(1a) [5 points] According to the four-largest firm concentration index, industry A is more concentrated than industry B since

$$I_4^A = 40 + 15 + 15 + 15 = 85 > 78 = 45 + 11 + 11 + 11 = I_4^B.$$

According to the Hirschman-Herfindahl concentration index, industry B is more concentrated than industry A since

$$I_{HH}^A = 40^2 + 4 \cdot 15^2 = 2500 < 2630 = 45^2 + 5 \cdot 11^2 = I_{HH}^B.$$

Country	Firms						Concentration Index	
	1	2	3	4	5	6	I_4	I_{HH}
Albania	40%	15%	15%	15%	15%	0%	85	2500
Bolivia	45%	11%	11%	11%	11%	11%	78	2630

(1b) [5 points] The post-merger $I_{\text{HH}} = 45^2 + (11 + 11)^2 + 3 \cdot 11^2 = 2872 > 1800$. The change in this index as a result of this merger is $2872 - 2630 = 242 > 50$. Therefore, the merger is likely to be challenged according to the merger guidelines.

(2a) [5 points]

$$p_G = BR_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H \end{cases} \quad \text{and} \quad p_F = BR_F(p_G) = \begin{cases} p^L & \text{if } p_G = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H \end{cases}$$

Note first that firm G does not have a dominant action. This follows from the above best-response function by observing that firm G sets a low price, p^L, if firm F sets p^L. However, firm G sets a high price, p^H, if firm F sets p^M.

Now, a pair of prices $\langle p_G, p_F \rangle$ constitutes an equilibrium in dominant actions if each firm plays its dominant action. However, since firm G does not have a dominant action, such as equilibrium does not exist.

(2b) [5 points] No, because $\pi_F(p^H, p^M) = 350 > 300 = \pi_F(p^H, p^H)$. Therefore, given that firm G sets $p_G = p^H$, firm F can increase its profit by deviating from $p_F = p^H$ to $p_F = p^M$.

(2c) [5 points] There are two NE outcomes in the restricted game: $\langle p_G, p_F \rangle = \langle p^L, p^L \rangle$ and $\langle p_G, p_F \rangle = \langle p^M, p^M \rangle$. This follows from

$$\pi_G(p^L, p^L) = 100 \geq 50 = \pi_G(p^M, p^L) \quad \text{and} \quad \pi_F(p^L, p^L) = 100 \geq 50 = \pi_F(p^L, p^M)$$

and

$$\pi_G(p^M, p^M) = 200 \geq 150 = \pi_G(p^L, p^M) \quad \text{and} \quad \pi_F(p^M, p^M) = 200 \geq 150 = \pi_F(p^M, p^L).$$

Another way of proving this would be to construct the following two best-response functions

$$p_G = BR_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^L \\ p^M & \text{if } p_F = p^M \end{cases} \quad \text{and} \quad p_F = BR_F(p_G) = \begin{cases} p^L & \text{if } p_G = p^L \\ p^M & \text{if } p_F = p^M \end{cases}$$

The two equilibria are on the firms' best-response functions.

(2d) [5 points] The equilibrium strategies are:

$$p_F = p^M \quad \text{and} \quad p_G = BR_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^L \\ p^M & \text{if } p_F = p^M \end{cases}$$

In this equilibrium $p_G = p^M$ and hence $\pi_F(p^M, p^M) = 200$ and $\pi_G(p^M, p^M) = 200$.

To prove that the above is a SPE, note that GM's strategy is its best-response function. Next, if Ford sets different prices then if

$$p_F = p^L \Longrightarrow p_G = p^L \Longrightarrow \pi_F(p^L, p^L) = 100 < 200.$$

So, $p_F = p^M$ yields a higher profit to Ford.

(3a) [10 points] In the absence of capacity constraint, the price discriminating monopoly solves $MR_1 = 120 - 2q_1 = c = 30$ and $MR_2 = 120 - 2q_2/3 = c = 30$ yielding $q_1 = 45$ and $q_2 = 135$. Hence, $p_1 = 120 - 45 = 75$ and $p_2 = 120 - 135/3 = 75$. Hence, total profit is

$$\Pi = \pi_1 + \pi_2 = (75 - 30)45 + (75 - 30)135 = 8100.$$

(3b) [10 points] The above computation showed that with no capacity limit $q_1 + q_2 = 180 > 160$. Hence, the capacity constraint is binding and the monopoly will produce at the maximum possible level, $Q = 160$.

Under capacity constraint, the monopoly solves for sales levels q_1 and q_2 that solve

$$MR_1 = 120 - 2q_1 = 120 - \frac{2q_2}{3} = MR_2 \quad \text{and} \quad q_1 + q_2 = 160$$

yielding $q_1 = 40$ and $q_2 = 120$. Hence, $p_1 = 120 - 40 = 80$ and $p_2 = 120 - 120/3 = 80$. The resulting profit is

$$\Pi = (p_1 - c)q_1 + (p_2 - c)q_2 = (80 - 30)40 + (80 - 30)120 = 8000 < 8100.$$

Clearly, the monopoly earns a lower profit if it is forced to reduce production below its profit-maximizing levels.

An alternative way of solving this problem is to substitute $q_2 = 160 - q_1$ into the monopoly's profit function and to solve

$$\max_{q_1} \Pi = (120 - q_1)q_1 + \left(120 - \frac{160 - q_1}{30}\right)(160 - q_1) - 30 \cdot 160$$
$$= (120 - q_1)q_1 - \frac{(q_1)^2}{3} - \frac{40q_1}{3} + \frac{32000}{3} - 30 \cdot 160$$

The first-order condition yields $0 = 8(40 - q_1)/3$, hence, $q_1 = 40$, etc...

