beta) - x_i.$$

$x_1 = x_2 \equiv x^{nc}$, where x^{nc} is the common noncooperative equilibrium

$$x^{nc} = \frac{50(2-\beta)}{4.5 - (2-\beta)(1+\beta)}. \tag{5.3}$$

5.3.2 R&D Coordination

The firms seek to jointly choose x_1 and x_2 to[1]

$$\max_{x_1,x_2}(\pi_1 + \pi_2),$$

where π_i, $i = 1, 2$ are given in (5.1). The first-order conditions are given by

$$0 = \frac{\partial(\pi_1 + \pi_2)}{\partial x_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_j}{\partial x_i}.$$

The first term measures the marginal profitability of firm i from a small increase in its R&D (x_i), whereas the second term measures the marginal increase in firm j's profit due to the spillover effect from an increase in i's R&D effort. Hence,

$$\begin{aligned}0 &= \frac{2}{9}[50 + (2-\beta)x_i + (2\beta-1)x_j](2-\beta) - x_i \\ &+ \frac{2}{9}[50 + (2-\beta)x_j + (2\beta-1)x_i](2\beta-1).\end{aligned}$$

Assuming that second order conditions for a maximum are satisfied, the first order conditions yield the cooperative R&D level

$$x_1^c = x_2^c = x^c = \frac{50(\beta+1)}{4.5 - (\beta+1)^2}. \tag{5.4}$$

We now compare the industry's R&D and production levels under noncooperative R&D and cooperative R&D.

Result 5.2

(a) Cooperation in R&D increases firms' profits.

(b) If the R&D spillover effect is large, then the cooperative R&D levels are higher than the noncooperative R&D levels. Formally, if $\beta > \frac{1}{2}$, then $x^c > x^{nc}$. In this case, $Q^c > Q^{nc}$.

(c) If the R&D spillover effect is small, then the cooperative R&D levels are lower than the noncooperative R&D levels. Formally, if $\beta < \frac{1}{2}$, then $x^c < x^{nc}$. In this case, $Q^c < Q^{nc}$.

[1] See Salant and Shaffer (1998) for a criticism of the symmetry of R&D assumption.

5.4 Patents

- We search for the socially optimal patent life T (is it 17 years?)
- Process innovation reduces the unit cost of the innovating firm by x
- For simplicity, we restrict our analysis to minor innovations only.
- market demand given by $p = a - Q$, where $a > c$.

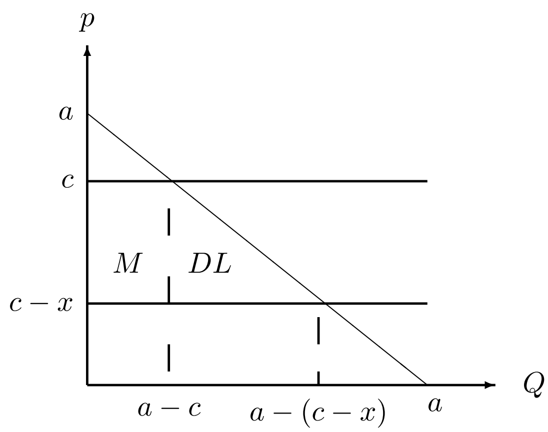


Figure 5.3: Gains and losses due to patent protection (assuming minor innovation)

$$M(x) = (a-c)x \quad \text{and} \quad DL(x) = \frac{x^2}{2}. \tag{5.5}$$

5.4.1 Innovator's choice of R&D level for a given duration of patents

Denote by $\pi(x;T)$ the innovator's present value of profits when the innovator chooses an R&D level of x. Then, in the second stage the innovator takes the duration of patents T as given and chooses in period $t = 1$ R&D level x to

$$\max_x \pi(x;T) = \sum_{t=1}^{T} \rho^{t-1} M(x) - TC(x). \tag{5.6}$$

That is, the innovator chooses R&D level x to maximize the present value of T years of earning monopoly profits minus the cost of R&D. We need the following Lemma.

Lemma 5.1

$$\sum_{t=1}^{T} \rho^{t-1} = \frac{1-\rho^T}{1-\rho}.$$

Proof.

$$\sum_{t=1}^{T}\rho^{t-1}=\sum_{t=0}^{T-1}\rho^{t} = \frac{1}{1-\rho}-\rho^T-\rho^{T+1}-\rho^{T+2}-\dots$$
$$= \frac{1}{1-\rho}-\rho^T(1+\rho+\rho^2+\rho^3+\dots)$$
$$= \frac{1}{1-\rho}-\frac{\rho^T}{1-\rho}=\frac{1-\rho^T}{1-\rho}.$$

□

Hence, by Lemma 5.1 and (5.5), (5.6) can be written as

$$\max_x \frac{1-\rho^T}{1-\rho}(a-c)x-\frac{x^2}{2},$$

implying that the innovator's optimal R&D level is

$$x^I=\frac{1-\rho^T}{1-\rho}(a-c). \tag{5.7}$$

Hence,

Result 5.3
(a) The R&D level increases with the duration of the patent. Formally, x^I increases with T.
(b) The R&D level increases with an increase in the demand, and decreases with an increase in the unit cost. Formally, x^I increases with an increase in a and decreases with an increase in c.
(c) The R&D level increases with an increase in the discount factor ρ (or a decrease in the interest rate).

