(1a) [10 points Firm A sets q_A to solve

$$\max_{p_A} \pi_A = \left(12 - \frac{q_A + q_B}{3}\right) q_A - 2q_A$$

The first order condition $0 = 12 - 2q_A/3 - q_B/3 - 2$ yields

$$q_A = BR_A(q_B) = 15 - \frac{1}{2}q_B.$$

Similarly, firm B chooses q_B to solve

$$\max_{p_B} \pi_B = \left(12 - \frac{q_A + q_B}{3}\right) q_B - 2q_B$$

The first order condition $0 = 12 - q_A/3 - 2q_B/3 - 2$ yields

$$q_B = BR_B(q_A) = 15 - \frac{1}{2}q_A.$$

Solving the above two best-response function yields $q_A^c = q_B^c = 10$.

(1b) [5 points Aggregate industry output is $Q^c = q_A^c + q_B^c = 20$. Therefore, the equilibrium price is $p^c = 12 - 20/3 = 16/3$. The profit of firm i ($i = A, B$) is:

$$\pi_a = p^c q_i^c - 2q_i = \left(\frac{16}{3} - 2\right) 10 = \frac{100}{3}.$$

(1c) [10 points Firm A takes B's best-best response function into account (instead of taking q_B as given) and chooses q_A to solve

$$\max_{p_A} \pi_A = \left[12 - \frac{q_A}{3} - \frac{1}{3}\left(15 - \frac{1}{2}q_A\right)\right] q_A - 2q_A = \left(7 - \frac{q_A}{6}\right) q_A - 2q_A.$$

The first-order condition yields $0 = 7 - q_A/3 - 2$, and hence $q_A^{sl} = 15$. Substituting into B's best-response function yields $q_B^{sl} = 15 - q_A/2 = 15/2$.

(1d) [5 points Total output is $Q^s = q_A^{sl} + q_B^{sf} = 45/2$. Substituting into the demand function obtains $p^s = 12 - 45/6 = 9/2 = \$4.5$. Substituting into the firms' profit functions yield

$$\pi_A^{sl} = \left(\frac{9}{2} - 2\right) 15 = \frac{75}{2} = \$37.5$$

$$\pi_B^{sf} = \left(\frac{9}{2} - 2\right) \frac{15}{2} = \frac{75}{4} = \$18.75$$

(2a) [10 points] Let ϵ (epsilon) denote a small number, or simply the smallest currency denomination. The firms' best-response functions are:

$$p_A = BR_A(p_B) = \begin{cases} 7 & \text{if } p_B > 7 \\ p_B - \epsilon & \text{if } 2 < p_B \leq 7 \\ 2 & \text{if } p_B \leq 2, \end{cases} \quad \text{and} \quad p_B = BR_B(p_A) = \begin{cases} 7 & \text{if } p_A > 7 \\ p_A - \epsilon & \text{if } 2 < p_A \leq 7 \\ 2 & \text{if } p_A \leq 2. \end{cases}$$

Note that $p^m = \$7$ is the monopoly price, which is computed by $MR^m = 12 - 2q^m/3 = 2$ implying that $q^m = 15$ units and hence $p^m = 12 - 15/3 = \$7$.

(2b) [5 points] The unique Bertrand-Nash equilibrium is $p^b_A = p^b_B = \$2$ (since the firms have identical unit costs, price competition leads to unit cost pricing). Clearly, each firm earns zero profit, so that $\pi^b_A = \pi^b_B = 0$. Total output is solved from $2 = 12 - Q/3$ implying that $Q^b = 30$. Hence, $q^b_A = q^b_B = 15$ units.

(2c) [5 points] There are "many" equilibria in the form of $p_A \geq 2$ (equilibrium strategy of firm A) and

$$p_B = BR_B(p_A) = \begin{cases} 7 & \text{if } p_A > 7 \\ p_A - \epsilon & \text{if } 2 < p_A \leq 7 \\ 2 & \text{if } p_A \leq 2, \end{cases}$$

which is the equilibrium strategy of firm B. Notice that firm A (first mover) is indifferent between setting $p_A = \$2$ and $p_A > \$2$ since it is being undercut by firm B in either case, and therefore makes zero profit.

