
(1a) [14 points] Since $p^A = \$40 > \$10 = S$ and $p^B = \$20 > \$10 = S$, the firm should accept any booking request. Formally, $d_4(\$40) = d_4(\$20) = 1$, provided that $k_4 = 1$. Therefore, the $t = 4$ expected value of capacity is

$$\mathsf{E}V_4(k_4) = \begin{cases} 0.4 \times 10 + 0.1 \times 40 + 0.5 \times 20 = \$18 & \text{if } k_4 \neq 0 \\ 0 & \text{if } k_2 = 0. \end{cases}$$

Working backwards, the period $t = 3$ booking decision rule is

$$d_3(P_3) = \begin{cases} 1 & \text{if } P_3 \geq \$18 \\ 0 & \text{otherwise} \end{cases} \quad \text{hence} \quad d_3(P_3) = \begin{cases} 1 & \text{if } P_3 = \$40 \\ 1 & \text{if } P_3 = \$20. \end{cases}$$

Thus, period $t = 3$ value of $k_3 = 1$ unit of capacity is

$$\mathsf{E}V_3(1) = 0.4 \times \mathsf{E}V_4(1) + 0.1 \times 40 + 0.5 \times 20 = 0.4 \times 18 + 4 + 10 = \$21.2.$$

(1b) [6 points]

$$\Pr\{\text{no bookings are made in } t = 3 \text{ and } t = 4\} = 0.4 \times 0.4 = 0.16.$$

$$\Pr\{A \text{ is booked in either } t = 3 \text{ or } t = 4\} = 0.1 + 0.4 \times 0.1 = 0.14.$$

$$\Pr\{B \text{ is booked in either } t = 3 \text{ or } t = 4\} = 0.5 + 0.4 \times 0.5 = 0.7.$$

Remark: Note that $0.16 + 0.14 + 0.7 = 1$.

(1c) [6 points] Working backwards, the period $t = 2$ booking decision rule is

$$d_2(P_2) = \begin{cases} 1 & \text{if } P_2 \geq \$21.2 \\ 0 & \text{otherwise} \end{cases} \quad \text{hence} \quad d_2(P_2) = \begin{cases} 1 & \text{if } P_2 = \$40 \\ 0 & \text{if } P_2 = \$20. \end{cases}$$

Thus, period $t = 2$ value of $k_2 = 1$ unit of capacity is

$$\mathsf{E}V_2(1) = (0.4 + 0.5) \times \mathsf{E}V_3(1) + 0.1 \times 40 = 0.9 \times 21.2 + 4 = \$23.08.$$

Working backwards, the period $t = 1$ booking decision rule is

$$d_1(P_1) = \begin{cases} 1 & \text{if } P_1 \geq \$23.8 \\ 0 & \text{otherwise} \end{cases} \quad \text{hence} \quad d_1(P_1) = \begin{cases} 1 & \text{if } P_2 = \$40 \\ 0 & \text{if } P_2 = \$20. \end{cases}$$

Thus, period $t = 1$ value of $k_1 = 1$ unit of capacity is

$$\mathsf{E}V_1(1) = (0.4 + 0.5) \times \mathsf{E}V_2(1) + 0.1 \times 40 = 0.9 \times 23.8 + 4 = \$24.772.$$

(1d) [6 points]

$$\Pr\{\text{no bookings are made in } t = 1, 2, 3, 4\} = 0.9 \times 0.9 \times 0.4 \times 0.4 = 0.1296.$$

$$\Pr\{A \text{ is booked in either } t = 1, 2, 3 \text{ or } 4\} = \underbrace{0.1}_{t=1} + \underbrace{0.9 \times 0.1}_{t=2} + \underbrace{(0.9)^2 \times 0.1}_{t=3} + \underbrace{(0.9)^2 \times 0.4 \times 0.1}_{t=4} = 0.3034.$$

$$\Pr\{B \text{ is booked in either } t = 1, 2, 3 \text{ or } 4\} = \underbrace{0}_{t=1} + \underbrace{0}_{t=2} + \underbrace{0.9^2 \times 0.5}_{t=3} + \underbrace{0.9^2 \times 0.4 \times 0.5}_{t=4} = 0.567.$$

Remark: Note that $0.1296 + 0.3034 + 0.567 = 1$.