5.4.2 Society's optimal duration of patents

Formally, the social planner calculates profit-maximizing R&D (5.7) for the innovator, and in period $t=1$ chooses an optimal patent duration T to

$$\max_T W(T) \equiv \sum_{t=1}^{\infty}\rho^{t-1}\left[CS_0+M(x^I)\right]+\sum_{t=T+1}^{\infty}\rho^{t-1}DL(x^I)-\frac{(x^I)^2}{2}$$
$$\textbf{s.t. } x^I = \frac{1-\rho^T}{1-\rho}(a-c). \tag{5.8}$$

Since

$$\sum_{t=T+1}^{\infty}\rho^{t-1}=\rho^T\sum_{t=0}^{\infty}\rho^t=\frac{\rho^T}{1-\rho}$$

and using (5.5), (5.8) can be written as choosing T^* to maximize

$$W(T)=\frac{CS_0+(a-c)x^I}{1-\rho}-\frac{(x^I)^2}{2}\frac{1-\rho-\rho^T}{1-\rho} \quad \textbf{s.t.} \quad x^I=\frac{1-\rho^T}{1-\rho}(a-c). \tag{5.9}$$

Thus, the government acts as a leader since the innovator moves after the government sets the patent length T, and the government moves first and chooses T knowing how the innovator is going to respond.

We denote by T^* the society's optimal duration of patents. We are not going to actually perform this maximization problem in order to find T^*. In general, computer simulations can be used to find the welfare-maximizing T in case differentiation does not lead to an explicit solution, or when the discrete nature of the problem (i.e., T is a natural number) does not allow us to differentiate at all. However, one conclusion is easy to find:

Result 5.4
The optimal patent life is finite. Formally, $T^* < \infty$.

Proof. It is sufficient to show that the welfare level under a one-period patent protection ($T = 1$) exceeds the welfare level under the infinite patent life ($T = \infty$). The proof is divided into two parts for the cases where $\rho < 0.5$ and $\rho \geq 0.5$.

First, for $\rho < 0.5$ when $T = 1$, $x^I(1) = a - c$. Hence, by (5.9),

$$W(1) = \frac{CS_0 + (a-c)^2}{1-\rho} - \frac{(a-c)^2}{2}\frac{1-2\rho}{1-\rho} = \frac{CS_0}{1-\rho} + \frac{(a-c)^2}{1-\rho}\frac{1+2\rho}{2}. \tag{5.10}$$

When $T = +\infty$, $x^I(+\infty) = \frac{a-c}{1-\rho}$. Hence, by (5.9),

$$W(+\infty) = \frac{CS_0}{1-\rho} + \frac{(a-c)^2}{(1-\rho)^2} - \frac{(a-c)^2}{2(1-\rho)^2} = \frac{CS_0}{1-\rho} + \frac{(a-c)^2}{2(1-\rho)^2}. \tag{5.11}$$

A comparison of (5.10) with (5.11) yields that

$$W(1) > W(\infty) \iff \frac{(a-c)^2}{1-\rho}\frac{1+2\rho}{2} > \frac{(a-c)^2}{2(1-\rho)^2} \iff \rho < 0.5. \tag{5.12}$$

Second, for $\rho \geq 0.5$ we approximate T as a continuous variable. Differentiating (5.9) with respect to T and equating to zero yields

$$T^* = \frac{\ln[3 + \sqrt{6+\rho^2-6\rho} - \rho] - \ln(3)}{\ln(\rho)} < \infty.$$

Now, instead of verifying the second-order condition, observe that for $T = 1$, $\mathrm{d}W(1)/\mathrm{d}T = [(a-c)^2\rho(1-5\rho)\ln(\rho)]/[2(1-\rho)^2] > 0$ for $\rho > 0.2$. □

5.5 Appropriable Rents from Innovation in the Absence of Property Rights

Following Anton an Yao (AER, 1994), we show that

- Even without patent law, a non-reproducible innovation can provide with a substantial amount of profit.
- This holds true also for innovators with no assets (poor innovators).