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(3) [10 points] The demand price elasticity is -2 in the nonstudents' market, and -3 in the students' market. In the nonstudents' market, the monopoly sets p_N to solve

$$p_N \left[1 + \frac{1}{-2}\right] = \$2 \quad \text{yielding } p_N = \$4 \text{ and hence } q_N = \frac{240}{4^2} = 15.$$

In the students' market, the monopoly sets p_S to solve

$$p_S \left[1 + \frac{1}{-3}\right] = \$2 \quad \text{yielding } p_S = \$3 \text{ and hence } q_S = \frac{540}{3^3} = 20.$$

(4a) [10 points] We must analyze two cases. $p > 50¢$, in which case only market 1 is served; and $p \leq \$50$ in which markets 1 and 2 are both served.

If only market 1 is served, the monopoly sets $MR_1 = 60 - 2q_1 = 30$ yielding $q_1 = 15$ and hence $p_1 = 60 - 15 - 45¢$. Since $p_1 < 50¢$, consumers in market 2 also buy, hence both markets are served and this computation becomes irrelevant.

If both market 1 and 2 are served, the aggregate direct demand function is $q_{12} = 60 - p + 50 - p = 110 - 2p$. The inverse aggregate demand is $p_{12} = 55 - q_{12}/2$. Therefore, the monopoly solves

$$MR_{12} = 55 - q_{12} = 30 \quad \text{yielding } q_{12} = 25 \text{ and hence } p_{12} = 55 - \frac{25}{2} = \frac{85}{2} = 42.5¢.$$

Observe that since $p_{12} < 50¢$, the monopoly indeed sells in both markets.

To compute the profit,

$$\pi_{12} = \left(\frac{85}{2} - 30\right) 25 = \frac{625}{2} = 312.5¢.$$

(4b) [10 points] The previous section demonstrated that the profit-maximizing price is $p_{12} = 42.5$ when selling to markets 1 and 2 only. Since $p_{12} = 42.5 > 40$, consumers in market 3 will not buy at this price. Therefore, it remains to investigate whether reducing the price below $40¢$ (thereby serving consumers in all 3 markets) would enhance the monopoly profit beyond the profit made when only markets 1 and 2 are served.

So, suppose that $p_{123} \leq 40$ so all 3 market are served. The aggregate direct demand function is $q_{123} = 60 - p + 50 - p + 40 - p = 150 - 3p$. The inverse demand function is $p_{123} = 50 - q_{123}/3$. Therefore, the monopoly solves

$$MR_{123} = 50 - \frac{2}{3}q_{123} = 30, \quad \text{yielding } q_{123} = 30 \text{ and hence } p_{123} = 50 - \frac{30}{3} = 40¢.$$

The resulting profit is

$$\pi_{123} = (40 - 30)30 = 300 < \frac{625}{2} = \pi_{12}.$$

Therefore, $p = 85/2 = 42.5$ is the profit-maximizing price. Under this price, the quantity sold in each market is

$$q_1 = 60 - \frac{85}{2} = 17.5, \quad q_2 = 50 - \frac{85}{2} = 7.5, \quad \text{and} \quad q_3 = 0.$$

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(5) [10 points]

$$t_A = BR_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O \end{cases} \quad \text{and} \quad t_B = BR_B(t_A) = \begin{cases} O & \text{if } t_B = N \\ N & \text{if } t_B = O. \end{cases}$$

There is no Nash equilibrium in this game (no outcome lies on both best-response functions). To see this, note that

$$t_A = N \Longrightarrow t_B = O \Longrightarrow t_A = O \Longrightarrow t_B + N \Longrightarrow t_A = N \ldots.$$

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(6) [10 points] Section 2 of the 1914 Clayton Act states that price discrimination is unlawful if its effect is "to lessen competition or tend to create a monopoly...or to injure destroy or prevent competition." In addition, price differentials are also allowed to account for "differences in the cost of manufactures, sale or delivery."

This, in part, implies that price discrimination that does not reduce competition should not be viewed as illegal.

THE END