The Economics of Network Industries
Graduate Lecture Notes

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Contents

Remarks

- Prepared for an 11-hour Ph.D. course, Finnish Postgraduate Program in Economics (FPPE), `http://www.valt.helsinki.fi/fppe` University of Helsinki, October 23–26, 2000.
- Slightly modified for an 8-hour Ph.D. course, Netherlands Network of Economics (NAKE) `www.nake.nl` in Groningen, the Netherlands, December 8–12, 2003.
- Corrected as part of an Industrial Organization Ph.D. course, University of Michigan, Ann Arbor, Fall 2007. Section 2.3 was added (Compatibility Decisions under Network Externalities).
- For a Syllabus see a separate file

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Topic 1

Overview of Network Industries

1.1 Major Issues

- Network industries include the telephone, e-mail, Internet, computer hardware, computer software, music players, music titles, video players, video movies, banking services, airline services, legal services, and many more.
- Main characteristics of these markets:
  - (i) Complementarity, compatibility and standards.
  - (ii) Consumption externalities.
  - (iii) Switching costs and lock-in.
  - (iv) Significant economies of scale in production.

1.1.1 Complementarity, compatibility and standards

- Examples: Computers and monitors, or software; CD players and CD titles, cameras and films; Stereo receivers and speakers or headphones; airline companies and a reservation system.
- Complementarity means that consumers in these markets are shopping for *systems*
- Question: Do firms benefit from designing machines that can work with machines produced by rival firms?
- This means that complementary products must operate on the same *standard*.
- Problem of *coordination* as how firms agree on the standards.
- Coordination may generate some antitrust problems.

1.1.2 Externalities

- Would anyone subscribe to a telephone service knowing that nobody else subscribes? Of course not!
- Would people use e-mail knowing that nobody else does?
- Would people purchase fax machines knowing that nobody else has such a machine?
- Thus, the utility derived from the consumption of these goods is affected by the number of other people using similar or compatible products
- This type of externalities is not found in the market for tomatoes, or the market for salt
- Such externalities are sometimes referred to as *adoption or network externalities*.

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- The reliance on joint-consumer expectations generates multiple equilibria (adoption vs. non-adoption, multiple standards)
- Examples: Histories of the fax machine, e-mail, and the internet
- In the presence of adoption externalities, what should be the minimal number of users (the critical mass) needed for inducing all potential consumers to adopt the technology?

1.1.3 Switching costs and lock-in

- Generated from, say, having to learn a particular operating system such as Windows, UNIX, DOS, or a Macintosh
- It is an established fact that users are very much annoyed by having to switch between operating systems.
- On the production side, producers heavily depend on the standards used in the production of other components of the system.
- For example, airline companies rely on spare parts and service provided by aircraft manufacturers.
- switching between banks (i.e., closing an account in one bank, and opening an account and switching the activities to a different bank) could reach 6 percent of the average account balance.
- In all of these cases, we say that users are *locked-in*.
- We call these costs *switching costs*
- Shapiro and Varian (1999) provide a nice classification of the various lock-ins.
  - (i) Contracts
  - (ii) Training and learning
  - (iii) Data conversion
  - (iv) Search cost
  - (v) Loyalty cost
- Switching costs affect price competition in two opposing ways:
  - (i) If consumers are already locked-in using a specific product, firms may raise prices knowing that consumers will not switch unless the price difference exceeds the switching cost to a competing brand
  - (ii) If consumers are not locked in, brand-producing firms will compete intensively by offering discounts and free complimentary products and services in order to attract consumers who later on will be locked in the technology
- In the presence of switching costs, once the critical mass is achieved we say that the seller has accumulated an *installed base* of consumers

1.1.4 Significant economies of scale

- Software production (or any information product): the production of the first copy involves a huge sunk cost (cost that cannot be recovered), whereas the second copy (third, fourth, and so on) costs almost nothing to reproduce.
- The cost of gathering the information for the Britannica encyclopedia involves more than one hundred years of research as well as the life-time work of a good number of authors. However, the cost of reproducing it on a set of CDs is less than five dollars.
- The cost of developing advanced software involves thousands of hours of programming time, however, the software can now be distributed without cost over the Internet.
- In economic terms: the average cost function declines sharply with the number of copies sold out to consumers.
- Hence, a competitive equilibrium does not exist

1.2 Welfare aspects & government intervention

- Since a competitive equilibria do not exist in markets for network products and services, the First-Welfare Theorem of classical economics cannot be applied.
- Moreover, even if a competitive equilibrium exists, the existence of consumption and production externalities would make this theorem inapplicable.
- A distortion could also be generated when the industry standardizes on the Pareto-inferior standard.
- However, I will argue that the *existence of market failures does not imply that government intervention is needed.*
- In fact, government intervention may make things even worse:
  - (i) The FCC's attempt to impose the CBS color TV standard in 1950
  - (ii) The Japanese Ministry of International Trade and Industry (MITI) poured millions of dollars into the research and development of a standard for a high-definition television (HDTV).
- It is clear why government intervention in standard setting is undesirable
- In fact, since politicians are financed partly by firms, governments may not consider the standard which is not supporting their campaigns
- I do *not* advocate government intervention in standard settings!
- Discuss the (outdate) concept of "Natural" monopolies
- Solution: Access pricing

1.3 Graphic illustrations of the 3 main approaches to network effects

1.3.1 The network externalities approach

Figure 1.1 illustrates the effect of compatibility.

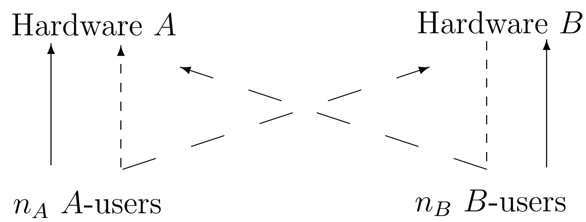

Figure 1.1: The network externalities approach: *Incompatible hardware* (solid arrows); *compatible hardware* (dashed arrows).

- Users' preferences exhibit *network externalities* if the utility of each user is enhanced with an increase in the number of users using the same or compatible brands.
- Remark: Users need not be only buyers; they could be illegal users (pirates) as well.
- *Incompatible brands*: Utility of A-users and B-users are: $U_A(n_A)$ and $U_B(n_B)$, respectively, where $U_i' > 0$.
- *Compatible brands*: Utility levels become $U_A(n_A + n_B)$ and $U_B(n_A + n_B)$.

1.3.2 The components approach

Figure 1.2 illustrates the effect of compatibility.

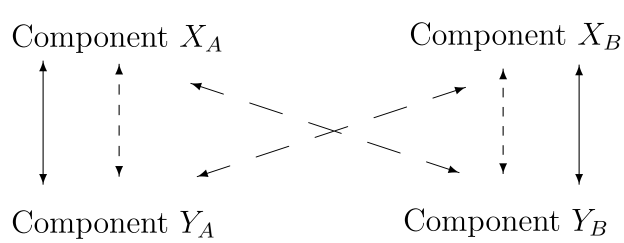

Figure 1.2: The components approach: *Incompatible hardware* (solid arrows); *compatible hardware* (dashed arrows).

- Consumers buy *systems* combining 2 components X and Y which are perfect complements.
- Examples: Computer hardware and monitors, stereo amplifiers and speakers.

- *Incompatible systems*: Consumers can choose between 2 systems $X_A Y_A$ or $X_B Y_B$ only.
- *Compatible systems*: Consumers can choose among 4 systems $X_A Y_A$ or $X_B Y_B$ or $X_A Y_B$ or $X_B Y_Y$.

1.3.3 The software variety approach

Figure 1.3 illustrates the effect of compatibility.

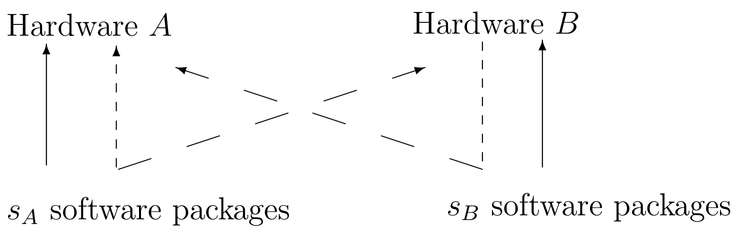

Figure 1.3: The software approach: *Incompatible hardware* (solid arrows); *compatible hardware* (dashed arrows).

- s_A is the variety (# software packages) written specifically for the A machine. s_B similarly defined.
- Computer users derive utility from the *variety* of software available for the machine they use.
- Incompatible system: A and B users derive $U_A(s_A)$ and $U_B(s_B)$, respectively.
- Compatible system: A and B users derive $U_A(s_A + s_B)$ and $U_B(s_A + s_B)$, respectively.

1.4 Why Do We Need a Special Theory?

Class Discussion: Why ordinary Microeconomics and Industrial Organization Theory must be modified to deal with these subjects?

Topic 2
The Network Externalities Approach

2.1 The Standardization-Variety Tradeoff

- Related Reference: Farrell & Saloner (1996).
- 2 brands: A and B.
- a ($0 < a < 1$) consumers prefer brand A over brand B, whereas b ($0 < b < 1$) consumers prefer brand B over brand A, where $a + b = 1$.
- $\delta > 0$ disutility from buying less desired brand (or switching costs from one brand to another)
- x_A (x_B) # of A (B) users, $x_A + x_B = 1$.
- There is *no coordination* among users.

Remark: This is an *adoption* model. That is, there are no firms and no prices in this model (see Section 2.2 for a model with firms).

$$U_A = \begin{cases} x_A & \text{buys brand } A \\ x_B - \delta & \text{buys brand } B \end{cases} \qquad U_B = \begin{cases} x_A - \delta & \text{buys brand } A \\ x_B & \text{buys brand } B \end{cases} \tag{2.1}$$

Assumption 2.1 *(a) Consumers' decision which brand to adopt are not coordinated.*

(b) Each consumer is "negligible" in the sense that consumers view x_A and x_B as unaffected by their own uncoordinated decision which brand to consumer.

Definition 2.1

*(a) If $x_A = 1$ and $x_B = 0$, we say that the product is **standardized** on A.*

*(b) If $x_A = 0$ and $x_B = 1$, we say that the product is **standardized** on B.*

*(c) If $x_A > 0$ and $x_B > 0$, we say that the product is produced with **incompatible standards**.*

(d) An allocation of buyers between brands $\hat{x}_A$ and $\hat{x}_B$ is called an equilibrium, if no single buyer would benefit from switching to the competing brand, given that all other consumers do not switch from their adopted brand.

Result 2.1

(a) If $\delta < 1$, then two equilibria exist: one in which A is the standard ($x_A = 1$) and one in which B is the standard ($x_B = 1$). Formally, $\langle \hat{x}_A, \hat{x}_B \rangle = \langle 1, 0 \rangle$ and $\langle \hat{x}_A, \hat{x}_B \rangle = \langle 0, 1 \rangle$ are equilibria.

(b) If $\delta > 1$, no single-standard equilibrium exists. Formally, $\langle 0, 1 \rangle$ and $\langle 1, 0 \rangle$ are not equilibria.