The model

- One innovator, financially broke
- π^M profit level if only one firm gets the innovation
- π^D profit level to each firm if two firms get the innovation

- Innovator reveals the innovation one firm
- then, the informed firm offers a take-it-or-leave-it contract $R = (R_M, R_D)$ = payment to innovator in case innovator does not reveal to a second firm (R_M), or he does reveal (R_D).
- Innovator approaches another firm and asks for a take-it-or-leave-it offer *before* revealing the innovation
- Innovator accepts/rejects the new contract

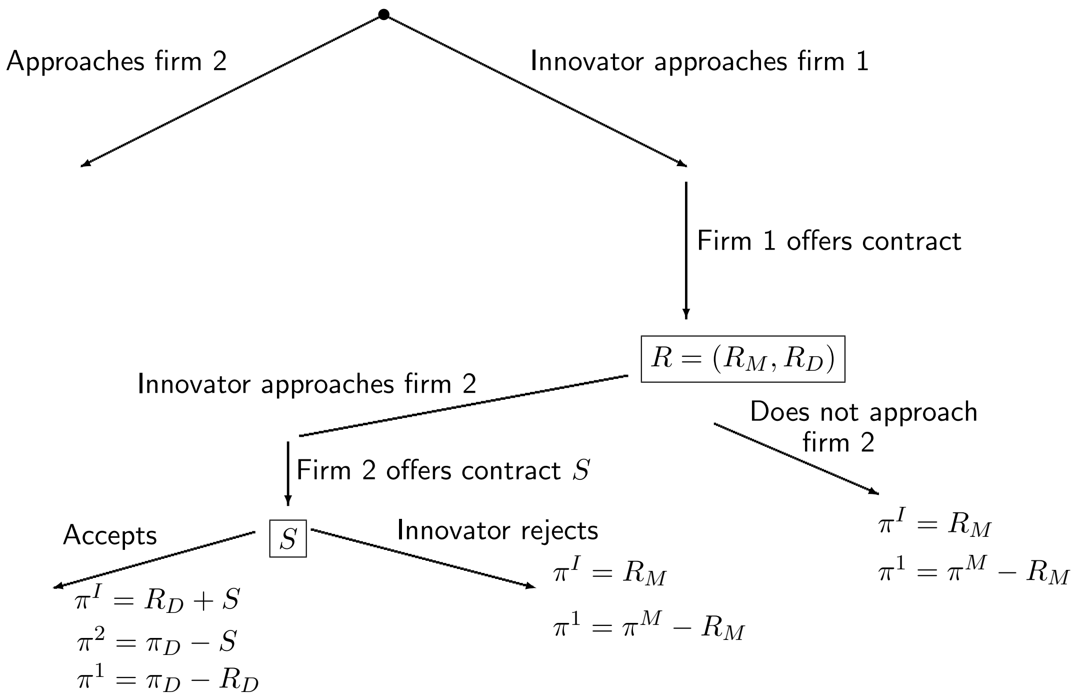


Figure 5.4: Sequence of moves: An innovator extracting rents from firms

Will the innovator reveal to a second firm?

Suppose that innovator already has a contract from firm 1, $R = (R_M, R_D)$.
Without revealing, innovator will accept a second contract, S from firm 2 if

$$R_D + S > R_M$$

Hence, firm 2 will offer a take-it-or-leave-it contact of $S = R_M - R_D + \epsilon$.
Firm 2 will offer a contract S if $\pi^2 = \pi^D - S > 0$, or $S < \pi_D$.

Firm 1's optimal contract offering

Hence, firm 1 will set contract R to satisfy $R_M - R_D \geq \pi^D > S$ to ensure rejection. Hence, firm 1 maximizes profit by offering $R = (R_M, R_D) = (\pi^D, 0)$.

Topic 6
Capacities and Preemption

6.1 Investment and entry deterrence

- Relaxing the Bain, Sylos-Labini postulate (Spence) using Dixit (1980)
- Two-stage game, Stage 1: firm 1 (incumbent) chooses a capacity level $\bar{k}$ that would enable firm 1 to produce without cost $q_1 \le \bar{k}$ units of output
- Stage 2: if incumbent chooses to expand capacity beyond $\bar{k}$ in the second stage, then the incumbent incurs a unit cost of c per each unit of output exceeding $\bar{k}$.
- Entrant makes entry decision in 2nd stage.

