(1) [10 pts.] The diaper industry in Albania consists of 5 firms producing identical diapers. Similarly, the diaper industry in Bolivia consists of 6 firms. It has been recently observed that firms' market shares in each country are given by

Country	Firms						Concentration Index	
	1	2	3	4	5	6	I_4	I_{HH}
Albania	40%	15%	15%	15%	15%	0%	85	2500
Bolivia	45%	11%	11%	11%	11%	11%	78	2630

Fill-in the missing items in the above table (show all your calculations). Then, conclude which industry is more concentrated (and according to which measure).

Answer: According to the four-largest firm concentration index, industry A is more concentrated than industry B since

$$I_4^A = 40 + 15 + 15 + 15 = 85 > 78 = 45 + 11 + 11 + 11 = I_4^B.$$

According to the Hirschman-Herfindahl concentration index, industry B is more concentrated than industry A since

$$I_{HH}^A = 40^2 + 4 \cdot 15^2 = 2500 < 2630 = 45^2 + 5 \cdot 11^2 = I_{HH}^B.$$

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(2) [10 pts.] Discuss whether it is illegal to price discriminate according to the U.S. Law. Explain which section of the law deals with price discrimination, and how this section should be interpreted.

Answer: Section 2 of the 1914 Clayton Act states that price discrimination is unlawful if its effect is "to lessen competition or tend to create a monopoly...or to injure destroy or prevent competition." In addition, price differentials are also allowed to account for "differences in the cost of manufactures, sale or delivery."

This, in part, implies that price discrimination that does not reduce competition should not be viewed as illegal.

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(3) Firms A and B can choose to adopt a new technology (N) or to adhere to their old technology (O). Formally, firms' action sets are: $t_A \in \{N, O\}$ and $t_B \in \{N, O\}$. The table below exhibits the profit made by each firm under different technology choices.

		Firm B: New Technology (N)		Firm B: Old Technology (O)	
Firm A	New	200	0	0	200
	Old	50	100	100	50

(3a) [10 pts.] Write down the best-response functions of firms A and B, $t_A = R_A(t_B)$ and $t_B = R_B(t_A)$

Answer:

$$t_A = BR_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O \end{cases} \quad \text{and} \quad t_B = BR_B(t_A) = \begin{cases} O & \text{if } t_B = N \\ N & \text{if } t_B = O. \end{cases}$$

(3b) [5 pts.] Draw the tree of a two-stage extensive-form game in which firm A chooses its technology t_A in stage I, and Firm B chooses its t_B in stage II (after observing the choice made by firm A). Make sure that you indicate firms' profits at the termination points on the tree. Solve for the subgame-perfect equilibrium of this game. Provide a short proof or an explanation justifying your answer.

Answer: The tree drawing is not provided here. From firm B's best response function given above, if $t_A = N$, $t_B(N) = O$, in which case firm A earns $\pi_A(N, O) = 0$. Instead, if $t_A = O$, $t_B(O) = N$, in which case firm A earns $\pi_A(O, N) = 50 > 0$. Therefore, the subgame-perfect equilibrium for this game is:

$$t_A = O \quad \text{and} \quad t_B = BR_B(t_A) = \begin{cases} O & \text{if } t_B = N \\ N & \text{if } t_B = O. \end{cases}$$

Note: Some students wrote that the is no SPE for this game since there is no Nash equilibrium of the normal-form game. This answer is incorrect, because a Nash equilibrium for the extensive-form game does exist.

(3c) [5 pts.] Draw the tree of a two-stage extensive-form game in which firm B chooses its technology t_B in stage I and Firm A chooses its t_A in stage II (after observing the choice made by firm B). Solve for the subgame-perfect equilibrium of this game. Provide a short proof or an explanation justifying your answer.

Answer: The tree drawing is not provided here. From firm A's best response function given above, if $t_B = N$, $t_A(N) = N$, in which case firm B earns $\pi_B(N, N) = 0$. Instead, if $t_B = O$, $t_A(O) = O$, in which case firm B earns $\pi_B(O, O) = 50 > 0$. Therefore, the subgame-perfect equilibrium for this game is:

$$t_B = O \quad \text{and} \quad t_A = BR_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O. \end{cases}$$

(3d) [5 pts.] Compare the equilibrium firms' profit levels of the games played in (3b) and in (3c). Conclude under which game firm A earns a higher profit. Briefly explain your answer.

Answer: In the game (3b) $\pi_A(O, N) = 50$ and $\pi_B(O, N) = 100$.
In the game (3c) $\pi_A(O, O) = 100 > 50$ and $\pi_B(O, O) = 50$.

This game is interesting because A has a first-mover *disadvantage*. Not every game yields this result. This is because A would like to "match" B's technology, whereas B gains from introducing a different technology. By letting B making the first choice, A is able to match its technology choice with B's choice.

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(4) [10 pts.] The market demand function for Marzipan in Frankenmuth Michigan has a constant elasticity of -3. More precisely the actual daily demand was estimated to be $Q = 34560\, p^{-3}$, where p is the price per pound. Each pound costs $c = \$8$ to produce. Frankenmuth is served by a local monopoly producer. Compute the monopoly's profit-maximizing price and the monopoly's profit level. Show your computations.

