(1a) [10 points Service provider A maximizes p_A subject to

$$\pi_B = 120p_B \geq 240(p_A - 60 + 0.5 \cdot 120).$$

Similarly, service provider B maximizes p_B subject to

$$\pi_A = 120p_A \geq 240(p_B - 90 + 0.5 \cdot 120).$$

Solving the two equation under equality yields

$$p_A^I = 20, \quad p_B^I = 40, \quad \pi_A^I = 120 \cdot 20 = 2400, \quad \text{and} \quad \pi_B^I = 120 \cdot 40 = 4800,$$

where superscript I indicates equilibrium values under incompatible networks.

(1b) [10 points Service provider A maximizes p_A subject to

$$\pi_B = 120p_B \geq 240(p_A - 60).$$

Similarly, service provider B maximizes p_B subject to

$$\pi_A = 120p_A \geq 240(p_B - 90).$$

Solving the two equation under equality yields

$$p_A^C = 140, \quad p_B^C = 160, \quad \pi_A^C = 120 \cdot 140 = 16,800, \quad \text{and} \quad \pi_B^C = 120 \cdot 160 = 19,200,$$

where superscript C indicates equilibrium values under incompatible networks.

(1c) [5 points Under incompatible networks:

$$U_A^I = \frac{1}{2}120 - 20 = 40 \quad \text{and} \quad U_B^I = \frac{1}{2}120 - 40 = 20.$$

Under compatible networks:

$$U_A^C = \frac{1}{2}240 - 140 = -20 < 40 \quad \text{and} \quad U_B^C = \frac{1}{2}240 - 160 = -40 < 20.$$

Hence, all types of consumers are worse off under compatible networks. This is because of the very high prices both service providers charge when they sell compatible services compared with the prices they charge when they sell incompatible services.

(1d) [5 points Social welfare under incompatible networks is

$$W^I = 120U_A^I + 120U_B^I + \pi_A^I + \pi_B^I = 120 \cdot 40 + 120 \cdot 20 + 2400 + 4800 = 14,400.$$

Social welfare under compatible networks is

$$W^C = 120U_A^C + 120U_B^C + \pi_A^C + \pi_B^C = 120(-20) + 120(-40) + 16800 + 19200 = 28,800 > 14,400.$$

Hence, social welfare is higher when the dating services are compatible. The profit firms gain from selling compatible services dominates the reduction in consumer welfare.

[Downloaded from `www.ozshy.com`]

(2) [15 points] The firm makes nonnegative profit as long as $TR(q) = pq \geq TC(q) = \phi + \mu q$. Hence, if

$$q \geq \frac{120000}{p-\mu} = \frac{120000}{45-1} = 2727.27.$$

Therefore, TAXME™ should sell at least 2728 copies in order to make nonnegative profit.

(3a) [10 points] Notice that all consumers are indifferent between the two systems when they sell of equal prices, $p_{AA} = p_{BB}$. In this case all consumers gain utility equal to $\beta - \delta$ (minus price) regardless of which system they buy. Therefore, if $p_{AA} < p_{BB}$ all consumers buy system $X_A Y_A$, and if if $p_{AA} > p_{BB}$ all consumers buy system $X_B Y_B$. This generates a price competition leading to marginal-cost pricing ($p_{AA} = p_{BB} = 0$ in the present case).

A formal proof for the above intuition is as follows:

$$\pi_B^I = 100 p_{BB}^I = 300(p_{AA}^I - 0) \quad \text{and} \quad \pi_B^I = 200 p_{AA}^I = 300(p_{BB}^I - 0)$$

yields a unique solution in which $p_{AA}^I = p_{BB}^I = 0$, where superscript "I" denotes incompatible systems. Therefore, both firms earn zero profits, $\pi_A^I = 0 \cdot q_{AA} = 0$ and $\pi_B^I = 0 \cdot q_{BB} = 0$.