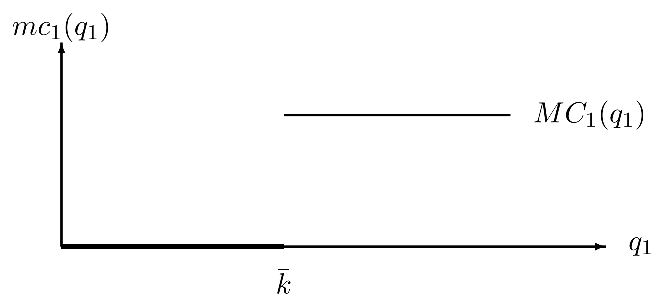


Figure 6.1: Capacity accumulation and marginal cost

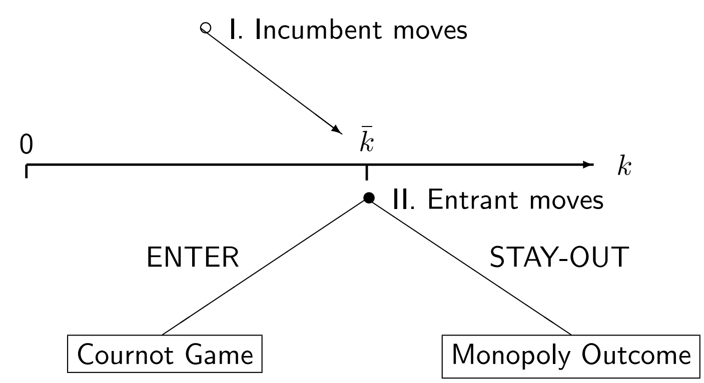


Figure 6.2: Relaxing the Bain-Sylos postulate

The 2nd stage BR functions are

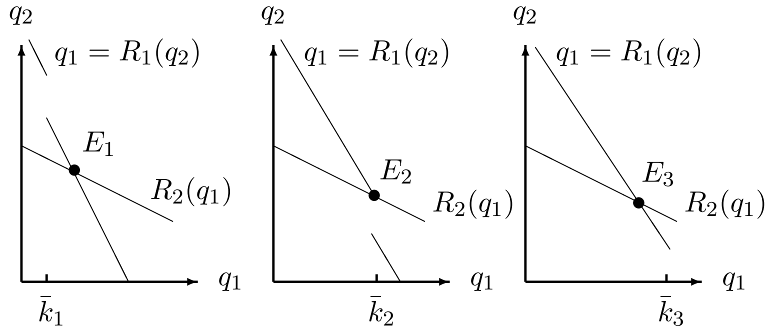


Figure 6.3: Best-response functions with fixed capacity: *Left:* low capacity; *Middle:* medium capacity; *Right:* High capacity

Result 6.1
An incumbent firm will not profit from investing in capacity that will not be utilized if entry occurs. In this sense, limit pricing will not be used to deter entry.

6.2 Spatial preemption

- How a differentiated brands monopoly provider reacts to partial entry?
- Judd (1985): monopoly firm (firm 1) which owns two restaurants, Chinese (denoted by C) and Japanese (denoted by J).
- zero production cost
- 2 types of consumers: Chinese-food oriented, Japanese-food oriented

$$U^C \equiv \begin{cases} \beta - p^C & \text{if eats Chinese food} \\ \beta - \lambda - p^J & \text{if eats Japanese food} \end{cases} \tag{6.1}$$

$$U^J \equiv \begin{cases} \beta - \lambda - p^C & \text{if eats Chinese food} \\ \beta - p^J & \text{if eats Japanese food} \end{cases}$$

  $\lambda > 0$ denotes the slight disutility a consumer has from buying his less preferred food.

- Assume $\lambda < \beta < 2\lambda$

Before entry

$p^C = p^J = \beta$ in each restaurant, and the monopoly's total profit $\pi_1 = 2\beta$.

Entry occurs in the market for Chinese food

If monopoly fights: $p_1^C = p_2^C = 0$.
Maximal price for Japanese: $p^J = \lambda$ since

$$U^J(J) = \beta - p^J = \beta - \lambda \geq \beta - \lambda - p^C = U^J(C).$$

Hence, $\pi_1 = \lambda$.

Incumbent withdraws from the Chinese restaurant

Lemma 6.1

The unique duopoly price game between the Chinese and the Japanese restaurants results in the consumer oriented toward Japanese food buying from the Japanese restaurant, the consumer oriented toward Chinese food buying from the Chinese restaurant, and equilibrium prices given by $p_1^J = p_2^C = \beta$.

