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Last Name (Please PRINT): .....

First Name (PRINT): .....

Your UM I.D. Number: .....

INSTRUCTIONS (please read!)

1. Please make sure that you have 8 pages, including this page. Complaints about missing pages will not be accepted.
2. Please answer all the questions. You are not allowed to use any course material. Calculators are permitted.
3. Maximum Time Allowed: 1 hour and 50 minutes (8:10–10:00).
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is "visible" and that your sentences are properly organized and fit into the designated blank space. If you think that your handwriting is poor, please print each word!
10. Good Luck !

Instructor's use only

Problem #	1	2	3 & 4	5	6	7	Total
Maximum	10	20	20	20	20	10	100
Points							

(1) [10 points] ChewMe is a monopoly in the market for sugarless chewing gum. There are only two types potential consumers, labeled as group 1 and group 2. Consumers' inverse demand functions are given by

$$p_1 = 8 - 2q_1 \quad \text{and} \quad p_2 = 4 - \frac{q_2}{2},$$

where q_i denotes the number of sticks consumed, and p_i the price. Each stick of chewing gum costs $c = 2$ to produce. Suppose ChewMe sells individual sticks at a uniform price of p per stick. That is, ChewMe cannot price discriminate between the two consumer groups. Compute the profit maximizing price, p, total quantity sold Q, and total profit.

(2) Consider a price discriminating monopoly selling in two markets. The market demand curves are given by

$$p_1 = 120 - \frac{q_1}{4} \quad \text{and} \quad p_2 = 240 - \frac{q_2}{2}.$$

The firm bears a marginal cost of $c = \$10$, and a fixed cost given by $F = \$10,000$, that is, the firm's cost function is $TC(Q) = 10,000 + 10Q$.

(2a) [10 points] Compute the profit-maximizing prices assuming that capacity is unlimited, and the resulting profit level.

(2b) [10 points] Now suppose that the firm has a limited production capacity in the sense that it cannot produce more than $K = 240$ units. Compute the profit-maximizing prices and the corresponding profit level.

(3) A monopoly faces an inverse demand function given by $p = 120 - \frac{Q}{2}$. Each unit costs $c = 40$ to produce. In addition, the State imposes a tax on the monopoly at the level of t per unit sold.

(3a) [5 points] Compute the monopoly's profit maximizing price and quantity sold as a function of the per-unit tax, t.

(3b) [5 points] Compute the tariff rate t^* which maximizes the government's revenue from this tax. How much revenue is collected under this tax rate?

(4) [10 points] The diaper industry is characterized by 10 firms producing identical diapers. Let s_i denote the percentage market share of firm i, $i = 1, 2, \ldots, 10$. It has been recently observed that the market shares are given by

Firm i	1	2	3	4	5	6	7	8	9	10
s_i	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

Suppose that firms 9 and 10 have decided to merge. Determine whether this merger is likely to be challenged by the FTC or DOJ. Answer this question using the merger guidelines, by first computing I^0_{HH}, I^1_{HH}, and ΔI_{HH}.

(5) The table below displays the profits from a price game between GM and FORD.

		FORD					
		Low Price (p^L)		Mid Price (p^M)		High Price (p^H)	
GM	Low (p^L)	100	100	250	50	400	0
	High (p^H)	150	200	150	350	300	300

(5a) [5 points] Derive GM's and FORD's best response functions.

(5b) [5 points] Conclude which pairs of prices (if any) constitute an equilibrium in *dominant* actions, and which pair constitutes a Nash equilibrium.

(5c) [5 points] Is the outcome $\langle p_G, p_F \rangle = \langle p^L, p^H \rangle$ *Pareto superior* to $\langle p^L, p^M \rangle$? Prove your answer.

(5d) [5 points] Solve for a subgame-perfect equilibrium price strategies of a two-stage game in which Ford announces its price before GM does. Also, compute the equilibrium profit level of each firm.

(6) In Ben Barber there are two suppliers of distilled water, labeled firm A and firm B. Distilled water is considered to be a homogenous good. Let p denote the price per gallon, q_A quantity sold by firm A, and q_B the quantity sold by firm B. Firm A and firm B bear unit production costs of $c_A = \$20$ and $c_B = \$100$ per one gallon of water, respectively. That is, firm A is more efficient than firm B. Ann Barber's inverse demand function for distilled water is given by

$$p = 140 - 2Q = 140 - 2(q_A + q_B),$$

where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Ben Barber.

(6a) [10 points] Assuming that the firms compete in prices p_A and p_B, write down each firm's price best response function, and solve for the price each firm sets in a Bertrand-Nash equilibrium.

(6b) [10 points] Suppose that firm B invests in R&D and manages to reduce its unit production cost to $c_B = \$60$. Firm A's unit cost stays the same, $c_A = \$20$. Consider a sequential quantity game in which firm B sets its output level q_B before firm A sets q_A. Compute the Stackelberg equilibrium output and profit levels of the two firms.

(7) [10 points]

		FORD			
		LOW PRICE (p^L)		HIGH PRICE (p^H)	
GM	LOW (p^L)	100	100	400	0
	HIGH (p^H)	150	200	300	300

Suppose the above game is repeated indefinitely in each period $t = 0, 1, 2, \ldots$. Let ρ, where $0 \leq \rho \leq 1$, denotes the firms' common time discount factor. Compute the minimum threshold value of ρ that would make it unprofitable for each firm to unilaterally deviate from the collusive outcome $\langle p_G, p_F \rangle = \langle p^H, p^H \rangle$.

THE END