Proof. (a) Look at $\langle 1,0\rangle$. Then, $U_B(1) = 1-\delta > 0 = U_B(0)$. Hence, no B-oriented user will deviate to buying a B-machine.
(b) Look again at $\langle 1,0\rangle$. Then, $U_B(1) = 1-\delta < 0 = U_B(0)$. Hence, B users will deviate and buy a B machine. ■

Result 2.2 *If the number of each type of consumers is sufficiently large (and not too unequal), then there exists a two-standard equilibrium. Formally, if $a, b > \frac{1-\delta}{2}$, then $x_A = a$, $x_B = b$ is an equilibrium.*

Proof.

$$U_A|_A = a > U_A|_B = b - \delta = 1 - a - \delta \Longrightarrow a > \frac{1-\delta}{2}$$

$$U_B|_B = b > U_B|_A = a - \delta = 1 - b - \delta \Longrightarrow b > \frac{1-\delta}{2}$$

■

Figure 2.1 illustrates the parameter range for which the two-standard equilibrium exists.

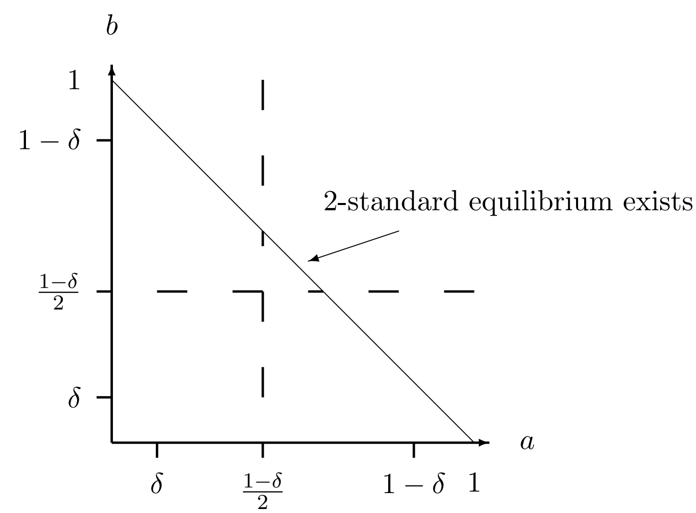

Figure 2.1: Two-standard (incompatibility) equilibrium

Welfare analysis

Since there are no firms, define social welfare by $W \equiv aU^A + bU^B$. In view of the three possible outcomes described above, we have it that

$$W = \begin{cases} a + b(1-\delta) & \text{if } A \text{ is the standard} \\ a^2 + b^2 & \text{if there are incompatible standards} \\ a(1-\delta) + b & \text{if } B \text{ is the standard.} \end{cases} \quad (2.2)$$

Result 2.3 *An equilibrium in which the industry produces two incompatible brands need not maximize social welfare.*

Proof. Let us take an example: $a = 0.6$ and $b = 0.4$ and $0.2 < \delta < 1.2$. Clearly, an incompatibility equilibrium exists.
Incompatibility: $W^I = 0.6^2 + 0.4^2 = 0.52$.
All on standard A: $W^A = 0.6 + 0.4(1 - \delta)$. Now,

$$0.6 + 0.4(1 - \delta) > 0.52 \quad \text{if} \quad \delta < \frac{6}{5} = 1.2.$$

Next, Proposition 2.2 implies that a two-standard equilibrium exists if $b = 0.4 > (1 - \delta)/2$, hence $\delta > 0.2$ must be assumed. ■

Finally, the opposite of Proposition 2.3 holds:

Result 2.4 *If incompatibility ($x_A = a$ and $x_B = b$) maximizes social welfare, then the incompatibility equilibrium exists and is unique.*

Proof.

Lemma 2.1 *If incompatibility maximizes social welfare, then it must be that $\delta > 1$.*

Proof. Incompatibility is socially-preferred over standardization on A, if $a^2+b^2 > a+b-b\delta = 1-b\delta$, or $\delta > \frac{1-a^2-b^2}{b}$. Using the fact that $b = 1 - a$, we see that this last condition is equivalent to $\delta > 2a$, or $a < \frac{\delta}{2}$. Similarly, incompatibility is socially preferred over standardization on B if $b < \frac{\delta}{2}$. However, these conditions cannot both hold if $\delta < 1$ since in this case $a + b < \frac{\delta}{2} + \frac{\delta}{2} < 1$. ■
Since $a > 0$ and $b > 0$, Proposition 2.2 implies that incompatibility is an equilibrium. Also, Proposition 2.1 implies that an equilibrium where an industry is standardized on a single standard does not exist. ■

2.2 Hardware Competition Under Network Externalities

Related Reference: Katz & Shapiro (1985). However, our approach relies a on differentiated-brands, price-competition model, whereas Katz & Shapiro rely on quantity competition with, seemingly, homogeneous products

- 2 (computer) brands: A (Apricot), B (Banana) [durable goods]
- n_A^0, n_B^0 are number of buyers who have already purchased A and B [*installed bases*] (history)
- n new buyers: n_A, n_B are the endogenously-determine, so $n_A + n_B = n$
- Constant population: $n_A + n_B = n = n_A^0 + n_B^0$
- paying p_A, p_B, respectively, buying one unit each
- indexed by x on $[0, 1]$ according to increased preference toward brand B

$$U_x \stackrel{\text{def}}{=} \begin{cases} \alpha(n_A^0 + n_A) - \delta x - p_A & \text{buys brand } A \\ \alpha(n_B^0 + n_B) - \delta(1 - x) - p_B & \text{buys brand } B \end{cases} \tag{2.3}$$

where $\alpha > 0$ is the *network effect* intensity parameter, and δ is measures the *degree of differentiation* between the computer brands

- Assumption: Sufficient differentiation, $\delta > 4\alpha n/3$. This assumption ensures that the prices (2.9) are nonnegative.

Indifferent consumer, $\hat{x}$ is defined by

$$\alpha(n_A^0+n_A)-\tau\hat{x}-p_A = \alpha(n_B^0+n_B)-\tau(1-\hat{x})-p_B \quad \text{hence,} \quad \hat{x} = \frac{\alpha(n_A+n_A^0-n_B-n_B^0)+\delta-p_A+p_B}{2\delta} \tag{2.4}$$

Figure 2.2 illustrates how consumers are divided between the two hardware brands. Substituting

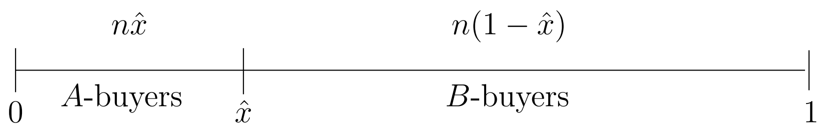

Figure 2.2: Hardware competition: Consumer allocation between brands

$n_A = nx$ and $n_B = n(1-x)$, and then solving for x yields

$$\hat{x} = \frac{\alpha(nx+n_A^0-n(1-x)-n_B^0)+\delta-p_A+p_B}{2\delta} \quad \text{hence,} \quad \hat{x} = \frac{\alpha(n_A^0-n_B^0-n)+\delta-p_A+p_B}{2(\delta-\alpha n)} \tag{2.5}$$

Looking for a Nash-Bertrand equilibrium in p_A and p_B. Firms solve

$$\max_{p_A} \pi_A = p_A n\hat{x} = p_A n\frac{\alpha(n_A^0-n_B^0-n)+\delta-p_A+p_B}{2(\delta-\alpha n)} \tag{2.6}$$

$$\max_{p_B} \pi_B = p_B n(1-\hat{x}) = p_B n\left[1-\frac{\alpha(n_A^0-n_B^0-n)+\delta-p_A+p_B}{2(\delta-\alpha n)}\right] \tag{2.7}$$

Best-response functions are given by

$$p_A = R_A(p_B) = \frac{\alpha(n_A^0-n_B^0-n)+\delta+p_B}{2}, \quad p_B = R_B(p_A) = \frac{\alpha(n_B^0-n_A^0-n)+\delta+p_A}{2} \tag{2.8}$$

- Plot the two best-response functions and conclude that: R_A shifts (i) outward with n_A^0, (ii) inward with n_B^0, (iii) inward with n, and (iv) inward with α.
- Hence, an increase in n is pro-competitive (typical result under network effects)

Solving (2.8) yields equilibrium prices

$$p_A = \frac{3\delta+\alpha(n_A^0-n_B^0-3n)}{3} \quad p_B = \frac{3\delta+\alpha(n_B^0-n_A^0-3n)}{3} \tag{2.9}$$

Result 2.5 *Equilibrium price of a hardware brand increases with its installed base, and decreases with the competitor's installed base.*

Substituting p_A and p_B into (2.5) yields

$$\hat{x} = \frac{\alpha(n_A^0 - n_B^0 - 3n) + 3\delta}{6(\delta - \alpha n)} \tag{2.10}$$

Result 2.6 *The equilibrium market share of a hardware brand increases with its installed base, and decreases with the competitor's installed base. Formally,* $n_A^0 \uparrow \Longrightarrow \hat{x} \uparrow$ *and* $n_B^0 \uparrow \Longrightarrow \hat{x} \downarrow$

Corollary 2.1 *The equilibrium profit level of a hardware brand increases with its installed base, and decreases with the competitor's installed base. Formally,* $n_i^0 \uparrow \Longrightarrow \pi_i \uparrow$, $i = A, B$.

Proof. Propositions 2.5 and 2.6 imply that, both, the price and the market share of a brand-producing firm increases with its own installed base. ■

- The equilibrium number of (new) buyers of each brand is:

$$n_A = n\hat{x} = n\frac{\alpha(n_A^0 - n_B^0 - 3n) + 3\delta}{6(\delta - \alpha n)}, \quad n_B = n(1 - \hat{x}) = n\frac{\alpha(n_B^0 - n_A^0 - 3n) + 3\delta}{6(\delta - \alpha n)}$$

- Question: Does firm A increase its market share relative to its installed base? Formally, $n_A > n_A^0$?

With no loss of generality, let $n_A^0 \geq n_B^0$ (brand A has initially higher installed base).

$$n_A - n_A^0 = \frac{\delta(n_A^0 - n_B^0)}{6[\delta - \alpha(n_A^0 + n_B^0)]} - \frac{2(n_A^0 - n_B^0)}{3} \geq 0 \Longleftrightarrow \delta \leq \frac{4\alpha(n_A^0 + n_B^0)}{3},$$

which never holds.

Result 2.7 *(a) The firm with the larger installed base less-than-doubles its clientele.*

(b) The firm with the smaller installed base more-than-double its clientele.

Proof. Part (a):

$$n_A \leq n_A^0 \Longleftrightarrow \delta \geq \frac{4\alpha(n_A^0 + n_B^0)}{3}$$

which holds by assumption. Part (b):

$$n_B \geq n_B^0 \Longleftrightarrow \delta \geq \frac{4\alpha(n_A^0 + n_B^0)}{3}$$

which holds by assumption. ■

Intuition: High differentiation means that network effects are less important so the market share will converge to 50%.