(3b) [10 points] In equilibrium, Firm A sells 200 units of X_A and 100 units of Y_A. Firm B sells 100 units of X_B and 200 units of Y_B.

First, we look at the market for component X. Firm A maximizes p_A^X subject to

$$\pi_B^X = 100 p_B^X \geq 300(p_A^X - \delta).$$

Firm B maximizes p_B^X subject to

$$\pi_A^X = 200 p_A^X \geq 300(p_B^X - \delta).$$

The UPE prices and profits (from component X only) are therefore

$$p_A^X = \frac{12\delta}{7}, \quad p_B^X = \frac{15\delta}{7}, \pi_A^X = 200 p_A^X = \frac{2400\delta}{7}, \quad \text{and} \quad \pi_B^X = 100 p_B^X = \frac{1500\delta}{7}.$$

Next, we explore the competition in the market for component Y. Firm A maximizes p_A^Y subject to

$$\pi_B^Y = 200 p_B^Y \geq 300(p_A^Y - \delta).$$

Firm B maximizes p_B^Y subject to

$$\pi_A^Y = 100 p_A^Y \geq 300(p_B^Y - \delta).$$

The UPE prices and profits (from component X only) are therefore

$$p_A^Y = \frac{15\delta}{7}, \quad p_B^Y = \frac{12\delta}{7}, \pi_A^X = 100 p_A^Y = \frac{1500\delta}{7}, \quad \text{and} \quad \pi_B^Y = 200 p_B^X = \frac{2400\delta}{7}.$$

Therefore, the total profit of each firm when both produce compatible components are

$$\pi_A = \pi_A^X + \pi_A^Y = \frac{3900\delta}{7} \quad \text{and} \quad \pi_B = \pi_B^X + \pi_B^Y = \frac{3900\delta}{7}.$$

Clearly, both firms earn higher profits when they produce compatible components (relative to incompatible components).

[Downloaded from `www.ozshy.com`]

(4) [15 points] In an UPE, firm A maximizes p_A subject to:

$$\pi_B^I = 1000(p_B^I - 120) \geq 2000(p_A - 10 - 120 + s_B - s_A) = 2000(p_A - 160).$$

firm B maximizes p_B subject to:

$$\pi_A^I = 1000(p_B^I - 120) \geq 2000(p_B - 10 - 120 + s_A - s_B) = 2000(p_B - 100).$$

Solving the above two equations (under equalities) yields $p_A^I = 160$ and $p_B^I = 120$. Hence, equilibrium profits are

$$\pi_A^I = 1000(160 - 120) = 40,000 \quad \text{and} \quad \pi_B^I = 1000(120 - 120) = 0.$$

(5a) [10 points] When software is unprotected, type I consumers will use the software but will not buy it. Therefore, type O will buy the software (rather than pirate it) if

$$400 + 0.5q - p \geq 0.5q, \quad \text{hence if} \quad p \leq 400.$$

Therefore, TAXME™ sells 100 packages for a price of $p = 400$ and earns a profit of $\pi^u = 100 \cdot 400 = 40,000$.

(5b) [10 points] If TAXME™ sets a low price (so all 300 consumers buy the software) it can sets $p = 150$. Notice that under this price type I consumers buy this software because $0.5 \cdot 300 - 150 \geq 0$. type O consumers will also buy this software because $400 + 0.5 \cdot 300 - 150 \geq 0$. The resulting profit is $\pi^p = 300 \cdot 150 = 45,000$.

If TAXME™ sets a high price (so only the 100 type O consumers buy the software) it sets $p = 450$. Under this price, type O consumers buy it because $400 + 0.5 \cdot 100 - 450 \geq 0$. Type I consumers don't buy it even if there are 300 users because $0.5 \cdot 300 - 450 < 0$. Under $p = 450$, $\pi^u = 450 \cdot 100 = 45,000$.

Hence, the seller earns a profit of $45,000$ in both cases. In this example, the seller earns a higher profit when software is protected.

THE END

[Downloaded from `www.ozshy.com`]