Proof. We have to show that no restaurant can increase its profit by undercutting the price of the competing restaurant. If the Japanese restaurant would like to attract the consumer oriented toward Chinese food it has to set $p^J = p^C - \lambda = \beta - \lambda$. In this case, $\pi_2 = 2(\beta - \lambda)$. However, when it does not undercut, $\pi_2 = \beta > 2(\beta - \lambda)$ since we assumed that $\beta < 2\lambda$. A similar argument reveals why the Chinese restaurant would not undercut the Japanese restaurant. □

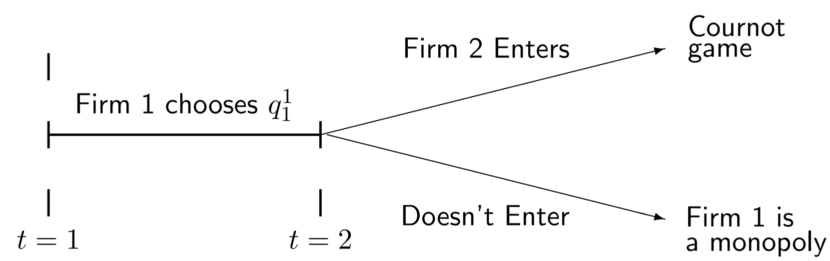
Result 6.2

When faced with entry into the Chinese restaurant's market, the incumbent monopoly firm would maximize its profit by completely withdrawing from the Chinese restaurant's market.

Proof. The profit of the incumbent when it operates the two restaurants after the entry occurs is $\pi_1 = \lambda$. If the incumbent withdraws from the Chinese restaurant and operates only the Japanese restaurants, Lemma 6.1 implies that $\pi_1 = \beta > \lambda$. □

Topic 7
Limit Pricing

- 2 periods, $t = 1, 2$.
- demand each period $p = a - bQ = 10 - Q$.
- Stage 1: firm 1 chooses q_1^1.
- Stage 2: firm 2 chooses to enter or not
- Stage 2: Assumption: Entry occurs: Cournot; Does not: Monopoly



- Firm 2: $c_2 =$ unit cost; $F =$ entry cost. Let $c_2 = 1$ and $F_2 = 9$.
- Firm 1:

$$c_1 = \begin{cases} 0 & \text{with probability } 0.5 \\ 4 & \text{with probability } 0.5. \end{cases} \tag{7.1}$$

- Profits: In the above table, the column labeled ENTER is based on the Cournot solution given by

Incumbent's cost:	Firm 2 (potential entrant)			
	ENTER		DO NOT ENTER	
Low ($c_1 = 0$)	$\pi_1^c(0) = 13.44$	$\pi_2^c(0) = -1.9$	$\pi_1^m(0) = 25$	$\pi_2 = 0$
High($c_1 = 4$)	$\pi_1^c(4) = 1$	$\pi_2^c(4) = 7$	$\pi_1^m(4) = 9$	$\pi_2 = 0$

Table 7.1: Profit levels for $t = 2$ (depending on the entry decision of firm 2). *Note:* All profits are functions of the cost of firm 1 (c_1); π_1^m is the monopoly profit of firm 1; π_i^c is the Cournot profit of firm i, $i = 1, 2$.

$$q_i^c = \frac{a - 2c_i + c_j}{3b}, \quad p^c = \frac{a + c_1 + c_2}{3}, \quad \text{and} \quad \pi_i^c = b(q_i)^2.$$

Solving the game assuming a high-cost incumbent

$$E\pi_2^c = \frac{1}{2}\pi_2^c(0) + \frac{1}{2}\pi_2^c(4) = \frac{1}{2}(-1.9) + \frac{1}{2}7 > 0,$$

Hence, firm 2 will enter.

Given that entry occurs at $t = 2$, firm 1 should play monopoly at $t = 1$.

$$q_1^1(4) = 5 \text{ and therefore earn } \pi_1(4) = \pi_1^m(4) + \pi_1^c(4) = 9 + 1 = 10. \tag{7.2}$$

Solving the game assuming a low-cost incumbent

If $c_1 = 0$, $\pi_2^c(0) < 0$, hence, no entry. However, entrant does not know for sure that 1 is a low-cost.

Result 7.1
A low-cost incumbent would produce $q_1^1 = 5.83$, and entry will not occur in $t = 2$.

Sketch of Proof. In order for the incumbent to convince firm 2 that it is indeed a low-cost firm, it has to do something "heroic." More precisely, in order to convince the potential entrant beyond all doubts that firm 1 is a low-cost one, it has to do something that a high-cost incumbent would never do – namely, it has to produce a first-period output level that is not profitable for a high-cost incumbent!

We look for first period incumbent's output level q_1^1 so that

$$(10 - q_1^1)q_1^1 - 4q_1^1 + \underbrace{\pi_1^m(4)}_{\text{entry deterred}} < \pi_1^m(4) + \underbrace{\pi_1^c(4)}_{\text{entry accommodated}} = 9 + 1 + 10.$$

Now, a high-cost incumbent would not produce $q_1^1 > 5.83$ since

$$9.99 = (10 - 5.83) \times 5.83 - 4 \times 5.83 + \pi_1^m(4) < \pi_1^m(4) + \pi_1^c(4) = 9 + 1 = 10. \tag{7.3}$$

That is, a high-cost incumbent is better off playing a monopoly in the first period and facing entry in the second period than playing $q_1^1 = 5.83$ in the first period and facing no entry in $t = 2$.