What happens to consumer's welfare? Using (2.4), we look at a "low-x" consumer who buys brand A:

$$U_x^A = \alpha \left[n_A^0 + nn\frac{\alpha(n_A^0 - n_B^0 - 3n) + 3\delta}{6(\delta - \alpha n)} \right] - \delta x - \frac{3\delta + \alpha(n_A^0 - n_B^0 - 3n)}{3}$$

$$\frac{\mathrm{d}U_x^A}{\mathrm{d}n_A^0} = \frac{\alpha(4\delta - 3\alpha n)}{6(\delta - \alpha n)} \geq 0 \iff \delta \geq \frac{3\alpha n}{4} \quad \text{always holds! By assumption}$$

Thus, despite the price increase, A-buyers are better off with an increase in brand A's installed base. Hence, the network effect dominates the price effect! In addition,

$$\frac{\mathrm{d}U_x^A}{\mathrm{d}n_B^0} = \frac{\alpha(2\delta - 3\alpha n)}{6(\delta - \alpha n)} \geq 0 \iff \delta \geq \frac{3\alpha n}{2}$$

2.3 Compatibility Decisions Under Network Externalities

- $\phi_A \in [0, 1]$ denote the degree of compatibility of A hardware with B hardware.
- $\phi_B \in [0, 1]$ denote the degree of compatibility of B hardware with A hardware.
- Special case: $\phi_A = \phi_B = 1$ means two-way full compatibility.
- Special case: $\phi_A = 1$ and $\phi_B = 0$ or $\phi_A = 0$ and $\phi_B = 1$ means one-way compatibility.
- Special case: $\phi_A = \phi_B = 0$ means fully incompatible brands.
- Now, (with no installed bases), with partial or full compatibility, (2.3) is modified to

$$U_x \stackrel{\text{def}}{=} \begin{cases} \alpha(n_A + \phi_A n_B) - \delta x - p_A & \text{buys brand } A \\ \alpha(n_B + \phi_B n_A) - \delta(1 - x) - p_B & \text{buys brand } B \end{cases} \tag{2.11}$$

- Assumption: $\delta > \alpha n$ (sufficient differentiation relative to network effects).
- Using $n_A = n\hat{x}$ and $n_B = n(1 - \hat{x})$,

$$\alpha(n_A + \phi_A n_B) - \tau\hat{x} - p_A = \alpha(n_B + \phi_A n_A) - \tau(1 - \hat{x}) - p_B \quad \text{yields}$$

$$\hat{x} = \frac{\delta - p_A + p_B - \alpha n(1 - \phi_A)}{2\delta - n\alpha(2 - \phi_A - \phi_B)}. \tag{2.12}$$

This model consists of a two stage game in which

Stage I: Firms decide (cooperatively or noncooperatively) about their degree of compatibility: ϕ_A and ϕ_B.

Stage II: Firms compete by setting prices p_A and p_B.

Stage II: ϕ_A and ϕ_B are given. Firm

$$\max_{p_A} \pi_A = p_A n\hat{x} \quad \text{and} \quad \max_{p_B} \pi_B = p_B n(1-\hat{x})$$

yielding

$$p_A = \frac{3\delta - n\alpha(3 - 2\phi_A - \phi_B)}{3} \quad \text{and} \quad p_B = \frac{3\delta - n\alpha(3 - \phi_A - 2\phi_B)}{3}. \tag{2.13}$$

Thus, the firm that produces the hardware with a higher degree of compatibility charges a higher price because

$$p_A - p_B = \frac{n\alpha(\phi_A - \phi_B)}{3} \geq 0 \iff \phi_A \geq \phi_B.$$

The equilibrium dividing consumers are indexed by

$$\hat{x} = \frac{3\delta - n\alpha(3 - 2\phi_A - \phi_B)}{3[2\delta - n_\alpha(2 - \phi_A - \phi_B)]} = \frac{1}{2} \quad \text{if} \quad \phi_A = \phi_B.$$

The equilibrium profits as functions of ϕ_A and ϕ_B are:

$$\pi_A(\phi_A, \phi_B) = \frac{n[3\delta - n\alpha(3 - 2\phi_A - \phi_B)]^2}{9[2\delta - n\alpha(2 - \phi_A - \phi_B)]} \quad \text{and} \quad \pi_B(\phi_A, \phi_B) = \frac{n[3\delta - n\alpha(3 - \phi_A - 2\phi_B)]^2}{9[2\delta - n\alpha(2 - \phi_A - \phi_B)]}.$$

Hence, the firm which makes its machine more compatible earns a higher profit since

$$\pi_A(\phi_A, \phi_B) - \pi_B(\phi_A, \phi_B) = \frac{n^2\alpha(\phi_A - \phi_B)}{3} \geq 0 \iff \phi_A \geq \phi_B.$$

Stage I: Suppose that firms are restricted to setting either $\phi_i = 0$ (incompatibility), or $\phi_i = 1$ (100% compatibility). Table 2.1 displays the profit levels for the four possible outcomes.

		Firm *B* $\phi_B = 0$		$\phi_B = 1$	
Firm *A*	$\phi_A = 0$	$\frac{n(\delta-n\alpha)}{2}$	$\frac{n(\delta-n\alpha)}{2}$	$\frac{n(3\delta-2n\alpha)^2}{9(2\delta-n\alpha)}$	$\frac{n(3\delta-n\alpha)^2}{9(2\delta-n\alpha)}$
	$\phi_A = 1$	$\frac{n(3\delta-n\alpha)^2}{9(2\delta-n\alpha)}$	$\frac{n(3\delta-2n\alpha)^2}{9(2\delta-n\alpha)}$	$\frac{n\delta}{2}$	$\frac{n\delta}{2}$

Table 2.1: Equilibrium profit levels under different degrees of hardware compatibility.

Result 2.8 *Both firms make a higher profit when they sell 100% compatible machines ($\phi_A = \phi_B = 1$) compared with incompatible machines ($\phi_A = \phi_B = 0$).*

Result 2.9 $\phi_A = \phi_B = 1$ *(full compatibility) is a unique subgame-perfect equilibrium.*

TOPIC 3
THE COMPONENTS' APPROACH

- Reference: Matutes & Regibeau (1988)
- System is defined as two components XY (perfect complements)
- Examples: Computer & monitor, stereo receiver & speakers, etc.
- Two brand-producing firms, A and B, each producing both components
- Can make components *compatible* or *incompatible*
- If systems are incompatible: only $X_A Y_A$ & $X_B Y_B$ are available to consumers, with *system* prices p_A and p_B
- If systems are compatible: $X_A Y_A$, $X_B Y_B$, $X_A Y_B$, $X_B Y_A$ are available, with *components'* prices p_A^X, p_A^Y, p_B^X, p_B^Y
- Consumers are indexed by x and y on the unit square

$$U_{xy} \stackrel{\text{def}}{=} \begin{cases} \beta - \delta x - \delta y - p_A^X - p_A^Y & \text{if buys system } X_A Y_A \\ \beta - \delta(1-x) - \delta(1-y) - p_B^X - p_B^Y & \text{if buys system } X_B Y_B \\ \beta - \delta(1-x) - \delta y - p_B^X - p_A^Y & \text{if buys system } X_B Y_A \\ \beta - \delta x - \delta(1-y) - p_A^X - p_B^Y & \text{if buys system } X_A Y_B \end{cases} \tag{3.1}$$

- Modeling methodology: (i) solve for an equilibrium under incompatible systems; (ii) under compatible systems

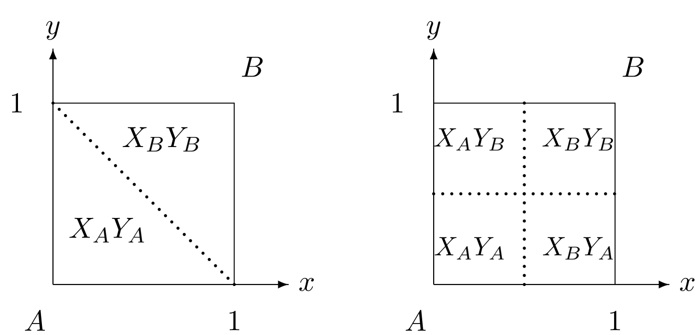

Figure 3.1: Equilibrium allocation of consumers among systems. *Left:* Incompatible systems. *Right:* Compatible systems.

- Compare: profits, utilities, and social welfare

3.1 Incompatible systems

- Two systems on the market: $X_A Y_A$ and $X_B Y_B$, priced p_A and p_B, respectively
- Let $(\hat{x}, \hat{y})$ denote all consumers who are indifferent between buying system $X_A Y_A$ and $X_B Y_B$
- In view of (3.1) they are defined by

$$\beta - \delta x - \delta y - p_A = \beta - \delta(1-x) - \delta(1-y) - p_B$$

Hence,

$$\hat{y} = \frac{2\delta - p_A + p_B}{2\delta} - \hat{x}$$

- Look at the following market division:

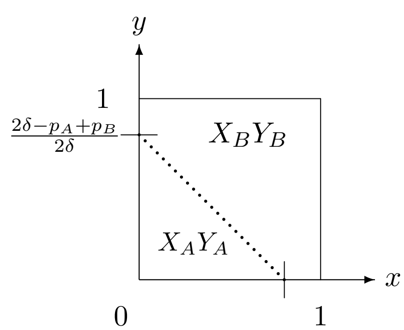

Figure 3.2: Possible market division under incompatibility.

- Let n_A and n_B denote the number of buyers of each system

$$n_A = \frac{(2\delta - p_A + p_B)^2}{8\delta^2}, \quad \text{and} \quad n_B = 1 - n_A = 1 - \left[\frac{(2\delta - p_A + p_B)^2}{8\delta^2}\right]$$

- We are looking for a Nash-Bertrand equilibrium in (p_A, p_B)
- The firms solve

$$\max_{p_A} \pi_A = p_A n_A = p_A \frac{(2\delta - p_A + p_B)^2}{8\delta^2}, \quad \max_{p_B} \pi_B = p_B(1-n_A) = p_B \left\{1 - \left[\frac{(2\delta - p_A + p_B)^2}{8\delta^2}\right]\right\}$$

- **Guess:** $p_A = p_B = \delta$ is a Nash-Bertrand equilibrium !
- Substitute $p_B = \delta$ into π_A to solve for A's best response to $p_B = \delta$

$$\max_{p_A} \pi_A = \frac{p_A(3\delta - p_A)^2}{8\delta^2} \Longrightarrow p_A \in \{\delta, 3\delta\} \quad \text{i.e., not unique}$$

However,

$$\frac{\mathrm{d}^2 \pi_A}{\mathrm{d}(p_A)^2} = \frac{3(p_A - 2\delta)}{4\delta^2} \Longrightarrow p_A = \delta \text{ is a unique maximum!}$$

- Substitute $p_A = \delta$ into π_B

$$\max_{p_B} \pi_B = p_B \left[1 - \frac{(\delta + p_B)^2}{8\delta^2}\right] \Longrightarrow \frac{\mathrm{d}\pi_B}{\mathrm{d}p_B} = \frac{7\delta^2 - 4\delta p_B - 3(p_B)^2}{8\delta^2} = \Big|_{p_B=\delta} 0$$

- Second-order condition for a maximum

$$\frac{\mathrm{d}^2\pi_B}{\mathrm{d}(p_B)^2} = -\frac{2\delta + 3p_B}{4\delta^2}\Big|_{p_B=\delta} < 0$$

- Equilibrium number of buyers and profit levels

$$n_A^I = n_B^I = \frac{1}{2}, \quad \text{and} \quad \pi_A^I = \pi_B^I = \frac{\delta}{2}, \tag{3.2}$$

  where I stands for *incompatibility.*

- Aggregate utility:

$$TU = \beta - \delta - 2 \times \int_0^1 \int_0^{1-x} (-\delta x - \delta y)\, dy\, dx = \beta - \delta - \frac{2\delta}{3} \tag{3.3}$$

- Social welfare under incompatibility:

$$W^I \stackrel{\text{def}}{=} \pi_A^I + \pi_B^I + TU^I = \beta - \frac{2\delta}{3} \tag{3.4}$$

3.2 Compatible systems

- Consumers who are indifferent between buying system X_AY_A and X_AY_B are defined by

$$\beta - \delta x - \delta y - p_A^X - p_A^Y = \beta - \delta x - \delta(1-y) - p_A^X - p_B^Y, \quad \text{or} \quad y = \frac{\delta - p_A^Y + p_B^Y}{2\delta}$$

- Consumers who are indifferent between buying system X_AY_A and X_BY_A are defined by

$$\beta - \delta x - \delta y - p_A^X - p_A^Y = \beta - \delta(1-x) - \delta(1-y) - p_B^X - p_A^Y, \quad \text{or} \quad y = \frac{\delta - p_A^X + p_B^X}{2\delta}$$

- Figure 3.3 displays how consumers are divided according to systems
- We look for a Nash-Bertrand equilibrium in p_A^X, p_A^Y, p_B^X, p_B^Y
- Figure 3.3 implies that hardware firm A sells components X_A and Y_A, hence solves the following profit-maximization problem:

$$\max_{p_A^X, p_A^Y} \pi_A = p_A^X \left(\frac{\delta - p_A^X + p_B^X}{2\delta} \times 1\right) + p_A^Y \left(1 \times \frac{-\delta - p_A^Y + p_B^Y}{2\delta}\right)$$

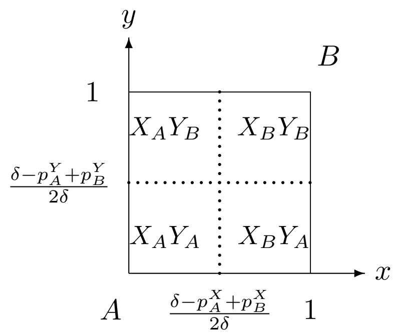

Figure 3.3: Possible market division under compatibility.

