(1) [10 points] For $p \le 4¢$, the aggregate demand facing the producer is

$$Q = q_1 + q_2 = \frac{8-p}{2} + 2(4-q) = \frac{24-5p}{2} \quad \text{or} \quad p = \frac{2(12-Q)}{5}.$$

For $p \le 4$, the monopoly solves

$$MR = \frac{2(12-2Q)}{5} = 2 = c \quad \text{yielding } Q = \frac{7}{2}, \quad p = \frac{17}{5} < 4.$$

Hence,

$$\pi_{1,2} = \left(\frac{17}{5} - 2\right)\frac{7}{2} = \frac{49}{10} = 4.9¢.$$

Selling at a price $p > 4¢$ would exclude consumer 2 from the market. In this case, the monopoly solves

$$MR_1 = 8 - 4q_1 = 2¢ = c, \quad \text{yielding } q_1 = \frac{3}{2} \quad \text{and} \quad p_1 = 5¢ > 4¢.$$

Under this price, the monopoly earns

$$\pi_1 = (5-2)\frac{3}{2} = \frac{9}{2} = 4.5¢ < 4.9¢.$$

Hence, the profit maximizing price is $p = 17/5 = 3.4¢$.

(2a) [10 points] Under unlimited capacity, solving $120 - 2 \cdot 0.25q_1 = 10$ yields $q_1 = 220$. Solving $240 - 2 \cdot 0.5q_2 = 10$ yields $q_2 = 230$.

Next, we compute profits. $p_1 = 120 - 0.25q_1 = \$65$. $p_2 = 240 - 0.5q_2 = \$125$. Hence,

$$\pi = \pi_1 + \pi_2 = (65-10)220 + (125-10)230 - 10,000 = 28,550 > 0.$$

(2b) [10 points] We first must check whether capacity is binding. In part (a), we found that quantity produced under unlimited capacity is $\bar{q} = 220 + 230 = 450 > 240 = K$. Thus, we proceed by assuming that capacity will be binding.

Under capacity constraint of $K = 240$, solving $120 - 2\,0.25q_1 = 240 - 2\,0.5q_2$ and $q_1 + q_2 = K = 240$ yields $q_1 = 80$ and $q_2 = 160$. Therefore, $p_1 = 120 - 0.25q_1 = \$100$ and $p_2 = 240 - 0.5q_2 = \$160$. The corresponding profit is

$$\pi = \pi_1 + \pi_2 = (100-10)80 + (160-10)160 - 10,000 = 21,200 > 0.$$

(3a) [5 points] Setting $MR = 120 - Q = c + t = 40 + t$ yields

$$Q = 80 - t \quad \text{and} \quad p = 120 - \frac{80-t}{2} = \frac{160+t}{2} \quad \text{hence} \quad \pi = \frac{(80-t)^2}{2}.$$

(3b) [5 points] The government selects a tax rate t^* to solve

$$\max_t G = t \cdot Q = t(80 - t).$$

The first- and second-order conditions for a maximum are

$$0 = \frac{dG}{dt} = 80 - 2t \quad \text{and} \quad \frac{d^2G}{dt^2} = -2 < 0.$$

Therefore, the tax rate which maximizes government revenue and total revenue collected by the government are

$$t^* = 40, \quad Q = 80 - 40 = 40, \quad \text{and} \quad G^* = t^* \cdot Q = 1600.$$

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(4) [10 points]

$$I^0_{HH} = 10 \cdot 10^2 = 1000, \quad I^1_{HH} = 8 \cdot 10^2 + (10 + 10)^2 = 1200 \quad \text{hence} \quad \Delta I_{HH} = 200.$$

The merger is likely to be challenged because $1000 < I^1_{HH} \le 1800$ and $\Delta I_{HH} > 100$.

(5a) [5 points] The firms' best-response functions are given by

$$p_G = R_G(p_F) = \begin{cases} p^H & \text{if } p_F = p^L \\ p^L & \text{if } p_F = p^M \\ p^L & \text{if } p_F = p^H \end{cases} \quad \text{and} \quad p_F = R_F(p_G) = \begin{cases} p^L & \text{if } p_G = p^L \\ p^M & \text{if } p_G = p^H. \end{cases}$$

(5b) [5 points] GM replies p^H to some of Ford's actions p^L for others. Therefore, GM does not have a dominant action. Hence, an equilibrium in dominant actions does not exist.

A Nash equilibrium does not exist because

$$p_F = p^L \Longrightarrow p_G = p^H \Longrightarrow p_F = p^M \Longrightarrow p_F = p^L \Longrightarrow \dots$$

so there is no outcome while satisfies both best-response functions simultaneously.

(5c) [5 points] No, because $\pi_F(p^L, p^H) = 0 < 50 = \pi_F(p^L, p^M)$ (Ford is worse off).

(5d) [5 points] The subgame-perfect equilibrium strategies are

$$p_F = p^L \quad \text{and} \quad s_G = R_G(p_F) = \begin{cases} p^H & \text{if } p_F = p^L \\ p^L & \text{if } p_F = p^M \\ p^L & \text{if } p_F = p^H \end{cases}$$

The resulting equilibrium path is $p_F = p^L$ and $p_G = p^H$. The resulting profit levels are $\pi_G(p^H, p^L) = 150$ and $\pi_F(p^H, p^L) = 200$.

(6a) [10 points] A's monopoly price can be found by solving $MR = 140 - q_A^m = c_A = 20$ yielding $q_A^m = 30$ and hence $p_A^m = \$80$.

B's monopoly price can be found by solving $MR = 140 - q_B^m = c_B = 100$ yielding $q_B^m = 10$ and hence $p_B^m = \$120$.

Hence, the firms best-response functions are:

$$p_A = BR_A(p_B) = \begin{cases} 80 & \text{if } p_B > 80 \\ p_B - \epsilon & \text{if } 20 < p_B \leq 80 \\ 20 & \text{if } p_B \leq 20. \end{cases} \quad \text{and} \quad p_B = BR_B(p_A) = \begin{cases} 120 & \text{if } p_A > 120 \\ p_B - \epsilon & \text{if } 100 < p_A \leq 120 \\ 100 & \text{if } p_A \leq 100. \end{cases}$$

Therefore, the unique Nash-Bertrand equilibrium is $p_A^b = \$80$ and $p_B^b = \$100$. The corresponding profit levels are $\pi_A^b = (80 - 20)30 = \$1800$ and $\pi_B^b = 0$.

(6b) [10 points] In stage II, firm A solves

$$\max_{q_A} \pi_A = (140 - 2q_A - 2q_B - 20)q_A \quad \text{yielding} \quad q_A = BR_A(q_B) = 30 - \frac{q_B}{2}.$$

In stage I, firm B solves

$$\max_{q_B} \pi_B = \left[140 - 2q_A - 2\left(30 - \frac{q_B}{2}\right) - 60\right] q_B \quad \text{yielding} \quad q_B = 10.$$

Thus, $q_A = 25$, $Q = 35$, $p = \$70$, so

$$\pi_A = (70 - 20)25 = \$1250 \quad \text{and} \quad \pi_B = (70 - 60)10 = \$100.$$

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(7) [10 points] Ford will not deviate because $\pi_F(p^H, p^H) = 300 > 200 = \pi_F(p^H, p^L)$.

GM will not deviate if

$$\frac{300}{1-\rho} \geq 400 + \frac{100\rho}{1-\rho} \quad \text{or} \quad \rho \geq \frac{1}{3}.$$

THE END