Finally, although we showed that $q_1^1 = 5.83$ indeed transmits the signal that the incumbent is a low-cost firm, why is $q_1^1 = 5.83$ the incumbent's profit-maximizing output level, given that the monopoly's output level is much lower, $q_1^m(0) = 5$. Clearly, the incumbent won't produce more than 5.83 since the profit is reduced (gets higher above the monopoly output level). Also (7.3) shows that any output level lower than 5.83 would induce entry, and given that entry occurs, the incumbent is best off playing monopoly in $t = 1$.

Hence, we have to show, for a low-cost firm, that deterring entry by producing $q_1^1 = 5.83$ yields a higher profit than accommodating entry and producing the monopoly output level $q_1^1 = 5$ in $t = 1$. That is,

$$\pi_1(0)|_{q_1^1=5} = 25 + 13 = 38 < 49.31 = (10 - 5.83) \times 5.83 - 0 \cdot 5.83 + 25 = \pi_1(0)|_{q_1^1=5.83},$$

hence, a low-cost incumbent will not allow entry and will not produce $q_1^1 < 5.83$. □

Topic 8
Predation

8.1 Judo Economics

- Focus on entrant's decision (rather than on only incumbent)
- Entrant adopts *judo economics* strategy, Gelman & Salop (1983), which means choosing to enter with a limited amount of capacity (small scale operation).
- Demonstrates that entry deterrence is costly
- 2 stage game:
  (1) Entrant Moves: decides whether to enter, capacity (max output) k, and p^e.
  (2) Incumbent Moves: decides on p^I.
- Homogeneous product: $p = 100 - Q$.

$$q^I = \begin{cases} 100 - p^I & \text{if } p^I \le p^e \\ 100 - k - p^I & \text{if } p^I > p^e \end{cases} \quad \text{and} \quad q^e = \begin{cases} k & \text{if } p^e < p^I \\ 0 & \text{if } p^e \ge p^I. \end{cases} \tag{8.1}$$

- Incumbent strategy: sets p^I (two options):
  (1) undercut entrant: $p^I = p^e$ or,
  (2) accommodate entrant $p^I > p^e$, facing residual demand $q^I = 100 - k - p^I$.

Incumbent deters entry

$\pi^I_D = p^e(100 - p^e)$.

Incumbent accommodates entry

$$\max_{p^I > p^e} \pi^I = p^I(100 - k - p^I),$$

yielding a first-order condition given by $0 = 100 - k - 2p^I$. Therefore, $p^I_A = (100 - k)/2$, hence $q^I_A = (100 - k)/2$ and $\pi^I_A = (100 - k)^2/4$.

Comparing deterrence with accommodation (for the incumbent)

$$\pi^I_A \ge \pi^I_D \quad \text{and} \quad \frac{(100 - k)^2}{4} \ge p^e(100 - p^e). \tag{8.2}$$

First stage: entrance chooses k and p^e

Under entry accommodation, the entrant earns $\pi^e = p^e k > 0$. The entrant chooses p^e to maximize $\pi^e = p^e k > 0$ subject to (8.2).

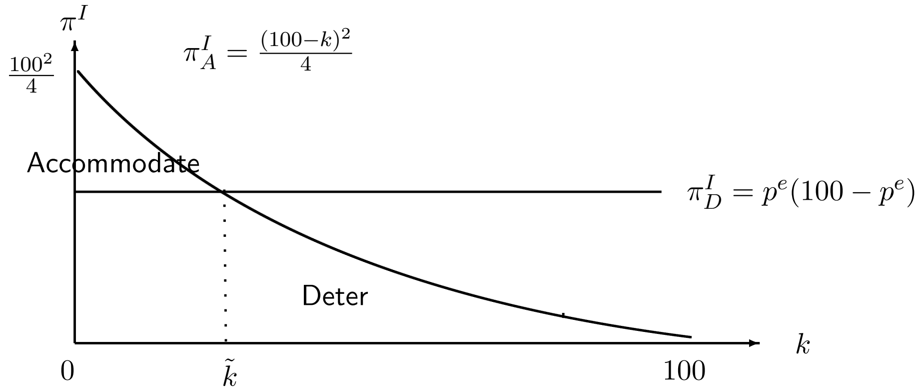


Figure 8.1: Judo economics: How an entrant secures entry accommodation

8.2 The Chain-Store Paradox

Selten (1978):

