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Last Name (Please PRINT): ....

First Name (PRINT): ....

Your UM I.D. Number: ....

INSTRUCTIONS (please read!)

1. Please make sure that you have 8 pages, including this page. Complaints about missing pages will not be accepted.
2. Please answer all the questions. You are not allowed to use any course material. Calculators are permitted.
3. Maximum Time Allowed: 1 hour and 20 minutes (11:40–13:00).
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is "visible" and that your sentences are properly organized and fit into the designated blank space. If you think that your handwriting is poor, please print each word!
10. Good Luck !

Instructor's use only

Problem #	1	2	3	4	5	Total
Maximum	30	15	20	15	20	100
Points						

(1) The online dating industry consists of two online service providers labeled as A and B. Assume production is costless ($\mu_A = \mu_B = 0$). Let p_A denote the price charged by provider A, and p_B the price charged by provider B.

There are 120 consumers who are A-oriented, and 120 consumers who are B-oriented. Let q_A be the number of consumers who subscribe to A, and q_B the number of consumers who subscribe to B. Formally, the utility of an A-oriented and B-oriented consumer is given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} \frac{1}{2}q_A - p_A & \text{subscribes to } A \\ \frac{1}{2}q_B - p_B - 60 & \text{subscribes to } B \end{cases} \quad \text{and} \quad U_B \stackrel{\text{def}}{=} \begin{cases} \frac{1}{2}q_A - p_A - 90 & \text{subscribes to } A \\ \frac{1}{2}q_B - p_B & \text{subscribes to } B \end{cases}$$

Notice that this model describes an online service industry in which A-oriented subscribers find it easier to switch from provider A to provider B, than for B-oriented to switch from B to A.

(1a) [10 points] Compute the UPE prices and the profit of each provider assuming that the two online providers are *incompatible* in the sense that A subscribers can date only other A subscribers and B subscribers can date only other B subscribers.

(1b) [10 points] Compute the UPE prices and profit level of each provider assuming that the two dating online providers are *compatible* in the sense that A and B subscribers can date all other A and B subscribers.

(1c) [5 points] Compute the utility levels of A and B subscribers when the dating services are incompatible and compatible. Which subscribers prefer compatible dating services and which are better off when these services are incompatible? Provide some intuition for your result.

(1d) [5 points] Compute and compare social welfare levels under incompatible and compatible online dating services.

(2) [15 points] Consider the $\text{T}\textsc{ax}\text{M}\textsc{e}^{\text{TM}}$ company which is a leader in tax preparation software. Suppose the software costs $\phi = \$120,000$ to develop, and in addition $\mu = \$1$ to duplicate and sell each copy.

Assuming that $\text{T}\textsc{ax}\text{M}\textsc{e}^{\text{TM}}$ sells each software package for $p = \$45$, compute the minimum number of copies that must be sold in order for $\text{T}\textsc{ax}\text{M}\textsc{e}^{\text{TM}}$ to make positive profit.

(3) Consider a system composed of two components labeled X and Y. There are two firms producing two different systems (different brands), at zero production cost. Firm A produces components X_A and Y_A, and firm B produces X_B and Y_B. In this market there are 200 (two-hundred) consumers labeled AB, and 100 (one-hundred) consumers labeled BA. The Utility function of a consumer (i,j) is

$$U_{i,j} = \begin{cases} \beta - \left(p_i^X + p_j^Y\right) & \text{buys system } X_i Y_j \\ \beta - \left(p_j^X + p_j^Y\right) - \delta & \text{buys system } X_j Y_j \\ \beta - \left(p_i^X + p_i^Y\right) - \delta & \text{buys system } X_i Y_i \\ \beta - \left(p_j^X + p_i^Y\right) - 2\delta & \text{buys system } X_j Y_i, \end{cases} \quad \text{where} \quad i, j = A, B.$$

(3a) [10 points] Let p_{AA} denote the price of system $X_A Y_A$, and p_{BB} denote the price of system $X_B Y_B$. Assuming that firms produce *incompatible* components, compute the Undercut-Proof equilibrium system prices and the equilibrium profit levels π_A and π_B.

(3b) [10 points] Compute the component prices in an Undercut-proof equilibrium assuming that the firms produce compatible components. Compute the profit levels and compare them to the profit levels when the firms produce incompatible components.

(4) [15 points] Consider the market for computers with two brand producing firms labeled as A and B. Assume that each unit of hardware costs 120 to produce, so $c_A = c_B = 120$. Let p_A and p_B denote the prices of hardware A and B, respectively. Suppose that there are $s_A = 90$ software packages written specifically for hardware A, and $s_B = 60$ specifically for hardware B. There are $\eta_A = 1000$ A-oriented consumers and $\eta_B = 1000$ B-oriented consumers whose utility functions are given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} s_A - p_A & \text{buys } A \text{ ; } A \text{ is incompatible} \\ s_B - p_B - 10 & \text{buys } B \text{ ; } B \text{ is incompatible} \\ s_A + s_B - p_A & \text{buys } A \text{ ; } A \text{ is } B\text{-compatible} \\ s_A + s_B - p_B - 10 & \text{buys } B \text{ ; } B \text{ is } A\text{-compatible,} \end{cases} \tag{1}$$

and

$$U_B \stackrel{\text{def}}{=} \begin{cases} s_A - p_A - 10 & \text{buys } A \text{ ; } A \text{ is incompatible} \\ s_B - p_B & \text{buys } B \text{ ; } B \text{ is incompatible} \\ s_A + s_B - p_A - 10 & \text{buys } A \text{ ; } A \text{ is } B\text{-compatible} \\ s_A + s_B - p_B & \text{buys } B \text{ ; } B \text{ is } A\text{-compatible.} \end{cases}$$

Compute the UPE hardware prices, p_A^I and p_B^I, and the firms' profits, π_A^I and π_B^I, assuming that the two hardware are *incompatible*, in the sense that hardware A can run only A-specific software whereas hardware B can run only B-specific software.

(5) Consider a market for a popular tax preparation software $\textsc{TaxMe}^{\text{TM}}$. There are 100 (one-hundred) support-oriented (type-O) users, and 200 (two-hundred) support-independent (type-I) users, with utility functions given by

$$U^O \stackrel{\text{def}}{=} \begin{cases} 400 + 0.5q - p & \text{buys the software} \\ 0.5q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases} \quad U^I \stackrel{\text{def}}{=} \begin{cases} 0.5q - p & \text{buys the software} \\ 0.5q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases}$$

where q denotes the number of users of this software (which includes the number of buyers and the number of pirates, if piracy takes place). Suppose that the software is costless to produce and costless to protect. Also, assume that $\textsc{TaxMe}^{\text{TM}}$ provides support only to those consumers who buy the software.

(5a) [10 points] Suppose that $\textsc{TaxMe}^{\text{TM}}$ is *not* copy protected, so piracy is an option for every consumer. Calculate the software seller's profit-maximizing price and the corresponding profit level.

(5b) [10 points] Suppose that $\textsc{TaxMe}^{\text{TM}}$ is copy protected, so piracy is impossible. Calculate the software seller's profit-maximizing price and the corresponding profit level. Does $\textsc{TaxMe}^{\text{TM}}$ benefit from protecting its software against piracy?

THE END