(2a) [6 points] $\mathsf{E}y(1,0) = (1/3)18 = \$6$. $\mathsf{E}y(0,1) = (2/3)9 = \$6$. Therefore, $\langle K_A, K_B \rangle = \langle 1, 0 \rangle$ and $\langle K_A, K_B \rangle = \langle 0, 1 \rangle$ generate the same level of expected profit.

(2a) [14 points]

$$\mathsf{E}y(1,0) = \underbrace{\frac{1}{3}\frac{1}{3}}_{\text{prob}\{AA\}} 18 + \underbrace{\frac{1}{3}\frac{2}{3}}_{\text{prob}\{AB\}} 18 + \underbrace{\frac{2}{3}\frac{1}{3}}_{\text{prob}\{BA\}} 18 + \underbrace{\frac{2}{3}\frac{2}{3}}_{\text{prob}\{BB\}} 0 = 2 + 4 + 4 + 0 = \$10.$$

$$\mathsf{E}y(0,1) = \underbrace{\frac{1}{3}\frac{1}{3}}_{\text{prob}\{AA\}} 0 + \underbrace{\frac{1}{3}\frac{2}{3}}_{\text{prob}\{AB\}} 9 + \underbrace{\frac{2}{3}\frac{1}{3}}_{\text{prob}\{BA\}} 9 + \underbrace{\frac{2}{3}\frac{2}{3}}_{\text{prob}\{BB\}} 9 = 0 + 2 + 2 + 4 = \$8.$$

Therefore, setting capacity according to $\langle K_A, K_B \rangle = \langle 1, 0 \rangle$ is more profitable.

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(3) [20 points] We first calculate the minimum refund levels that would induce each type of passengers to book a flight with Air Flint.

$$U_S = 0.9 \cdot 20 - 19 + 0.1r \geq 0 \quad \Longleftrightarrow \quad r_S \geq 10.$$

$$U_B = 0.3 \cdot 40 - 19 + 0.7r \geq 0 \quad \Longleftrightarrow \quad r_B \geq 10.$$

Thus, for refund levels $r < 0$ no one would book a flight. For $r \geq 10$, both passenger types book. At this level, Air Flint's expected profit is

$$\begin{aligned}\mathsf{E}y(r = 10) &= (100 + 200)(P - \mu_k) - (0.9 \cdot 100 + 0.3 \times 200)\mu_O - (0.1 \times 100 + 0.7 \times 200)r - \phi \\ &= (100 + 200)(19 - 2) - (0.9 \cdot 100 + 0.3 \times 200)2 - (0.1 \times 100 + 0.7 \times 200)10 - 3000 = \$300.\end{aligned}$$

(4a) [15 points] If only one consumer is booked,

$$\mathsf{E}y(b=1) = \pi(P-\mu_O) = \frac{2}{3}(9-5) = \frac{8}{3} \approx \$2.66$$

If two consumers are booked,

$$\mathsf{E}y(b=2) = \underbrace{\pi(1-\pi)(P-\mu_O) + (1-\pi)\pi(P-\mu_O)}_{\text{1 consumer shows up}} + \underbrace{\pi^2(P-\mu_O-\psi)}_{\text{2 consumers show up}}$$
$$= 2\times\frac{2}{3}(9-5) + \left(\frac{2}{3}\right)^2(9-5-2) = \frac{8}{3} \approx \$2.66$$

Therefore, Ann Arbor Massage (AAM) earns the same expected profit when it books $b=1$ and $b=2$ consumers.

(4b) [5 points] Expected penalty cost of booking $b=3$ consumers is:

$$\underbrace{\Pr\{s(3)=2\}\psi}_{\text{1 denied service}} + \underbrace{\Pr\{s(3)=3\}2\psi}_{\text{2 denied service}} = 3\pi^2(1-\pi)\psi + \pi^3 2\psi$$
$$= 3\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)2 + \left(\frac{2}{3}\right)^3 2\times 2 = \frac{56}{27} \approx \$2.07$$

(4c) [5 points] Expected penalty cost of booking $b=4$ consumers is:

$$\underbrace{\left(\frac{2}{3}\right)^4(4-1)2}_{\text{3 denied service}} + \underbrace{\frac{4!}{3!1!}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)(3-1)2}_{\text{2 denied service}} + \underbrace{\frac{4!}{2!2!}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2(2-1)2}_{\text{1 denied service}} = \frac{272}{81} \approx \$3.35.$$

THE END