- An incumbent firm has 20 chain stores in different locations
- Different potential entrant in *each* location :

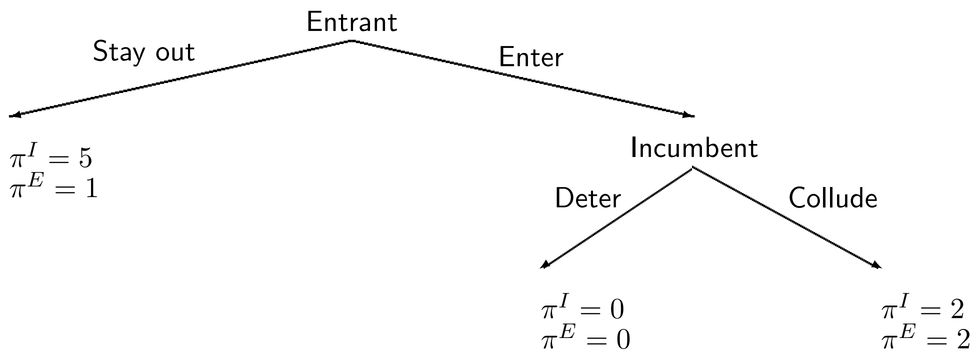


Figure 8.2: Chain-store paradox: The game in each location

		Potential Entrant			
		Enter		Stay Out	
Incumbent	Accommodate	2	2	5	1
	Fight	0	0	5	1

Table 8.1: Chain-store paradox: the game in each location

- Working backwards, after 19 stores enter, the 20th should enter and the incumbent should accommodate
- Working backwards, entrant 19 should enter, and incumbent should accommodate.
- That is, in a game in which the entrant decides before the incumbent, in the unique SPE, Accommodate–Enter will be played in each period.
- In reality, an incumbent will probably fight the first few stores that enter.

Topic 9
Facilitating Practices

9.1 A Meeting Competition Clause

- A sales agreement (like a warranty) between a seller and a buyer
- Three types:

Most Favored Nation (MFN): two types:

Rectroactive: Any future price discounts to other buyers before delivery takes place will be rebated to consumers. Common in industries where delivery comes long after ordering takes place.

Contermporaneous MFN: Agreement made only with repeated purchases allowing them to benefit from temporary price cuts made to other consumers. This is a lower commitment by the firm since it protects only a selected group of buyers.

Meeting the Competition Clause: Two types:

Meet or Release (MOR): If a buyer discovers a lower price elsewhere, the seller will either match the discounted price, or release the customer to buy elsewhere. Purpose: to detect any secret price cuts made by other sellers, and avoiding deterction cost.
Note: It is likely that the seller will choose to release.

No-release MCC: Here there is no release. Seller must match (and even take a loss). Provide much stronger threat on rivals not to reduce prices.

A Example of MCC Game

In a market for luxury cars there are two firms competing in prices. Each firm can choose to set a high price given by p_H, or a low price given by p_L, where $p_H > p_L \geq 0$. The profit levels of the two firms as a function of the prices chosen by both firms is given in Table 9.1. The rules of this two-stage market

		Firm 2: p_H		Firm 2: p_L	
Firm 1	p_H	100	100	0	120
	p_L	120	0	70	70

Table 9.1: Meet the competition clause

game are as follows:

Stage I.: Firm 1 sets its price $p_1 \in \{p_H, p_L\}$.

Stage II.: Firm 1 cannot reverse its decision, whereas firm 2 observes p_1 and then chooses $p_2 \in \{p_H, p_L\}$.

Stage III.: Firm 1 is allowed to move only if firm 2 played $p_2 = p_L$ in stage II. This stage demonstrates the MCC commitment.

We can derive the SPE directly by formulating the extensive form game which is illustrated in Figure 9.1. In this case, the SPE if given by

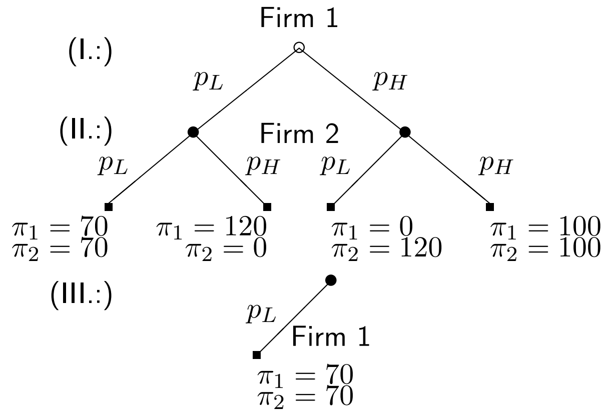


Figure 9.1: Sequential price game: Meet the competition clause

$$p_2 = \begin{cases} p_H & \text{if } p_1 = p_H \\ p_L & \text{if } p_1 = p_L \end{cases} \quad \text{and} \quad p_1 = p_H,$$

implying that the firms charge the industry's profit maximizing price and earn a profit of 100 each.

