Econ 403: Midterm Examination — February 18, 2009

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Last Name (Please PRINT): ..........................................

First Name (PRINT): ..............................................

Your UM I.D. Number: ............................................

INSTRUCTIONS (please read!)

1. Please make sure that you have 6 pages, including this page. Complaints about missing pages will not be accepted.
2. Please answer all the questions. You are <u>not</u> allowed to use any course material. Calculators are permitted.
3. Maximum Time Allowed: 1 hour and 20 minutes (10:10–11:30).
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is "visible" and that your sentences are properly organized and fit into the designated blank space. If you think that your handwriting is poor, please print each word!
10. Good Luck !

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Instructor's use only

| Problem # | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| Maximum | 40 | 20 | 20 | 20 | 100 |
| Points | | | | | |

**(1)** A service provider has one unit of capacity (capable of booking at most one consumer). Formally, $K = 1$. This provider offers two service classes ($A$ and $B$). The booking prices and the probability of booking each class are given in the table below.

| Class ($i$) | 0 | $A$ | $B$ | $S$ |
|---|---|---|---|---|
| Proportion ($\pi^i$) | 0.4 | 0.1 | 0.5 | |
| Price/fare ($P^i$) | \$0 | \$40 | \$20 | \$10 |

Unused capacity can be sold for a salvage value of $S = \$10$. There are $T = 2$ booking periods labeled as $t = 3, 4$. In each booking period there is one booking request for one of the classes $0$, $A$, or $B$.

**(1a) [14 points]** Compute the service provider's profit-maximizing decision rules, $d_4(P_4)$ and $d_3(P_3)$, as well as the values of capacity $\mathsf{E}V_4(k_4)$ and $\mathsf{E}V_3(1)$.

**(1b) [6 points]** Under the decision rules you computed above, compute the probabilities (separately) that (i) no one will be booked, (ii) a consumer will be booked on class $A$, and (iii) on class $B$.

**(1c) [14 points]** Suppose now that there are four booking periods $t = 1, 2, 3, 4$ $(T = 4)$. Compute the service provider's profit-maximizing decision rules, $d_2(P_2)$ and $d_1(P_1)$, as well as the values of capacity $\mathsf{E}V_1(1)$ at the beginning of the booking process, and $\mathsf{E}V_2(k_2)$ in booking period $t = 2$.

**(1d) [6 points]** Solve the 3 problems given in (1b) for the case of $T = 4$ booking periods.

**(2)** A service provider has solve a fixed-class allocation problem by allocating one unit of capacity $K = 1$ to either to class $A$ or to class $B$. That is, the firm must allocate either $\langle K_A, K_B \rangle = \langle 1, 0 \rangle$, or $\langle K_A, K_B \rangle = \langle 0, 1 \rangle$ before the booking process begins. The service provider bases her decision on the following information:

| Class ($i$) | $A$ | $B$ |
|---|---|---|
| Proportion ($\pi^i$) | $1/3$ | $2/3$ |
| Price/fare ($P^i$) | \$18 | \$9 |

**(2a) [6 points]** Assuming only one booking period ($T = 1$), compute the expected profit $\mathsf{E}y(1, 0)$ (capacity allocated to class $A$) and $\mathsf{E}y(0, 1)$ (capacity allocated to class $B$) and conclude to which class should the provider allocate the unit of capacity.

**(2b) [14 points]** Answer the above question assuming two booking periods ($T = 2$).

**(3) [20 points]** AIR FLINT operates daily flights. The airfare is $P = \$19$ which must be pre-paid at the time when the booking is made. The airline bears a fixed cost $\phi = \$3000$, marginal operating and capacity costs $\mu_O = \mu_k = \$2$. There are two types of potential passengers:

$N_S = 100$ **Students:** Maximum willingness to pay $V_S = \$20$ and a survival probability $\pi_S = 0.9$.
$N_B = 200$ **Business passengers:** $V_B = \$40$ and a survival probability $\pi_B = 0.3$.

The utility function of type $\ell$ consumer ($\ell = S, B$) is given by

$$U_\ell(p, r) \stackrel{\text{def}}{=} \begin{cases} \pi_\ell V_\ell - p + (1 - \pi_\ell) r & \text{books the flight} \\ 0 & \text{does not book this flight,} \end{cases}$$

where $r$ $(0 \leq r \leq p)$ is the lump-sum refund level paid to a booked passenger who does not show up for the flight. Using this information, compute the airline's profit-maximizing refund level and the resulting expected profit.

**(4)** ANN ARBOR MASSAGE (AAM) offers one-hour therapy for a price of $\$9$. Consumers book in advance for free, and then pay only if they show up for the service. The marginal operating cost (per-hour labor cost) is $\mu_O = \$5$. There are no fixed and marginal capacity costs ($\phi = \mu_k = 0$) because the service is delivered in a public place (Burns Park to be exact).

AAM's capacity is limited to one patient per hour, so capacity can be written as $K = 1$. As it turned out, consumers who book massage therapy are not very reliable in the sense that they show up only $2/3$ of the times (their survival probability is $\pi = 2/3$). Therefore, AAM is considering overbooking one consumer ($b = 2$) instead of booking up to capacity $(b = 1)$. AAM must pay a compensation (penalty) of $\psi = \$2$ to each booked consumer who is denied service.

**(4a) [10 points]** Using this information, compute AAM's expected profit when it books $b = 1$ customer and when it books $b = 2$ customers.

**(4b) [5 points]** Compute the expected total penalty cost to be paid to booked consumers who are denied service assuming that AAM books $b = 3$ customers.

**(4c) [5 points]** Compute the expected total penalty cost to be paid to booked consumers who are denied service assuming that AAM books $b = 4$ customers.

**THE END**