Answer: The monopoly equates marginal revenue to marginal cost, c. Therefore,

$$p^m\left(1+\frac{1}{\text{elas}}\right)=p^m\left(1+\frac{1}{-3}\right)=\frac{2p^m}{3}=c=8.$$

Therefore, $p^m=\$12$. Next, $Q=34560\,p^{-3}=20$ units. Hence, $\pi=(p-c)Q=(12-8)20=\$80$.

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(5) Consider the market for the G-Jeans (the latest fashion among people in their late thirties). G-Jeans are sold by a single firm that carries the patent for the design. On the demand side, there are $n^H=200$ high-income consumers who are willing to pay a maximum amount of $V^H=\$20$ for a pair of G-Jeans, and $n^L=300$ low-income consumers who are willing to pay a maximum amount of $V^L=\$10$ for a pair of G-Jeans. Each consumer chooses whether to buy one pair of jeans or not to buy at all.
(5a) [10 pts.] Draw the market aggregate-demand curve facing the monopoly.

Answer: The aggregate demand curve should be drawn according to the following formula:

$$Q(p)=\begin{cases}0 & \text{if } p>\$20\\ 200 & \text{if } \$10<p\le\$20\\ 200+300 & \text{if } p\le\$10.\end{cases}$$

(5b) [10 pts.] The monopoly can produce each unit at a cost of $c=\$5$. Suppose that the G-Jeans monopoly cannot price discriminate and is therefore constrained to set a uniform market price. Find the profit-maximizing price set by G-Jeans, and the profit earned by this monopoly.

Answer: Setting a high price, $p=\$20$ generates $Q=200$ consumers and a profit of $\pi_H=(20-5)200=\$3000$.

Setting a low price, $p=\$10$ generates $Q=200+300$ consumers and a profit of $\pi_H=(10-5)500=\$2500<\$3000$. Hence, $p=\$20$ is the profit-maximizing price. Type L consumers will not buy under this prices.

(5c) [5 pts.] Compute the profit level made by this monopoly assuming now that this monopoly can price discriminate between the two consumer populations. Does the monopoly benefit from price discrimination. Prove your result!

Answer: The monopoly will charge $p=\$20$ in market H and $p=\$10$ in market L. Hence, total profit is given by

$$\Pi=pi_H+\pi_L=(20-5)200+(10-5)300=3000+1500=\$4500>\$3000.$$

Clearly, the ability to price discriminate cannot reduce the monopoly profit since even with this ability, the monopoly can always set equal prices in both markets. The fact that the monopoly chooses different prices implies that profit can only increase beyond the profit earned when the monopoly is unable to price discriminate.

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(6) In Waterville there are two suppliers of distilled water, labeled as firm A and firm B. Distilled water is considered to be a homogenous good. Let p denote the price per gallon, q_A quantity sold by firm A, and q_B the quantity sold by firm B.

Both firms are located close to a spring so the only production cost is the cost of bottling. Formally, each firm bears a production cost of $c_A = c_B = \$3$ per one gallon of water. Waterville's aggregate inverse demand function for distilled water is given by $p = 12 - Q = 12 - q_A - q_B$, where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Waterville.

(6a) [10 pts.] Solve for firm A's best-response function, $q_A = R_A(q_B)$. Also solve for firm B's best-response function, $q_B = R_B(q_A)$. Show your derivations.

Answer: See the solution to the homework assignment for the method of solving for the best-response functions in a Cournot market structure. In the present example,

$$q_A(q_B) = \frac{9 - q_B}{2} \quad \text{and} \quad q_B(q_A) = \frac{9 - q_A}{2}.$$

(6b) [5 pts.] Solve for the Cournot equilibrium output levels q_A^c and q_B^c. State which firm sells more water (if any) and why.

Answer: The above best-response function constitute two linear equations with two variables, q_A and q_B. The unique solution is $q_A^c = q_B^c = 3$ gallons. Both firms produce the same amount since there equally efficient in the sense that the bear identical production costs.

(6c) [5 pts.] Solve for the aggregate industry supply and the equilibrium price of distilled water in Waterville.

Answer: $Q = q_A + q_B = 3 + 3 = 6$ gallons. The equilibrium price is $p = 12 - Q = \$6$.

(6d) [5 pts.] Solve for the profit level made by each firm, and for the aggregate industry profit. Which firm earns a higher profit and why?

Answer: Since there are no fixed costs, $\pi_A = (p - c_A)q_A = (6-3)3 = \$9$. Similarly, $\pi_B = (p - c_B)q_B = (6-3)3 = \$9$. Industry profit is then $\Pi = \pi_A + \pi_B + 9 + 9 = \$18$. Both firms earn the same profit since the bear identical production costs.

Remark: Students who have assumed $p = 120 - Q$ instead of $p = 12 - Q$ should obtain $q_A = (117 - q_B)/2$, $q_B = (117 - q_A)/2$, $q_A^c = q_B^c = 39$, $p^c = 120 - 39 - 39 = \$42$, and $\pi_A^c = \pi_B^c = \$1521$.

THE END