- First-order conditions:

$$0=\frac{\partial \pi_A}{\partial p_A^X}=\frac{\delta-2p_A^X+p_B^X}{2\delta} \quad \text{and} \quad 0=\frac{\partial \pi_A}{\partial p_A^Y}=\frac{\delta-2p_A^Y+p_B^Y}{2\delta}$$

- To prove strict concavity of π_A note that

$$\frac{\partial^2 \pi_A}{\partial (p_A^X)^2}=\frac{\partial^2 \pi_A}{\partial (p_A^Y)^2}=-\frac{1}{\delta}<0, \quad \text{and} \quad \frac{\partial^2 \pi_A}{\partial p_A^X \partial p_A^Y}=0$$

- By symmetry, the unique equilibrium component prices and profit levels are

$$p_A^X=p_B^X=p_A^Y=p_B^Y=\delta \quad \text{and} \quad \pi_A^C=\pi_B^C=\delta \tag{3.5}$$

where superscript C stands for *compatibility*.

- Aggregate consumer utility:

$$TU^C \stackrel{\text{def}}{=} \beta-\delta-\delta-4\int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}}(-\delta x-\delta y)d\,y\,d\,x=\beta-2\delta-4\times\frac{\delta}{8}=\beta-\frac{5\delta}{2} \tag{3.6}$$

- Social welfare:

$$W^C \stackrel{\text{def}}{=} \pi_A^C+\pi_B^C+TU^C=\beta-\frac{\delta}{2} \tag{3.7}$$

3.3 Comparing incompatible and compatible systems

- Comparing the profit levels under incompatibility (3.2) with profits under compatibility (3.5): $\pi_i^C > \pi_i^I$, $i=A,B$ (firms are better off under compatibility)
- Why? The system price is twice under compatibility than under incompatibility
- Comparing aggregate consumer utility under incompatibility (3.3) with compatibility (3.6): $TU^I > TU^C$ (consumers are better off under incompatible systems).

- Thus, despite the fact that consumers are exposed to system combinations closer to their taste, the price hike extracts more surplus than what they gain.
- Comparing social welfare under incompatibility (3.4) with compatibility (3.7): $W^C > W^I$.
- Society is better off under compatibility (gain to the firm dominates the loss to consumers)

Topic 4
The Supporting-Services (Software) Approach

Basic features of this approach:

- No externality is assumed!
- Key assumption: Users gain utility from the variety of supporting services (software), supporting the specific machine they buy
- Examples: Intel-based computers: variety of specific software;
- Apple machine: variety of Macintosh-specific software DVD: variety of DVD movies; etc.

4.1 The software industry under monopolistic competition[1]

- Consider a consumer who already has the hardware
- Facing a continuum of (machine-specific!) software packages indexed by x, where $x \in X \subset [0, +\infty)$
- $s(x)$ consumption level of software named x (e.g., if $x =$ Elvis, $s(x)$ is the number of Elvis CDs the consumer buys)
- The software service (utility from all software packages) is

$$S \stackrel{\text{def}}{=} \left\{ \int_X [s(x)]^\rho \, d\,x \right\}^{\frac{1}{\rho}}, \quad 0 < \rho < 1 \tag{4.1}$$

- Let $p(x)$ denote the price of one unit of software x
- $E_S =$ a consumer's expenditure on software
- Each consumer solves

$$\max_{x \in X} \left\{ \int_X [s(x)]^\rho \, d\,x \right\}^{\frac{1}{\rho}} \quad \textbf{s.t.} \quad \int_X p(x)\, s(x)\, d\,x = E_S$$

- Let λ be the Lagrangian multiplier. First-order condition:

$$[s(x)]^{\rho-1} = \lambda p(x), \quad \text{or} \quad s(x) = [\lambda p(x)]^{\frac{1}{\rho-1}}$$

- So, $e = 1/(\rho - 1)$ is the demand-price elasticity of *a single brand*

[1]Pioneering papers on monopolistic competition: Dixit, A., and J. Stiglitz (1977) "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67: 297–308. Krugman, P. (1979) "Increasing Returns, Monopolistic Competition, and International Trade," *Journal of International Economics* 9: 469–479.

- Let $\mu \stackrel{\text{def}}{=} \mu(X)$ denote the variety of available software (Lebesgue measure)

Substituting into the budget constraint, letting $p(x) = p$ for all x:

$$\int_0^\mu p(x)\,[\lambda p(x)]^{\frac{1}{\rho-1}}\,d\,x = E_S \Longrightarrow \lambda = \left(\frac{E_S}{\mu}\right)^{\rho-1}[p(x)]^{-\rho} \Longrightarrow s(x) = \frac{E_S}{\mu}[p(x)]^{-1}$$

- Software producer x: marginal cost $= 1$, fixed development cost f
- With n consumers, software producer x solves

$$\max_{p(x)} \pi(x) = [p(x)-1]ns(x)-f \Longrightarrow MR(x) = p(x)\left[1+\frac{1}{e}\right] = p(x)\left[1+\frac{1}{\frac{1}{\rho-1}}\right] = p(x)\rho = 1 = MC(x)$$

- Hence, for all $x \in X$

$$p(x) = 1/\rho \Longrightarrow s(x) = \frac{E_S}{\mu}\rho$$

- More substitutable, $\rho \to 1 \Longrightarrow p(x) \to 1 = MC$
- Free entry implies zero profit, yields the software variety

$$0 = \pi(x) = [p(x)-1]ns(x) - f = \left[\frac{1}{\rho}-1\right]n\frac{E_S}{\mu}\rho^{\frac{\rho}{1-\rho}} - f \Longrightarrow \mu = \frac{(1-\rho)nE_S}{f} \tag{4.2}$$

- Example: $\rho = 1/2$ implies $\mu = nE_S/(2f)$, $\rho = 1/3$ implies $\mu = nE_S/(3f)$, hence

$$\rho \uparrow \Longrightarrow \mu \downarrow$$

- Consumer's utility

$$S = \left\{\int_0^\mu [s(x)]^\rho\,d\,x\right\}^{\frac{1}{\rho}} = \rho(1-\rho)^{\frac{1-\rho}{\rho}}\left(\frac{n}{f}\right)^{\frac{1-\rho}{\rho}}(E_S)^{\frac{1}{\rho}} \tag{4.3}$$

- Thus, the utility from a system, S, is a concave function of consumers' expenditure, E_S.
- For example: if $\rho = 1/2$,

$$S = \frac{nE^2}{4f}$$

4.2 Hardware Competition

- Related Reference: Chou & Shy (1990)
- Two hardware brands, A and B supported by brand-specific software
- n consumers, uniformly indexed by $\delta \in [0,1]$ according to increase preference for hardware B

- (4.3) implies that the utility (service) from system (actually software) i, $i = A, B$ is:

$$S_i \stackrel{\text{def}}{=} K(n_i)^{\frac{1-\rho}{\rho}}(E_i)^{\frac{1}{\rho}}, \quad \text{where } \frac{1}{2} < \rho < 1 \text{ and} \tag{4.4}$$

  n_i is the number of i-brand users (buyers), and
- E_i is the expenditure on software by a hardware i user
- *Note:* n_i is NOT an network externality component! n_i is the number of consumer spending money on buying software running for machine i which affects the variety according to (4.2)
- Each consumer has \$$m$ to spend on hardware and software
- Let p_A, p_B be hardware prices, then
- expenditure on software: $E_A = m - p_A$ and $E_B = m - p_B$
- Utility of consumer δ, $\delta \in [0, 1]$ is:

$$U_\delta \stackrel{\text{def}}{=} \begin{cases} (1-\delta)S_A & \text{If buys system } A \\ \delta S_B & \text{If buys system } B \\ 0 & \text{Not buying any system} \end{cases}$$

4.2.1 Equilibrium hardware price under incompatibility

- There exists a $\hat{\delta} \in [0, 1]$ for which all consumers $\delta \in [0, \hat{\delta}]$ buy A, and all $\delta \in (\hat{\delta}, 1]$ buy B
- Hence, $n_A = \hat{\delta}n$ and $n_B = (1 - \hat{\delta})n$
- The "indifferent" consumer, $\hat{\delta}$ is defined by $(1 - \hat{\delta})S_A = \hat{\delta}S_B$, hence,

$$(1-\hat{\delta})K(\delta n)^{\frac{1-\rho}{\rho}}(m-p_A)^{\frac{1}{\rho}} = \hat{\delta}K(\delta n)^{\frac{1-\rho}{\rho}}(m-p_B)^{\frac{1}{\rho}}$$

$$\hat{\delta} = \frac{(m-p_A)^{\frac{1}{2\rho-1}}}{(m-p_A)^{\frac{1}{2\rho-1}} + (m-p_B)^{\frac{1}{2\rho-1}}}$$

- Hardware producers' profits: $\pi_A = \hat{\delta}np_A$ and $\pi_B = (1 - \hat{\delta})np_B$
- Looking for a Nash-Bertrand equilibrium in $\langle p_A, p_B\rangle$

Solving $\max_{p_A} \pi_A$, and evaluating the first-order condition at $p_A = p_B = p$ yields

$$p^I = \frac{2m(2\rho-1)}{4\rho-1} \quad \text{and} \quad \pi^I = \frac{mn(2\rho-1)}{4\rho-1} \tag{4.5}$$

where "I" stands for incompatibility. The second order condition evaluated at p^I is

$$\left.\frac{\partial^2 \pi_A}{\partial (p_A)^2}\right|_{p^I=p_A=p_B} = \frac{n\rho(4\rho-1)}{m(1-2\rho)} < 0 \quad \text{since } \rho > \frac{1}{2}$$

Finally, to get an idea about consumer welfare, substitute $n_i = n/2$ and $E_i = m - p^I$, $i = A, B$ into (4.4) to obtain

$$S_A^I = S_B^I = \left(\frac{n}{2}\right)^{\frac{1-\rho}{\rho}} K\left(\frac{m}{4\rho-1}\right)^{\frac{1}{\rho}} \tag{4.6}$$

4.2.2 Equilibrium hardware price under compatibility

Here, $n_A = n_B = n$ meaning that the variety of software running on any machine is determined by the aggregate consumer spending on software. So, the service (utility) from software is independent of which computer is chosen. That is, from consumers' view point

$$S_A = S_B$$

$(1-\hat{\delta})S_A = \hat{\delta}S_B$ implies that

$$\hat{\delta} = \frac{1}{2}$$

Hence, market shares are independent of hardware prices. Hence, each hardware producer can raise his price until consumers become indifferent between buying and not buying, meaning that Solving $\max_{p_A} \pi_A = \hat{\delta} n p_A$, yields

$$p^C = m - \epsilon \quad \text{and} \quad \pi^C = \frac{m-\epsilon}{2} \tag{4.7}$$

where "C" stands for compatibility, and ϵ is a "small" real number.