9.2 Tying as a Facilitating Practice

- Tying as a tool to differentiate brands, Seidmann (1991), Horn and Shy (1996)
- Segmenting markets by tying service with products
- 2 firms, homogeneous product, p^S price with service, p^N w/o service
- Continuum of consumers indexed by $s \in [0, 1]$

$$U^s = \begin{cases} B - p^N & \text{if the product is bought without services} \\ B + s - p^S & \text{if bought tied with services.} \end{cases} \tag{9.1}$$

- m unit production cost, w unit service cost (wage rate)

Market-dividing condition: $B + \hat{s} - p^S = B - p^N$

$$\hat{s} = \begin{cases} 1 & \text{if } p^S - p^N \geq 1 \\ p^S - p^N & \text{if } 0 < p^S - p^N \leq 1 \\ 0 & \text{if } p^S \leq p^N. \end{cases} \tag{9.2}$$

An equilibrium: one firm ties and the other does not as the pair $(\bar{p}^S, \bar{p}^N)$, such that for a given $\bar{p}^N$, the bundling firm chooses $\bar{p}^S$ to maximize $\pi^S = (p^S - m - w)(1 - \hat{s})$, subject to $\hat{s}$ satisfying (9.2); and for a given $\bar{p}^S$, the nontying firm chooses $\bar{p}^N$ to maximize $\pi^N = (p^N - m)\hat{s}$, subject to $\hat{s}$ satisfying (9.2).

$$0 = \frac{\partial \pi^S}{\partial p^S} = 1 - 2p^S + p^N + m + w \quad \text{and} \quad 0 = \frac{\partial \pi^N}{\partial p^N} = p^S - 2p^N + m. \tag{9.3}$$

Therefore, the reaction functions are given by, respectively,

$$p^S = \begin{cases} p^N & \text{if } p^N > m + w + 1 \\ \frac{1}{2}(1 + m + w + p^N) & \text{if } m + w - 1 \le p^N \le m + w + 1 \\ [p^N + 1, \infty) & \text{if } p^N < m + w - 1 \end{cases} \tag{9.4}$$

$$\text{and} \quad p^N = \begin{cases} p^S - 1 & \text{if } p^S > m + 2 \\ \frac{1}{2}(m + p^S) & \text{if } m \le p^S \le m + 2 \\ [p^S, \infty) & \text{if } p^S < m. \end{cases}$$

Solving the "middle" parts of the reaction functions given in (9.4) shows that an interior solution exists and is given by

$$\bar{p}^S = \frac{2}{3}(1 + w) + m; \;\; 1 - \bar{s} = \frac{1}{3}(2 - w); \;\; \bar{\pi}^S = \frac{1}{9}(2 - w)^2 \tag{9.5}$$

$$\bar{p}^N = \frac{1}{3}(1 + w) + m; \;\; \bar{s} = \frac{1}{3}(1 + w); \;\; \bar{\pi}^N = \frac{1}{9}(1 + w)^2.$$

Result 9.1

(a) *In a two-stage game where firms choose in the first period whether to tie their product with services, one firm will tie-in services while the other will sell the product with no service.*

(b) *An increase in the wage rate (in the services sector) would*

  (i) *increase the market share of the nontying firm (the firm that sells the product without service) and decrease the market share of the tying firm (decreases $1 - \bar{s}$).*

  (ii) *increase the price of the untied good and the price of the tied product (both $\bar{p}^S$ and $\bar{p}^N$ increase).*

(c) $\bar{\pi}^S \ge \bar{\pi}^N$ *if and only if* $w \le \frac{1}{2}$.

The socially optimal provision of service

The socially optimal number of consumers purchasing the product without service, denoted by $s^\star$, is obtained under marginal-cost pricing. Thus, let $p^S = m + w$ and $p^N = m$. Then, $s^\star \equiv p^S - p^N = w$. It can easily be verified that $\bar{s} \le s^\star$ if and only if $w \ge \frac{1}{2}$. Hence,

Result 9.2

(a) *If the wage rate in the services sector is high, that is, when $w > \frac{1}{2}$, the equilibrium number of consumers purchasing the product tied with service exceeds the socially optimal level. That is, $1 - \bar{s} > 1 - s^\star$.*

(b) *If the wage rate is low, that is, when $w < \frac{1}{2}$, the equilibrium number of consumers purchasing the product tied with service is lower than the socially optimal level. That is, $1 - \bar{s} < 1 - s^\star$.*

