file=io-w08-final.tex

Last Name (Please PRINT): .....

First Name (PRINT): .....

Your UM I.D. Number: .....

INSTRUCTIONS (please read!)

1. Please make sure that you have 9 pages, including this page. Complaints about missing pages will not be accepted.
2. Please answer all the questions. You are not allowed to use any course material. Calculators are permitted.
3. Maximum Time Allowed: 2 hours (16:00–18:00).
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is "visible" and that your sentences are properly organized and fit into the designated blank space. If you think that your handwriting is poor, please print each word!
10. Good Luck !

Instructor's use only

Problem #	1	2	3	4	5	6	Total
Maximum	20	20	10	10	20	20	100
Points							

(1) Aike (Brand A) and Beebok (Brand B) are leading brand names of fitness shoes. The inverse demand functions for these brands are

$$p_A = 80 - \frac{3}{2}\,q_A - q_B \quad \text{and} \quad p_B = 80 - \frac{3}{2}\,q_B - q_A.$$

Assume that firms A and B do not bear any production costs (that is, $c_A = c_B = 0$). Solve the following problems:

(1a) [10 points] Solve for the Nash-Cournot equilibrium quantity levels q_A^c and q_B^c. Also, compute the resulting equilibrium market prices p_A^c and p_B^c, and profits π_A^c and π_B^c.

(1b) [5 points] Invert the above system of inverse demand functions to obtain the direct demand functions which map prices to quantities (instead of prices as functions of quantities sold). That is compute the coefficients a, b and c of the equations given by $q_A = a - bp_A + cp_B$ and $q_B = a - bp_B + cp_A$.

(1c) [5 points] Solve for the Nash-Bertrand equilibrium price levels p_A^b and p_B^b. Also, compute the resulting equilibrium quantity levels q_A^b and q_B^b, and profits π_A^b and π_B^b. Compare the equilibrium quantities sold, prices, and profit level under the Cournot game to the levels obtained under the Bertrand game. Explain the differences.

(2) Three separate labs consider engaging in R&D for developing an anti-laziness pill (intended to used mainly by students). The value of the patent on this pill is estimated to be $V = \$640$. If more than one firm discovers the patent, the firms equally share the prize ($640/2$ or $640/3$).

These profit-maximizing labs are privately and separately owned. Each lab has a probability $1/4$ of discovering the cure provided that it invests an amount of $I = \$120$ in constructing a research lab. If a firm does not invest, the probability of discovery is zero. Solve the following problems.

(2a) [10 points] Compute the equilibrium number of labs engaging in R&D.

(2c) [10 points] Now suppose that all three labs have been purchased by a single investor. How many labs will be operated under the new single ownership?

(3) Assume that there are two technologies for producing batteries for heavy trucks:
Long-lasting batteries: Each battery lasts for 60 months. Each costs $c_L = \$120$ to produce.
Short-lasting batteries: Each battery lasts for 40 months. Each costs $c_S = \$80$ to produce.

Suppose that all car owners are identical. Each truck owner is willing to pay no more than $v = \$30$ for a one-month service obtained from a car battery. Assume that truck owners do NOT bear any cost of time and transportation when they go to the shop to replace a battery. Solve the following problems:

(3a) [5 points] Which type of battery will be produced by a monopoly seller?

(3b) [5 points] Which type of battery will be produced and sold by a competitive industry?

(4) A monopoly offers a product for sale. The product costs $C = \$60$ to produce. The product may fail with probability $1/4$, hence it is fully operative with probability $\rho = 3/4$. This probability is public information in the sense that it is known to the seller and to all buyers.

The product can be either fully functioning or totally defective. Consumers are willing to pay up to $V = \$120$ for a fully-functioning product. If the product is found to be defective, consumers do not gain any utility. Solve the following problems.

(4a) [4 points] Compute the monopoly's price and profit level assuming that the monopoly does not provide any warranty.

(4b) [6 points] Suppose now that the monopoly provides a repair warranty for the case where the product is found to be defective after purchase. The cost of a repair (borne by the monopoly) is $R = \$40$. Compute the monopoly price and profit level under this repair warranty. Assume that a repaired product becomes a fully-functioning product (which cannot break).

(5) [20 points] Electata is the sole provider of electricity in a remote island near Africa. The company debates how much to invest in electricity generation capacity (measured in Kilowatt/hour). There are no operating costs ($c = 0$) because Electata's generators are installed on river dams. However, the capacity cost is $r = 4$ per Kilowatt/hour generation capacity.

The daytime and nighttime inverse demand functions for electricity are $p_D = 12 - 0.5\,q_D$ and $p_N = 24 - 2q_N$, respectively. Compute Electata's profit-maximizing investment level in capacity K, the daytime price of electricity p_D, and nighttime price, p_N.

(6) The problem facing the manager of the PARADISE Hotel is whether to tie breakfast and a visit to the gym with the room rental (pure tying) or whether to sell the three services separately (no tying). The guests' willingness to pay for each service and the hotel's marginal cost of providing each service are given in the Table below.

Type	Room (R)	Breakfast (B)	Gym (G)	# Guests
Type 1	$\$100$	$\$5$	$\$10$	200
Type 2	$\$60$	$\$10$	$\$10$	800
Marginal Cost	$\mu_R = \$40$	$\mu_B = \$2$	$\mu_G = \$0$	

Solve the two problems on the next page.

(6a) [10 points] Compute the hotel's profit-maximizing room rate p_R, breakfast price p_B, gym entrance fee p_G, and resulting profit π^{NT}, given that each service is sold separately (no tying).

(6b) [10 points] Suppose now that the hotel sells all three services in one package (pure tying). Compute the package's profit-maximizing price p_{RBG} and the corresponding profit level π^{PT}. Conclude whether the hotel should tie the three services in a single package or sell them separately.

THE END