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(4) [20 points] First, we should solve for the direct demand functions: $q_1 = 36 - p_1$, and $q_2 = 48 - 2p_2$, and $q_3 = 24 - 2p_3$. Next, we should examine three possible price ranges, and compare the resulting profit levels.

Let $p > 24$, which means that $q_2 = q_3 = 0$. Solving $MR_1 = 36 - 2q_1$ yields $q_1 = 18$ and $p = 36 - 18 = 18 < 24$. A contradiction to our assumption that $p > 24$.

Let $12 < p \leq 24$, in which case $q_3 = 0$. Aggregate demand facing this monopoly is therefore $q_{12} = q_1 + q_2 = 84 - 3p$. Thus, $p = (84 - q_{12})/3$ and hence $MR_{12} = (84 - 2q_{12})/3 = c = 0$ yields $q_{12} = 42$. Hence, $p = (84 - 42)/3 = 14$. The resulting profit (revenue, since production is costless) is $\pi_{12} = 14 \cdot 42 = 588$.

Lastly, let $p < 12$. Aggregate demand is $q_{123} = q_1 + q_2 + q_3 = 108 - 5p$. Inverse demand is therefore $p = (108 - q_{123})/5$. Solving $MR_{123} = (108 - 2q_{123})/5 = c = 0$ yields $q_{123} = 54$ and hence $p = 54/5$. Profit (revenue) is therefore $\pi_{123} = 583.2 < 588$.

To summarize, the profit-maximizing price of this non-discriminating monopoly is $p = 14$. The monopoly sells in markets 1 and 2 only and earns a profit of $\pi = 588$.

(5a) [10 points] First, compute the monopoly's price. $MR = 140 - 4Q = 20$ yields $Q^m = 30$, and hence $p^m = 120 - 2Q = 80$. Next, the price best-response function of each firm i to the price set by firm j is

$$p_i = BR_i(p_j) = \begin{cases} 80 & \text{if } p_j > 80 \\ p_j - \epsilon & \text{if } 20 < p_j \leq 80 \\ 20 & \text{if } p_j < 20. \end{cases} \quad i, j = A, B; \quad i \neq j.$$

The unique Nash-Bertrand equilibrium is therefore $p_A^b = p_B^b = 20$ (each firm replies to a price of 20 by setting also a price of 20).

(5b) [10 points] When both firm cooperate by setting the monopoly price $p = 80$, they jointly produce $Q = (140 - 80)/2 = 30$ units. Assuming equal production, each firm produces $q_A = q_B = 15$. Hence, each firm earns a profit of $\pi_i(t) = (80 - 20)15 = 900$ in each period of cooperation t. Thus, if both firms cooperate, they earn a discounted profit of

$$\sum_{t=0}^{\infty} = \rho^t \cdot 900 = \frac{900}{1-\rho}.$$

Now, suppose that firm A deviates and undercut firm B by setting $p'_A = 80 - \epsilon$. Then, firm A sells to the entire market, so $q'_A = 30$. In the period of deviation, the firm earns $\pi'_A \approx (80 - 20)30 = 1800$. But, according to the trigger strategy, both firms set $p_A = p_B = 20$ in all subsequent period.

Deviation is not profitable for firm A (by symmetry, also for firm B) if

$$\frac{900}{1-\rho} \geq 1800 + \rho\frac{0}{1-\rho} = 1800$$

hence if $\rho > 0.5$.

(6) [10 points] In stage $t = 3$, firm 3 takes q_1 and q_2 as given and solves

$$\max_{q_3} \pi_3 = (120 - q_1 - q_2 - q_3)q_3 \quad \text{yielding} \quad q_3 = BR_3(q_1, q_2) = \frac{120 - q_1 - q_2}{2}.$$

In stage $t = 2$, firm 2 takes q_1 and $BR_3(q_1, q_2)$ as given and solves

$$\max_{q_2} \pi_2 = \left[120 - q_1 - q_2 - \frac{120 - q_1 - q_2}{2}\right] q_2 \quad \text{yielding} \quad q_2 = BR_2(q_1) = \frac{120 - q_1}{2}.$$

Substituting $q_2 = BR_2(q_1)$ into $BR_3(q_1, q_2$ yields

$$q_3 = BR_3(q_1) = \frac{120 - q_1}{4}.$$

In stage $t = 1$, firm 1 chooses q_1 to solve

$$\max_{q_1} \pi_1 = (120 - q_1 - q_2 - q_3)q_1 = \left(120 - q_1 - \frac{120 - q_1}{2} - \frac{120 - q_1}{4}\right) q_1.$$

The solution to firm 1's profit maximization problem is

$$q_1 = 60, \quad q_2 = \frac{120 - q_1}{2} = 30, \quad \text{and} \quad q_3 = \frac{120 - q_1}{4} = 15.$$

Aggregate industry production and the market price are therefore

$$Q = q_1 + q_2 + q_3 = 105 \quad \text{hence} \quad p = 120 - 105 = 15.$$

Profits (same as revenue because production is costless) are therefore

$$\pi_1 = 15 \cdot 60 = 900, \quad \pi_2 = 15 \cdot 30 = 450, \quad \text{and} \quad \pi_3 = 15 \cdot 15 = 225.$$

THE END