Substituting $n_i = n$ and $E_i = m - p^C = \epsilon$, $i = A, B$ into (4.4) to obtain

$$S_A^C = S_B^C = n^{\frac{1-\rho}{\rho}} K(\epsilon)^{\frac{1}{\rho}} \tag{4.8}$$

4.2.3 Comparing incompatibility with compatibility equilibria

4.2.3.1 Comparing hardware prices and profits

Subtracting (4.5) from (4.7) yields

$$p^C - p^I = m - \frac{2m(2\rho-1)}{4\rho-1} = \frac{m}{4\rho-1} > 0 \quad \text{hence,} \quad \pi^C - \pi^I > 0$$

Hence, hardware producers earn a higher profit under compatibility!

4.2.3.2 Comparing software variety

By (4.2),

$$\mu^C = \frac{1-\rho}{f} n\epsilon < \mu^I = \frac{1-\rho}{f}\frac{n}{2}\left(m - \frac{2m(2\rho-1)}{4\rho-1}\right)$$

Surprising result: Software variety if *lower* under compatibility.

4.2.3.3 Comparing consumer welfare

Since prices are higher, and software variety is lower under compatibility, it must be that $S^C < S^I$.

4.3 Partial Compatibility

- Related Reference: Chou & Shy (1993)
- Purpose: define and model the concept of partial compatibility in an industry producing differentiated brands that are supported by brand-specific supporting services

- To analyze the effects of increasing a brand's degree of compatibility with the supporting services of the competing brand on firms' market shares and the variety of brand-specific supporting services.
- It is often observed that different brands of the same product are only *partially* compatible with each other
- For example, not all Windows/DOS computers are one hundred percent compatible in the sense that there always exist some software packages that can run on one machine but not on the others
- Compatibility is not a symmetric relation. For example, some Apple machines can read DOS diskettes, but DOS machines cannot read the Apple format
- Consumers can choose between two computer brands A and B
- Ignoring consumers' expenditure on computer hardware, each consumer is endowed with Y dollars to be spent on software.
- N_i is the total number of software packages that can be run on an i-machine.
- The service to a system i user, denoted by S_i, is defined as an increasing function of both her expenditure on software and the number of software packages compatible with machine i, $i = A, B$. Formally, let

$$S_i \stackrel{\text{def}}{=} Y(N_i)^\theta, \quad \text{where } 0 < \theta < 1 \text{ and } i = A, B. \tag{4.9}$$

- Consumers are uniformly indexed by δ on the interval $[0, 1]$ according to their relative preference towards computer brand B

$$U^\delta \stackrel{\text{def}}{=} \begin{cases} (1-\delta)S_A & \text{if she is an } A\text{-user} \\ \delta S_B & \text{if she is a } B\text{-user} \end{cases} \tag{4.10}$$

- n_i is the number of software packages written *specifically* for machine i
- Let ρ_i, $0 < \rho_i < 1$, measure the *exogenously given* degree of compatibility of computer i with respect to j's software
- ρ_i measures the proportion of machine j-software that can be run on an i-machine, $i, j = A, B$ and $i \neq j$.
- Therefore, the total number of software packages available to an i-machine user is equal to

$$N_i = n_i + \rho_i n_j \quad i, j = A, B, \ i \neq j. \tag{4.11}$$

- The consumer who is indifferent between system A and system B is

$$(1-\hat{\delta})Y(N_A)^\theta = \hat{\delta}Y(N_B)^\theta \Longrightarrow \hat{\delta} = \frac{(N_A)^\theta}{(N_A)^\theta + (N_B)^\theta}. \tag{4.12}$$

- Thus, a consumer indexed by $\delta < \hat{\delta}$ is an A-user while a consumer indexed by $\delta > \hat{\delta}$ is a B-user
- An i-user (purchasing n_i software packages designed for the i-machine and $\rho_i n_j$ software packages designed for the j-machine) spends $n_i Y/N_i$ on i-software and $\rho_i n_j Y/N_i$ on j-software
- Assumption (production-side): n_i equal the total expenditure on i-specific software:

$$n_A = \hat{\delta}\frac{n_A Y}{N_A} + (1-\hat{\delta})\frac{\rho_B n_A Y}{N_B} \quad \text{and} \quad n_B = (1-\hat{\delta})\frac{n_B Y}{N_B} + \hat{\delta}\frac{\rho_A n_B Y}{N_A} \tag{4.13}$$

Solving the system of equations (4.13) for N_A and N_B yields

$$N_A = \frac{Y\hat{\delta}(1-\rho_A\rho_B)}{1-\rho_B} \quad \text{and} \quad N_B = \frac{Y(1-\hat{\delta})(1-\rho_A\rho_B)}{1-\rho_A}$$

Substituting N_A and N_B into (4.12),

$$\hat{\delta} = \frac{1}{1+\left(\frac{1-\rho_B}{1-\rho_A}\right)^{\frac{\theta}{1-\theta}}} \tag{4.14}$$

yielding our main proposition:

Result 4.1 *An increase in the degree of compatibility of machine A with B's software, reduces the market share of the A machine. Formally,*

$$\rho_A \uparrow \Longrightarrow \hat{\delta} \downarrow, \quad \textit{and} \quad \rho_B \uparrow \Longrightarrow \hat{\delta} \uparrow$$

- The paper also shows that $\rho_A \uparrow \Longrightarrow N_A \downarrow$ and even $n_A \downarrow$
- Intuition: An increase in ρ_A induces software writers to write for machine B (since it will be purchases by more A users anyway
- Example: Apple's introduction of the hyperdrive (able to read Windows/DOS-formatted diskettes)
- Example: Explains why SUN and Silicon Graphics workstations are not Windows compatible

4.4 Software Piracy[2]

Related Reference: Shy & Thisse (1999).

- Actually, this analysis relies on pure network externalities, and therefore relates to Lecture 2.
- Major observation: Since the widespread introduction of personal computers in the early 1980s, software firms began gradually removing protection against copying.
- firms realized that consumers were annoyed by the consequences protective devices had on the effectiveness of their products
- When the market expands and competition intensifies, due to large network effects, firms have strategic incentives to remove protection in order to increase the number of consumers using their packages
- Purpose of the present analysis: (i) Model strategic incentives for removing copy protection (ii) Claim that the software industry estimates on lost sales due to piracy are exaggerated
- A single monopoly software firm which supplies one piece of software
- Consumers are divided according to

  Support-oriented consumers (type O): who gain an extra utility from services and support provided by software firms to their legal customers. We assume that there are η support-oriented potential consumers.

  Support-independent consumers (type I): who do not derive utility from the services and support provided by the software firms to their legal customers. We assume that there are η support-independent potential consumers.

- Altogether, the total population in the economy is 2η.
- Each consumer has 3 options: (i) buy the software, (ii) pirate the software, or (iii) not use any software
- Assumption:
  (a) The software firm bundles the support with purchase.
  (b) Illegal software users cannot obtain support from an independent supplier.
- Let q denote the number of *users* of this software (who *legally and illegally* use this software)
- The utility of a consumer of support-oriented consumers is given by

$$U^O \stackrel{\text{def}}{=} \begin{cases} (1+\sigma)q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases} \tag{4.15}$$

[2] The pioneering paper is Conner, K., and R. Rumelt. 1991. "Software Piracy: An Analysis of Protection Strategies." *Management Science* 37: 125–139. For an empirical support see: Givon, M., V. Mahajan, and E. Muller (1995) "Software Piracy: Estimation of Lost Sales and the Impact on Software Diffusion," *Journal of Marketing* 59: 29–37.

- p is the price set by the software monopoly producer, $\sigma > 0$ measures the value of service to a type O consumer
- The utility of a support-independent consumer is

$$U^I \stackrel{\text{def}}{=} \begin{cases} q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software.} \end{cases} \tag{4.16}$$

- Clearly,

  (a) Support-oriented consumers would prefer buying software over pirating software if $p \le \sigma q$

  (b) If software is not copy protected, support-independent consumers never buy software

- Assumption: In addition to price, the monopoly software firm has two options in choosing its protection policy:

  Nonprotection policy (n): Any consumer can costlessly pirate the software, but cannot obtain any service from the software firm.

  Protection policy (p): Installing devices and/or implementing an enforcement policy in order to make software piracy practically impossible.

4.4.1 No copy protection

The highest price the software monopoly can set and the resulting profit level are

$$p^n = \sigma 2\eta, \quad \text{and} \quad \pi^n = p^n \eta = 2\sigma\eta^2, \tag{4.17}$$

where superscript n stands for nonprotection policy.

In this equilibrium each support-oriented consumer buys the software and gains a utility of $U^O = (1+\sigma)2\eta - p = 2\eta$, and each support-independent consumer pirates the software and gains $U^I = 2\eta$.

4.4.2 Copy protection

- Piracy is not an option
- High price equilibrium: For $\sigma > 1$

$$p^{p,H} = (1+\sigma)\eta \quad \text{and} \quad \pi^{p,H} = (1+\sigma)\eta^2, \tag{4.18}$$

  where superscript p stands for a copy protection policy.

- Low price equilibrium:

$$p^{p,L} = 2\eta \quad \text{and} \quad \pi^{p,L} = 4\eta^2. \tag{4.19}$$

- $$\pi^{p,H} \ge \pi^{p,L} \quad \text{if and only if} \quad \sigma \ge 3.$$

  Therefore, if software is protected, the monopoly's price and profit levels are

$$p^p = \begin{cases} (1+\sigma)\eta & \text{if } \sigma \ge 3 \\ 2\eta & \text{if } \sigma < 3, \end{cases} \text{ and } \pi^p = \begin{cases} (1+\sigma)\eta^2 & \text{if } \sigma \ge 3 \\ 4\eta^2 & \text{if } \sigma < 3. \end{cases} \tag{4.20}$$

4.4.3 Should the software firm choose to protect its software?

Result 4.2 *When software users' preferences exhibit network externalities,*

(a) no copy protection yields a higher profit than copy protection if support-oriented consumers place a high value on service offered by the software firm to its legal users, i.e., when $\sigma \geq 2$*;*

(b) copy protection yields a higher profit than no copy protection when support-oriented consumers place a low value for service, $\sigma < 2$*.*

Topic 5

Technology Replacement

5.1 A simple Standardization Game

Consider a technology-adoption game played by two users (or firms) displayed in Table 5.1.

