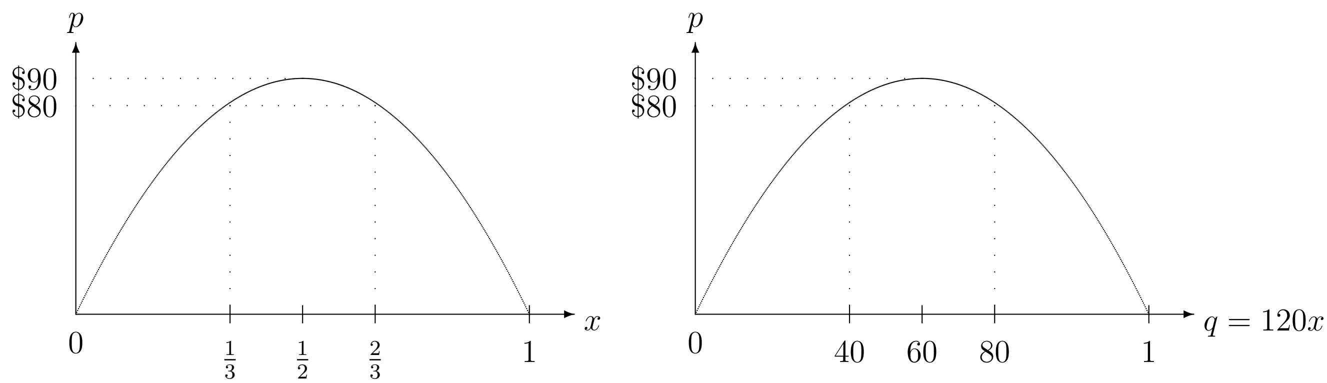
(1a) [5 points] The utility function implies that the market inverse demand function (as a function of the # types subscribed) is $p = (3 - 3x)120x$, which is drawn below on the left.



The figure on the right "stretches" the horizontal axis by 120 to obtain price as a function of aggregate number of subscriptions (instead of the number of types). To find the maximum solve

$$0 = \frac{dp}{dx} = \frac{d[(3-3x)120x]}{dx} = 360(1-2x) \Longrightarrow x = \frac{1}{2} \Longrightarrow p = (3 - 3 \cdot \frac{1}{2})120 \cdot \frac{1}{2} = \$90.$$

(1b) [5 points] To find the critical mass solve for x satisfying $80 = (3-3x)120x$ or, using a quadratic form, $360x^2 - 360x - 80 = 0$. Therefore,

$$x = \frac{360 \pm \sqrt{360^2 - 4 \cdot 360 \cdot 80}}{2 \cdot 360} = \frac{360 \pm 120}{720} \in \left\{ \frac{1}{3} \,;\, \frac{2}{3} \right\}.$$

Hence, the critical mass is

$$q^{cm} = \frac{1}{3}\, 120 = 40 \text{ subscribers.}$$

(1c) [5 points] A monopoly service provider maximizes profit by solving

$$\max_x \pi = p(x)120x = [(3-3x)120x]120x = 43200(x^2 - x^3).$$

The first- and second-order condition for a maximum are

$$0 = \frac{d\pi}{dx} = 43200(2x - 3x^2) \quad \text{and} \quad \frac{d^2\pi}{dx^2} = 43200(2 - 6x).$$

The first-order condition yields two solutions: $x = 0$ and $x = 2/3$. But,

$$\frac{d^2\pi}{dx^2}(0) = 43200(2 - 6 \cdot 0) > 0 \quad \text{and} \quad \frac{d^2\pi}{dx^2}(1/3) = 43200\left(2 - 6 \cdot \frac{2}{3}\right) < 0.$$

Hence, $x = 2/3$ is a unique maximum. The number of subscribers, price, and profit are

$$q = 120 \cdot \frac{2}{3} = 80, \quad p = \left(3 - 3 \cdot \frac{2}{3}\right) 120 \cdot \frac{2}{3} = \$80, \quad \text{and} \quad \pi = p \cdot q = 80 \cdot 80 = \$6400.$$

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(2) [5 points] The total traffic on the incumbent's local network is $Q = q_L^I + d^I + d^E = 600,000$ phone calls. The access fee under the fully distributed costs rule is

$$a = \mu_L^I + \frac{\phi}{Q} = 0.5 + \frac{1,200,000}{600,000} = \frac{5}{2} = 2.5.$$

That is, the entrant pays the incumbent's marginal cost of terminating a LD call on the local network plus the share in the maintenance cost of the local network.

The access fee under the ECPR is

$$a = p^I - \mu^I = 4 - 3 = 1.$$

Under this rule, the entrant "compensates" for the loss of having consumers placing the phone call on the entrant's LD network rather than on the incumbent's LD network

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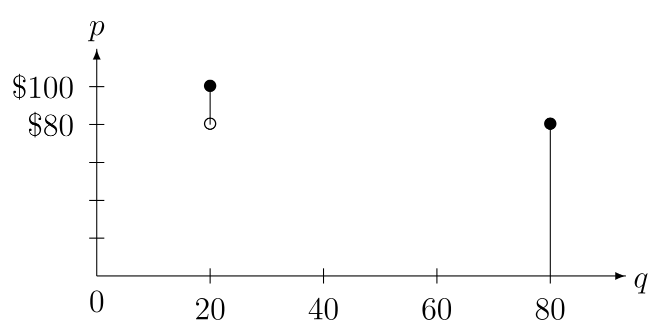
(3a) [10 points] Suppose only type H subscribe. Hence, it must be that $5 \cdot 20 - p \geq 0$, or $p \leq 100$. Now, suppose that both types subscribe. Then, $20 + 60 - p \geq 0$, or $p \leq 80$. Hence,



Formally,

$$q(p) = \begin{cases} 0 & \text{if } p > 100 \\ 20 & \text{if } 80 < p \leq 100 \\ 20 + 60 & \text{if } p \leq 80. \end{cases}$$

(3b) [10 points] In view of the above demand function, the monopoly's profit as function of the connection fee is

$$\pi(p) = \begin{cases} 0 & \text{if } p > 100 \\ (100 - 75)20 = \$500 & \text{if } p = 100 \\ (80 - 75)80 = \$400 & \text{if } p = 80. \end{cases}$$

Therefore, $p = \$100$ is the profit-maximizing price. Under this price, only 20 consumers subscribe to this service.

