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Last Name (Please PRINT): .....

First Name (PRINT): .....

Your UM I.D. Number: .....

INSTRUCTIONS (please read!)

1. Please make sure that you have 8 pages, including this page. Complaints about missing pages will not be accepted.
2. Please answer all the questions. You are not allowed to use any course material. Calculators are permitted.
3. Maximum Time Allowed: 1 hour and 20 minutes (11:40–13:00).
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip it and go to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is 'visible' and that your sentences are properly organized and fit into the designated blank space. If you think that your handwriting is poor, please print each word!
10. Good Luck !

(1) Two firms have technologies for producing identical paper clips. Assume that all paper clips are sold in boxes containing 100 paper clips. Firm A can produce each box at unit cost of $c_A = \$6$ whereas firm B (less efficient) at a unit cost of $c_B = \$8$.

(1a) [10 pts.] Suppose that the aggregate market demand for boxes of paper clips is $p = 12 - Q/2$, where p is the price per box and Q is the number of boxes sold. Solve for the Nash-Bertrand equilibrium prices p_A^b and p_B^b, and the equilibrium profits π_A^b and π_B^b. Explain your reasoning!

(1b) [10 pts.] Answer the previous question assuming that firm A has developed a cheaper production technology so its unit cost is now given by $c_A = \$2$.

(2) Consider an infinitely-repeated price competition game between GM and FORD. Each firm can set a high price or a low price in each period $t = 0, 1, 2, \ldots$. The profit of each outcome are given in the following matrix:

		FORD	
		Low Price (p^L)	High Price (p^H)
GM	Low (p^L)	4 | 3	5 | 1
	High (p^H)	1 | 6	5 | 4

Suppose that each firm adopts a trigger-price strategy under which the firms may be able to implicitly collude on setting the high price. Let ρ $(0 < \rho < 1)$ denote the time discount factor.

(2a) [10 pts.] Compute the minimum threshold value of ρ which would ensure that GM sets p^H in every period t. Show and explain your derivations.

(2b) [10 pts.] Compute the minimum threshold value of ρ which would ensure that FORD sets p^H in every period t. Show and explain your derivations.

(3) Aike (Brand A) and Beebok (Brand B) are leading brand names of fitness shoes. The direct demand functions facing each producer are given by

$$q_A(p_A, p_B) = 180 - 2p_A + p_B \quad \text{and} \quad q_B(p_A, p_B) = 120 - 2p_B + p_A.$$

Assume zero production cost ($c_A = c_B = 0$).

(3a) [10 pts.] Derive the price best-response function of firm A as a function of the price set by firm B, $p_A = BR_A(p_B)$. Show your derivations, and draw the graph associated with this function.

(3b) [10 pts.] Derive the price best-response function of firm B as a function of the price set by firm A, $p_B = BR_B(p_A)$. Show your derivations, and draw the graph associated with this function.

(3c) [10 pts.] Solve for the Nash-Bertrand equilibrium prices, $\langle p_A^b, p_B^b \rangle$. Then, compute the equilibrium output levels $\langle q_A^b, q_B^b \rangle$, the equilibrium profits $\langle \pi_A^b, \pi_B^b \rangle$, and aggregate industry profit $\Pi^b = \pi_A^b + \pi_B^b$.

(3d) [10 pts.] Suppose now that the two producers hold secret meetings in which they discuss fixing the price of shoes to a uniform (brand-independent) level of $p = p_A = p_B$. Compute the price p which maximizes joint industry profit, $\pi_A + \pi_B$. Then, compute aggregate industry profit and compare it to the aggregate industry profit made under Bertrand competition which you computed in part (3c).

(4) Ann Arbor and Ypsilanti are very similar cities, because each city has exactly one McDonald's. Ann Arbor has $N_A = 200$ residents and Ypsilanti has $N_Y = 200$ residents. Each resident demands one hamburger. A resident of Ann Arbor who wishes to buy a hamburger in Ypsilanti must bear a transportation cost of $T_A = \$3$. Similarly, a resident of Ypsilanti who wishes to buy a hamburger in Ann Arbor must bear a transportation cost of $T_Y = \$3$.

(4a) [10 pts.] Solve for the undercut-proof equilibrium prices p_A^U and p_Y^U and profit levels π_A^U and π_Y^U assuming that McDonald's has the technology for producing hamburgers at no cost. Show your derivation.

(4b) [10 pts.] Answer the previous question assuming now that McDonald's in Ann Arbor bears a cost of $\$1$ of producing each hamburger, whereas McDonald's in Ypsilanti bears a cost of $\$4$ of producing each hamburger. Show your derivation.

THE END