		User ***B***			
		New Technology		Old Technology	
User *A*	New	α	α	γ	δ
	Old	δ	γ	β	β

Table 5.1: The static new technology adoption game

- Assumption: Both users exhibit network externalities for both technologies. Formally, in terms of Table 5.1, we assume that $\alpha > \delta$ and $\beta > \gamma$.
- That is, using the same technology as the other user yields a higher utility (or profit) than using any technology alone.
- There exist two Nash equilibria for the static technology adoption game displayed in Table 5.1 given by (New, New) and (Old, Old).
- The existence of multiple equilibria in this game raises the question how the two firms coordinate their actions?
- Farrell and Saloner (1985) provided the following terminology for two commonly observed *market failures.*

Definition 5.1 *(a) If (*Old, Old*) is the played Nash equilibrium outcome, and if the outcome (*New, New*) Pareto dominates the outcome (*Old, Old*), then we call this situation* ***excess inertia****.*

*(b) If (*New, New*) is the played Nash equilibrium outcome, and if the outcome (*Old, Old*) Pareto dominates the outcome (*New, New*), then we call this situation* ***excess momentum****.*

- *excess momentum* occurs when a new technology replaces an old technology, but the old technology yields a higher utility (or profit) to both users than the new technology.
- *excess inertia* occurs when a new technology yields a higher utility (profit) to both users, however, in equilibrium all users stay with the old technology.
- Using the example displayed in Table 5.1, if $\beta > \alpha$ and if (New, New) is played, then we have excess momentum

- if $\beta < \alpha$ and if (Old, Old) is played, then we have excess inertia
- Operating systems present an example of excess inertia (Windows)
- The 1982 Reagan's deregulation of car bumpers present an example of excess momentum

5.2 Preferences and Technology Advance

- Related Reference: Shy (1996).[1]
- A discrete time overlapping generations (OLG) economy,
- in each period t, $t = 1, 2, \ldots$, there are N_t young consumers of generation $\tau = t$, and N_{t-1} old consumers of generation $\tau = t - 1$
- Denote by T_t, ($T_t > 0$, $t = 1, 2, 3, \ldots$) the period t quality of the (potential) state-of-the-art technology, and assume that T_t is *exogenously* given and is strictly increasing over time, (i.e., $T_t > T_{t-1}$ for every t)
- Assume $T_t = \lambda t$, where $\lambda > 0$ can be interpreted as the quality advance parameter.
- Denote by V_t the *actual* quality level (stand-alone value) of the period t technology embodied into the product to a period t young who purchases this product
- Hence, $V_t \le T_t$ for all t.
- The actual technological quality consumed by the young consumers in period t is given by

$$V_t = \begin{cases} T_t & \text{if the young at } t \text{ adopt the new technology} \\ V_{t-1} & \text{otherwise.} \end{cases} \tag{5.1}$$

- Investment in the context of this paper means spending resources on converting the state-of-the-art technology into actual production. Thus, it is always possible to catch up with the latest technology.
- Let c ($0 < c < 1$) denote the degree of compatibility between new and old technologies.
- To each newly adopted technology we attach a "serial" number denoted by g, $g = 1, 2, \ldots$.
- The utility of a young of generation τ is given by

$$U^\tau = \begin{cases} u(T_\tau, cN_{\tau-1} + N_\tau) - p_\tau^g & \text{if generation } \tau \text{ buys generation } g \text{ technology product} \\ \\ u(V_{t_{g-1}}, N_{\tau-1} + N_\tau) - p_\tau^{g-1} & \text{if generation } \tau \text{ buys generation } g-1 \text{ technology product.} \end{cases} \tag{5.2}$$

$u(,)$ is monotonically increasing in both arguments

[1]For an infinite-horizon technology-replacement growth model see: Chou, C., and O. Shy. 1993. "Technology Revolutions and the Gestation of New Technologies." *International Economic Review* 34(3): 631–45.

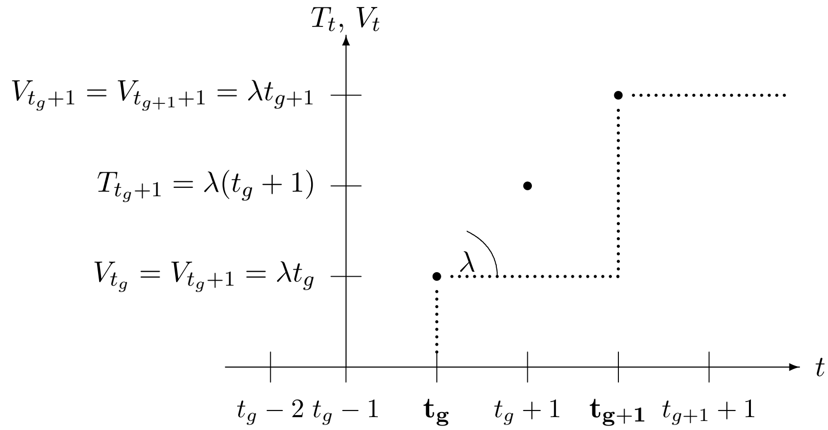

Figure 5.1: Exogenous technology development path (T_t), and the actually adopted technology path (V_t).

- generation $t = \tau$ young consumers would choose to purchase the new technology product if and only if
$$u(T_{t_g}, cN_{\tau-1} + N_\tau) - p^g_\tau \geq u(V_{t_{g-1}}, N_{\tau-1} + N_\tau) - p^{g-1}_\tau. \tag{5.3}$$

- A monopoly would find it profitable to introduce a new technology in period t if and only if
$$u(T_{t_g}, cN_{\tau-1} + N_\tau) - u(V_{t_{g-1}}, N_{\tau-1} + N_\tau) \geq 0. \tag{5.4}$$

- Approximate the duration of generation g technology by a real number d_g, where d_g is determined by the implicit condition
$$u(T_{t_g+d_g}, cN_{t_g+d_g-1} + N_{t_g+d_g}) = u(T_{t_g}, N_{t_g+d_g-1} + N_{t_g+d_g}). \tag{5.5}$$

- Definition 5.2 *(a) The* **duration** *of generation g technology, denoted by Δ_g, is defined by*
$$\Delta_g \stackrel{def}{=} \lceil d_g \rceil$$
where $\lceil d_g \rceil$ is the smallest integer greater or equal to d_g.
(b) The **frequency** *of introduction of new technologies evaluated at the introduction date of generation $g + 1$, denoted by f_g, is defined by $f_g \stackrel{def}{=} 1/\Delta_g$.*

- Thus, if new technologies are 100-percent compatible with old technologies, then new technologies are adopted each period

- Formally, if $c = 1$, then $\Delta_g = f_g = 1$ for all $g = 1, 2, \dots$

5.2.1 An example for the case of complements

To simplify the exposition of the next two examples, set $c = 0$ (new technologies are always incompatible with old technologies); and ignore prices by setting $p^g_\tau = 0$ for all t and g.

$$U^\tau = \begin{cases} \min\{V_{t_{g-1}}; N_{\tau-1} + N_\tau\} & \text{if generation } \tau \text{ purchases an old technology product} \\ \\ \min\{T_{t_g}; N_\tau\} & \text{if generation } \tau \text{ purchases the new technology product.} \end{cases} \tag{5.6}$$

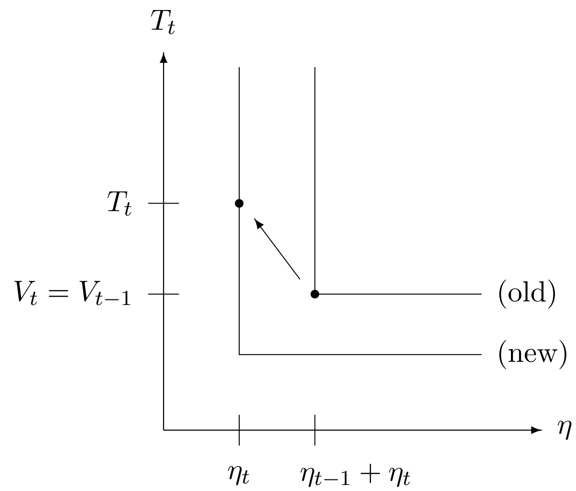

Figure 5.2: Perfect complements

In this example, the economy stagnates in the sense that new technologies are never adopted.

5.2.2 An example for the case of substitutes

$$U^\tau = \begin{cases} V_{t_{g-1}} + N_{\tau-1} + N_\tau & \text{if generation } \tau \text{ purchases an old technology product} \\ \\ T_{t_g} + N_\tau & \text{if generation } \tau \text{ purchases the new technology product.} \end{cases} \tag{5.7}$$

In this example, the economy does not stagnate for a long period because as T_t keeps growing which would eventually move new users to a higher indifference curve.

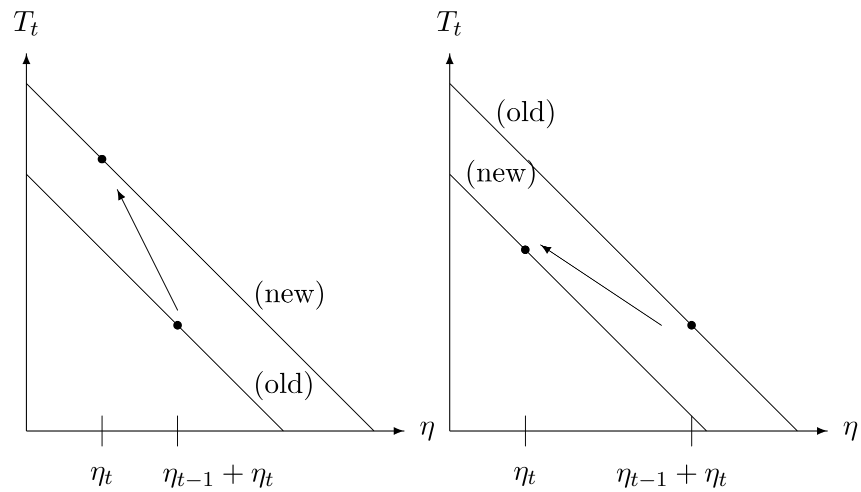

Figure 5.3: Perfect substitutes

Topic 6
Switching Costs

Related reference: Shy (2002)[1]

- Markets for durable goods & services: Consumers don't switch brands very often
- This observation holds for private consumption & commercial
- Examples: Operating systems, keyboards, airline, aircrafts, banks, HMOs, travel agencies, lawyers, phone service
- Why is it so? Answer: invested human capital!
- Switching costs result in *lock-in*
- Problem: Switching costs are not observable
- Why?
  1. involves invested human capital
  2. consumer specific (individual human-capital needs)
- Hence, nonobservability of switching costs implies that we need a model that uses *easily-observed* brand prices & market shares
- Classifications of lock-in and switching costs (see Shapiro & Varian (1999, Ch.5)

  Conctratual Commitments: Compensatory & liquidated damages

  Brand-specific training Learning new systems: direct and lost productivity

  Information databases: Converting data files to a new format

  Search costs: Learning quality of alternatives

  Loyality programs: Lost benefits

A Simple Method for Estimating Switching Costs

- Two firms, two brands: A and B
- N_α consumers are A-users;
  N_β consumers are B-users
- prices: p_A and p_B. Switching Cost: S

[1]Original formulation is Klemperer (1987)

Utility from next purchase:

$$U_\alpha \stackrel{\text{def}}{=} \begin{cases} -p_A & \text{staying with brand } A \\ -p_B - S & \text{switching to brand } B \end{cases}$$

$$U_\beta \stackrel{\text{def}}{=} \begin{cases} -p_A - S & \text{switching to brand } A \\ -p_B & \text{staying with brand } B. \end{cases}$$

Resulting demand functions for the brands:

$$n_A = \begin{cases} 0 & \text{if } p_A > p_B + S \\ N_A & \text{if } p_B - S \le p_A \le p_B + S \\ N_A + N_B & \text{if } p_A < p_B - S \end{cases}$$

$$n_B = \begin{cases} 0 & \text{if } p_B > p_A + S \\ N_B & \text{if } p_A - S \le p_B \le p_A + S \\ N_A + N_B & \text{if } p_B < p_A - S. \end{cases}$$