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(4) [15 points] Bank A (the largest) maximizes its fee f_A subject to

$$\pi_C = 200 \cdot 60 \geq (200 + 600)(60 - \delta_A) \Longrightarrow \delta_A = \$45.$$

Bank B maximizes its fee f_B subject to

$$\pi_C = 200 \cdot 60 \geq (200 + 400)(60 - \delta_B) \Longrightarrow \delta_B = \$40.$$

Bank C (the smallest) maximizes its fee f_C subject to

$$\pi_A = 600 \cdot 60 \geq (200 + 600)(60 - \delta_C) \Longrightarrow \delta_C = \$15.$$

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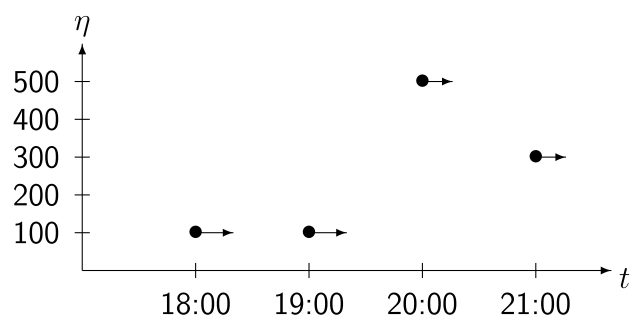
(5) [15 points] The distribution of viewers' ideal watching time is plotted on the figure below.



The arrows indicate that viewers can postpone their watching but are unable advance it due to work obligations. The broadcasting time for network i as a function of broadcasting time of network j is

$$t_i = BR_i(t_j) = \begin{cases} 21 & \text{if } t_j = 18 \text{ (hence, } \pi_i = 900\rho) \\ 21 & \text{if } t_j = 19 \text{ (hence, } \pi_i = 800\rho) \\ 20 & \text{if } t_j = 20 \text{ (hence, } \pi_i = 350\rho) \\ 20 & \text{if } t_j = 21 \text{ (hence, } \pi_i = 700\rho). \end{cases}$$

The unique Nash equilibrium is $\langle t_A, t_B \rangle = \langle 20, 20 \rangle$. The networks split the $100 + 100 + 500$ viewers whose ideal watching time are: 18:00, 19:00, and 20:00 (other viewers cannot watch). Therefore, $\pi_A(20, 20) = \pi_B(20, 20) = 350\rho$.

(6a) [5 points] Yes, $n_{HE} = 40$ and $n_{BE} = 60$ is a Nash equilibrium. To prove, we show that no one can benefit from deviating from $n_{HE} = 40$ and $n_{BE} = 60$. Given $n_{HE} = 40$ and $n_{BE} = 60$, the utility of a Bengali native speaker is

$$U_B = \begin{cases} 60 & \text{does not study} \\ 70 & \text{studies Hindi} \\ 60 + 0 + 40 - 30 = 70 & \text{studies English} \end{cases}$$

Hence, Bengali speakers don't have an incentive to deviate and they will all study English, $n_{BE} = 60$. Similarly, the utility of an Hindi native speaker is

$$U_H = \begin{cases} 60 & \text{does not study} \\ 70 & \text{studies Bengali} \\ 40 + 0 + 60 - 30 = 70 & \text{studies English} \end{cases}$$

Hence, Hindi speakers don't have an incentive to deviate and they will all study English, $n_{HE} = 40$.

(6b) [10 points]

I. $CS^I = 60(40 + 0 + 60 - 30) + 40(60 + 0 + 40 - 30) = 7000$.

II. $CS^{II} = 60(100 - 30) + 40(40 + 60) = 8200$.

III. $CS^{III} = 60(60 + 40) + 40(100 - 30) = 8800.$

(7a) [10 points] Since route 1 and route 2 passengers fly directly, a monopoly airline would set $p_1 = p_2 = 12$. To set the profit-maximizing p_3, two cases must be analyzed, First, $p_3 = 1$ in which case $q_3 = 50 + 10$ and hence,

$$\Pi(p_3 = 8) = 2(50 + 10)12 + (50 + 10)8 - 2 \cdot 200 = 1520.$$

The second option is to set a high price, $p_1 2$, so $q_3 = 10$. In this case,

$$\Pi(p_3 = 12) = 2(50 + 10)12 + 10 \cdot 12 - 2 \cdot 200 = 1160.$$

Therefore, under the HS network, the monopoly airline sets $p_1 = p_2 = 12$ and $p_3 = 8$, and earns $\Pi^{HS} = 1520$.

(7b) [5 points] Under FC network, all passengers fly directly to destination, hence the monopoly airline can charge $p_i = 12$. Total profit is

$$\Pi^{FC} = \pi_1 + \pi_2 + \pi_3 = 3(50 + 10)12 - 3 \cdot 200 = 1560.$$

Comparing (7a) with (7b) reveals that the FC is the most profitable network of operation.

THE END