Firms maximize:

$$\pi_A(p_A, p_B) = p_A n_A \text{ and } \pi_B(p_A, p_B) = p_B n_B$$

Definition 6.1 *firm i is said to **undercut** firm j, if it sets its price to $p_i < p_j - S$, $i = A, B$ and $i \neq j$. That is, if firm i 'subsidizes' the switching cost of firm j's customers.*

Definition 6.2 *A pair of prices $\langle p_A^U, p_B^U \rangle$ is said to satisfy the **Undercutproof Property (UPP)** if*

(a) For given p_B^U and n_B^U, firm A chooses the highest price p_A^U subject to

$$\pi_B^U = p_B^U n_B^U \ge (p_A - S)(N_A + N_B).$$

(b) For given p_A^U and n_A^U, firm B chooses the highest price p_B^U subject to

$$\pi_A^U = p_A^U n_A^U \ge (p_B - S)(N_A + N_B).$$

(c) The distribution of consumers between the firms is determined by the demand functions.

$$p_A^U = \frac{(N_A + N_B)(N_A + 2N_B)S}{(N_A)^2 + N_A N_B + (N_B)^2}$$

$$p_B^U = \frac{(N_A + N_B)(2N_A + N_B)S}{(N_A)^2 + N_A N_B + (N_B)^2}$$

$$\Delta p^U \stackrel{\text{def}}{=} p_B^U - p_A^U = \frac{[(N_A)^2 - (N_B)^2]S}{(N_A)^2 + N_A N_B + (N_B)^2}$$

$$\Delta \pi^U \stackrel{\text{def}}{=} \pi_B^U - \pi_A^U = \frac{(N_A + N_B)^2 (N_B - N_A)S}{(N_A)^2 + N_A N_B + (N_B)^2}.$$

Conclusions:

1. Firm serving more customers charges a lower price
2. But, earns a higher profit

Company's Name:	Pelephone	Cellcom
Profit (mil.NIS):	$\pi_p = 283.5$	$\pi_c = 310$
Sales (mil.NIS):	$R_p = 2400$	$R_c = 2370$
Subscribers (mil.):	$N_p = 1$	$N_c = 1.15$
Price (Sales/Subs):	$p_p = 2400$	$p_c = 2061$

Yielding

$$S_p = \frac{(N_p + N_c)p_p - N_c p_c}{N_p + N_c} = 1298$$

$$S_c = \frac{(N_c + N_p)p_c - N_p p_p}{N_c + N_p} = 945$$

Topic 7

Telecommunication

- The telecommunication industry is the fastest growing industry in almost every country.
- Major technology advances in the telephony industry in general, and in the wireless technology in particular, as well as technology advance of the
- The Internet contributed the most for the fast growth of this industry.
- Telecommunication services constitute the most natural example of network externalities, since by definition, the nature of these services involves communicating with a large number of people.

7.1 The interdependent demand for communication services

- Related Reference: Rohlfs (1974)
- $\eta > 0$ potential phone users indexed by x on the unit interval $[0, 1]$.
- low x as those who love to subscribe to a phone system (high willingness to pay)
- n, $0 \le n \le 1$ the total number of consumers who actually subscribe to the phone system
- p the price of subscribing to the phone system.

$$U^x \stackrel{\text{def}}{=} \begin{cases} n(1-x) - p & \text{if he or she subscribes to the phone system} \\ 0 & \text{if he or she does not subscribe.} \end{cases} \quad x \in [0, 1]. \tag{7.1}$$

Let consumer $\hat{x}$ be, at a given price p, indifferent to the alternatives of subscribing to the phone system and not subscribing.

$$0 = n(1 - \hat{x}) - p.$$

Since the number of consumers is given by $n = \eta\hat{x}$,

$$0 = \eta\hat{x}(1 - \hat{x}) - p \quad \text{or} \quad p = \eta\hat{x}(1 - \hat{x}), \tag{7.2}$$

which is drawn in Figure 7.1. The point $\eta\hat{x}_0^L$ is defined in the literature as the *critical mass.*

7.1.1 The problem of the monopoly phone company

One monopoly firm, marginal cost of adding a subscriber is negligible, PTT's profit-maximization problem, choose $\hat{x}$

$$\max_{\hat{x}} \pi(\hat{x}) \stackrel{\text{def}}{=} p(\hat{x})\eta\hat{x} = \eta\hat{x}(1 - \hat{x})\eta\hat{x} = \eta^2(\hat{x})^2(1 - \hat{x}). \tag{7.3}$$

The profit function (7.3) is drawn in Figure 7.1.1.

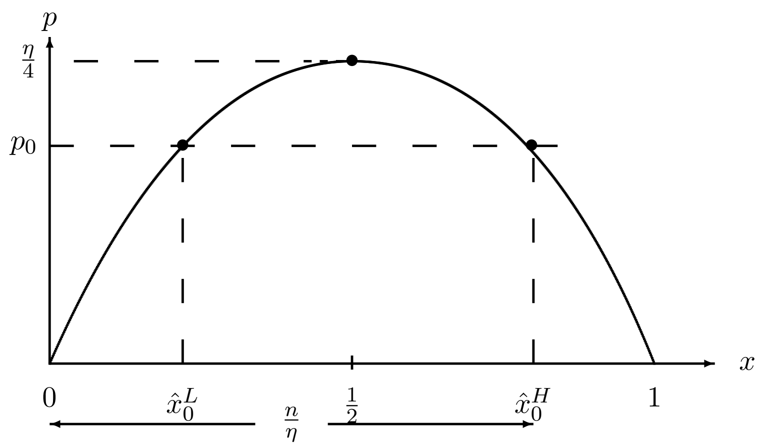

Figure 7.1: Deriving the demand for telecommunication services

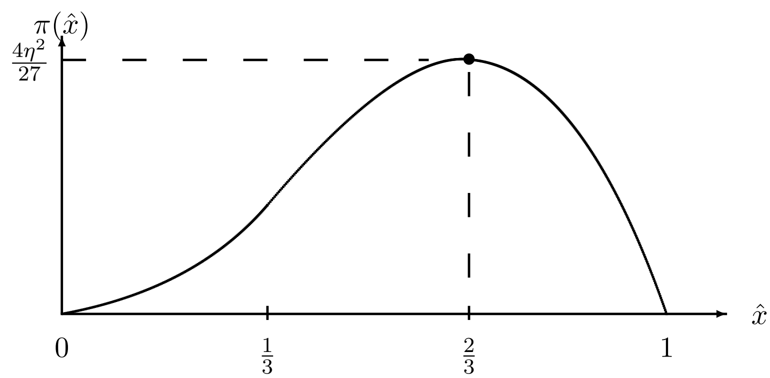

Figure 7.2: The PTT profit function in the presence of network externalities

$$0 = \frac{\partial \pi}{\partial x} = \eta^2(2x - 3x^2) \quad \text{and} \quad \frac{\partial^2 \pi}{\partial x^2} = \eta^2(2 - 6x). \tag{7.4}$$

$\hat{x} = 2/3$ is a global maximum point.
To obtain the connection fee and the resulting profit:

$$p = \eta(1 - \hat{x})\hat{x} = \frac{2\eta}{9}, \quad \text{and} \quad \pi = \eta^2(1 - \hat{x})(\hat{x})^2 = \frac{4\eta^2}{27}. \tag{7.5}$$

Hence,

Result 7.1 *A monopoly phone company maximizes its profit by setting its connection fee so that the number of customers exceeds half of the consumer population but is less than the entire population.*

7.1.2 Critical mass

After constructing the demand curve we wish to define a concept which telecommunication firms find very useful when marketing a new telecommunication service.

DEFINITION 7.1 *Let p_0 be a given connection fee for this service. The a* ***critical mass*** *at a price (connection fee) p_0 is the minimal number of customers needed to ensure that at least this number of consumers will benefit from subscribing to the service at the fee p_0.*

- The "low demand" point $\hat{x}_0^L$ has a special characteristic (Definition 7.1). Figure 7.1 shows that at p_0 the critical mass is $\eta\hat{x}_0^L$ customers.
- In telecommunication the *critical mass is always a function of the market price*, meaning that a rise in price would imply an increase in the critical mass and a decrease in the market price will decrease the critical mass as since at a lower price customers would be "satisfied" with a reduced network size.
- Social life interpretation: in order to organize a party or a trip during the weekend, the organizer has to convince the potential participants that a certain minimum number of people would indeed attend this party, which would then imply that even a greater number will join due to the increasing network effects.

7.1.3 Entry of new firms into the telecommunication industry

- The entrant can potentially hook all those $(1 - 2/3)\eta$ potential users who are not connected to the system via the incumbent firm.
- Figure 7.3 demonstrates how the residual demand facing the entrant is constructed by subtracting the 2/3 of the consumer population who have already connected via the incumbent firm.

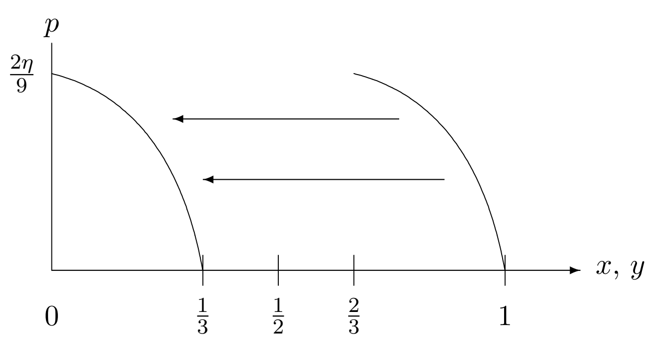

Figure 7.3: Residual demand for telecommunication connection facing the entrant.

- Inverting the inverse-demand curve to obtain the "indifferent" type as a function of the connection fee, and then subtracting the 2/3 consumers who are already connected

$$\hat{y} \stackrel{\text{def}}{=} \hat{x} - \frac{2}{3} = \frac{\eta + \sqrt{\eta(\eta - 4p_0)}}{2\eta} - \frac{2}{3}. \tag{7.6}$$

- Inverting (7.6), we obtain the residual-inverse-demand facing the entrant, and the implied profit function of the entrant,

$$p = \frac{\eta(2 - 3\hat{y} - 9\hat{y}^2)}{9}, \quad \text{hence} \quad \pi = \frac{\eta(2 - 3\hat{y} - 9\hat{y}^2)}{9}\,\eta\hat{y}. \tag{7.7}$$

  The first- and second-order condition for profit maximization are

$$0 = \frac{\mathrm{d}\pi}{\mathrm{d}\hat{y}} = \frac{\eta^2(2 - 6\hat{y} - 27\hat{y}^2)}{9}, \quad \text{and} \quad 0 > -\frac{2\eta^2(9\hat{y} + 1)}{3},$$

  which holds for all nonnegative values of $\hat{y}$.

- Extracting the positive root of the first-order condition, we obtain the consumer type who is indifferent between connecting to the entrant's services, or staying disconnected.

- Then, substituting into (7.7) we obtain the entrant's connection fee and profit level.

- Altogether, we have that

$$\hat{y} = \frac{\sqrt{7} - 1}{9} \approx 0.182, \quad p = \frac{\eta(23 - \sqrt{7})}{81} \approx 0.128, \quad \pi = \eta^2\frac{14\sqrt{7} - 20}{729}. \tag{7.8}$$

- Labeling the entrant's variables with a superscript "E" and the incumbent's variable by a superscript "I" and comparing (7.8) with (7.5) implies that $p^E \approx 0.128 < 0.222 \approx p^I$ and $\pi^E \approx 0.023\eta^2 < 0.148\eta^2 \approx \pi^I$.

- The entrant charges a lower connection fee and earns a lower profit than the incumbent.

- Telecommunication provider serves all consumer types indexed on $[0, 2/3]$ and the entrant serves all consumers indexed on $[2/3, 2/3 + \hat{y}] = [2/3, (\sqrt{7} + 5)/9] \approx [0.67, 0.85]$.

- 

Figure 7.4: Division of market shares after entry into the telecommunication market.

  Figure 7.4 shows that the despite the 67% market share captured by the incumbent during the monopoly era, the entrant can capture about 18% of the market.

- Clearly, we can now allow for a third entrant which will further reduce the connection fee.

- Who benefits from entry into the telecommunication industry?

Result 7.2 *Entry into the telecommunication industry increases the utility of old and newly connected consumers, as well as the profit of the entering firm.*

The proposition follows from that fact that old users gain because of the increase in the network size; new users gain because now they are connected to this service; and the entering firm makes above normal profit.

Topic 8

Social Interaction

- Related reference: Grilo, I., O. Shy, and J. Thisse. 2001. "Price Competition when Consumer Behavior is Characterized by Conformity or Vanity." *Journal of Public Economics.*

- Here, we formulate it for linear transportation cost (quadratic cost is needed for the vanity case only, not solved here!)

- Location of 2 stores:

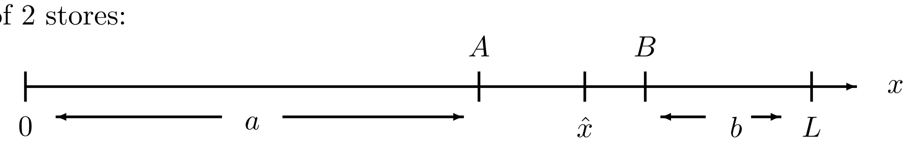

Figure 8.1: Hotelling's linear city model

- There is a continuum of N consumers uniformly distributed on the interval $[0, L]$

- n_A denotes the number of consumers buying from store A and n_B the number of consumers buying from B

- Each consumer buys 1 unit, so $n_A + n_B = N$.

- The utility of consumer x, $x \in [0, L]$

$$U_x \stackrel{\text{def}}{=} \begin{cases} \alpha n_A - p_A - \tau|x - a| & \text{when buying from } A \\ \\ \alpha n_B - p_B - \tau|L - b - x| & \text{when buying from } B. \end{cases} \tag{8.1}$$

  where α (which can be positive or negative) measures the intensity of network effects

- Note that for $\alpha = 0$, the model reduces to the standard Hotelling model

- Definition 8.1 *Let $\beta \stackrel{def}{=} \alpha N/L$. Consumer preferences are said to exhibit*

  - **negative network (vanity) effects** *if $\beta < 0$,*
  - **weakly positive network (conformity) effects** *if $0 < \beta < \tau$,*
  - **strong positive network (conformity) effects** *if $\beta > \tau$.*

- $\hat{x}$ is the consumer so that all consumers indexed by $x \in [0, \hat{x}]$ purchase from A and all consumers indexed by $x \in (\hat{x}, L]$ buy from B

- $n_A = \hat{x}N/L$ and $n_B = (L - \hat{x})N/L$.

$$\frac{\alpha N\hat{x}}{L} - p_A - \tau(\hat{x} - a) = \frac{\alpha(L - \hat{x})N}{L} - p_B - \tau(L - b - \hat{x}).$$

Setting $\beta = \alpha N/L$ and solving for $\hat{x}$ yields

$$\hat{x}(p_A, p_B) = \begin{cases} 0 & \text{if } p_A - p_B > \tau(L - b + a) - \beta L \equiv \phi_H \\ L & \text{if } p_A - p_B < -\tau(L + b - a) + \beta L \equiv \phi_L \\ \dfrac{p_B - p_A}{2(\tau - \beta)} + \dfrac{\tau(L - b + a)}{2(\tau - \beta)} - \dfrac{\beta L}{2(\tau - \beta)} & \text{if } \phi_L \le p_A - p_B \le \phi_H. \end{cases} \tag{8.2}$$

- Definition 8.2 *An equilibrium is a pair* (p^h_A, p^h_B), *such that, given* p^h_B, p^h_A *solves* $\max_{p_A} \pi_A \equiv p_A\hat{x}(p_A, p^h_B)$ *and, given* p^h_A, p^h_B *solves* $\max_{p_B} \pi_B \equiv p_B[L - \hat{x}(p^h_A, p_B)]$; *where* $\hat{x}(p_A, p_B)$ *is given in (8.2).*

- Best-response functions

$$p_A = R_A(p_B) = \frac{1}{2}[p_B + \tau(L - b + a) - \beta L] \text{ and } p_B = R_B(p_A) = \frac{1}{2}[p_A + \tau(L + b - a) - \beta L]. \tag{8.3}$$

They are drawn in Figure 8.2.

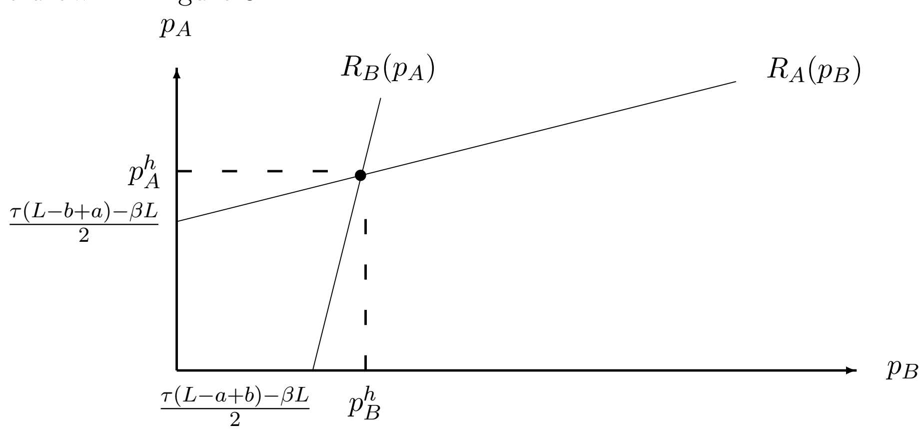

Figure 8.2: Negative and weakly positive network effects: best-response functions ($a > b$)

- solving (8.3) yields

$$p^h_A = \frac{\tau(3L + a - b)}{3} - \beta L \quad \text{and} \quad p^h_B = \frac{\tau(3L - a + b)}{3} - \beta L. \tag{8.4}$$

Substituting into (8.2) yields the market share of each store:

$$\hat{x} = \frac{L}{2} + \frac{\tau(a - b)}{6(\tau - \beta)} \quad \text{and} \quad L - \hat{x} = \frac{L}{2} + \frac{\tau(b - a)}{6(\tau - \beta)}. \tag{8.5}$$

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Recalling that $n_A = \hat{x}N/L$ and $n_B = (L - \hat{x})N/L$, the profit of each store is given by

$$\pi_A^h = p_A^h n_A = \frac{N[\tau(3L + a - b) - 3\beta L]^2}{18L(\tau - \beta)} \quad \text{and} \quad \pi_B^h = p_B^h n_B = \frac{N[\tau(3L + b - a) - 3\beta L]^2}{18L(\tau - \beta)}. \tag{8.6}$$

Result 8.1

1. *Suppose that consumer preferences exhibit negative network effects. Then, both equilibrium prices p_A^h and p_B^h increase when*
   - (a) *negative network effects become more significant in consumers' preferences (α becomes more negative);*
   - (b) *there are more consumers per unit of area (N increases).*
2. *Suppose that consumers' preferences exhibit weakly positive network effects. Then, both equilibrium prices p_A^h and p_B^h are reduced when*
   - (a) *network effects become more significant in consumers' preferences (α increases),*
   - (b) *there are more consumers per unit of area (N increases).*

Topic 9
References

There is *no* intention to survey the literature in these short notes, and therefore there is no attempt to provide the student with comprehensive reading lists. For the most-comprehensive reading list known to me see Nicholas Economides' webpage: `http://raven.stern.nyu.edu/networks/`

9.1 Selected Literature for this Course

The following list is more-or-less related to the lecture notes. I don't claim that this list is representative, unbiased, or whatever. To the contrary, this list reflects my way of thinking about network industries and network behavior.

Chou, C., and O. Shy. 1990. "Network Effects without Network Externalities." *International Journal of Industrial Organization* 8(2): 259–270.

Chou, C., and O. Shy. 1993. "Partial Compatibility and Supporting Services." *Economics Letters* 41(2): 193–197.

Farrell, J., and G. Saloner. 1986. "Standardization and Variety." *Economics Letters* 20: 71–74.

Grilo, I., O. Shy, and J. Thisse. 2001. "Price Competition When Consumer Behavior is Characterized by Conformity and Vanity" *Journal of Public Economics*, 80(3):385–408

Katz, M., and C. Shapiro. 1985. "Network Externalities, Competition, and Compatibility." *American Economic Review* 75: 424–440.

Klemperer, P. 1987. "The Competitiveness of Markets with Switching Costs," *RAND Journal of Economics* 18(1): 138–150.

Laffont, J-J., P. Ray, and Tirole, J. 1998. "Network Competition: I. Overview and Nondiscriminatory Pricing." *RAND Journal of Economics* 29: 1–37.

Matutes, C., and P. Regibeau. 1988. "Mix and Match: Product Compatibility Without Network Externalities." *RAND Journal of Economics* 19: 221–234.

Rohlfs, J. 1974. "A Theory of Interdependent Demand for Communication Service." *Bell Journal of Economics* 5: 16–37.

Shy, O. 1996. "Technology Revolutions in the Presence of Network Externalities." *International Journal of Industrial Organization* 14(6): 785–800.

Shy, O. 2002. "A Quick-and-Easy Method for Estimating Switching Costs." *International Journal of Industrial Organization*, 20(1): 71–87.

Shy, O., and J. Thisse. 1999. "A Strategic Approach to Software Protection." *Journal of Economics & Management Strategy* 8(2): 163–190.

9.2 Literature Survey Articles and Relevant Books

Besen, S., and J. Farrell. 1994. "Choosing How to Compete: Strategies and Tactics in Standardization." *Journal of Economic Perspectives* 2: 117–131.

David, P., and S. Greenstein. 1990. "The Economics of Compatibility Standards: An Introduction to Recent Research." *Economics of Innovation and New Technology* 1: 3–41.

Economides, N. 1996. "The Economics of Networks." *International Journal of Industrial Organization* 14: 673–699.

Farrell, J., and G. Saloner. 1987. "The Economics of Horses, Penguins, and Lemmings." In *Production Standardization and Competitive Strategies,* edited by L. G. Gable. Amsterdam: North-Holland.

Gilbert, R. 1992. "Symposium on Compatibility." *Journal of Industrial Economics* 40: 1–8.

Katz, M., and C. Shapiro. 1994. "Systems Competition and Network Effects." *Journal of Economic Perspectives* 2: 93–115.

Leibowitz, S., and S. Margolis. 1994. "Network Externality: An Uncommon Tragedy." *Journal of Economic Perspectives* 2: 133–150.

Shapiro, C., and H. Varian. 1999. *Information Rules: A Strategic Guide to the Network Economy.* Boston: Harvard Business School Press.

Shy, O. 2001. *The Economics of Network Industries.* Cambridge: Cambridge University Press.

Remark: The Leibowitz & Margolis paper is a criticism of the literature, and is worth studying.