(1a) [7 points] Each resident takes the demand by all other residents q_j for $i \neq j$ as given and chooses her Internet usage level q_i to maximize the above utility. The first and second-order conditions are

$$0 = \frac{dU_i}{dq_i} = \frac{1}{2\sqrt{q_i}} - \frac{1}{30} - p \quad \text{and} \quad \frac{d^2U_i}{d(q_i)^2} = \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)(q_i)^{-\frac{3}{2}} < 0.$$

Because, Internet is free, setting $p = 0$ yields $q_i = 225$ which is the equilibrium demand by each resident in Mbps. Hence, aggregate demand is $Q = 3 \cdot 225 = 675$ Mbps. Clearly, the network is heavily congested because $Q = 625 > 30 = \bar{Q}$.

(1b) [7 points] The social planner chooses a uniform consumption level q by each resident to maximize social welfare given by

$$\max_q W = 12U_i + \pi^{\text{ISP}} = 3\left[\sqrt{q} - \frac{Q}{30} - pq\right] + 3pq = 3\left[\sqrt{q} - \frac{3q}{30}\right].$$

Clearly, total consumer expenditure cancels out with total profit of the ISP. Also, note that $Q = 3q$. The first-order condition for a maximum is

$$0 = \frac{dW}{dq} = 3\left[\frac{1}{2\sqrt{q}} - \frac{3}{30}\right] \Longrightarrow q^* = 25 \quad \text{and} \quad Q^* = 3q = 75.$$

The network is still congested because $Q^* = 75 > 30$, but much less congested compared with the equilibrium level.

(1c) [6 points] To find the price which would induce all resident to consume at the socially-optimal level note from part (a) that residents choose their consumption level q_i to satisfy

$$p = \frac{1}{2\sqrt{q_i}} - \frac{1}{30} = \frac{1}{2\sqrt{25}} - \frac{1}{30} = \frac{1}{15} \approx 0.066.$$

(2a) [5 points] Total cost of operating a fully-connected network is

$$TC^{FC} = 50 + 40 + 50 + \sqrt{1800} + \sqrt{700} + \sqrt{1800} \approx \$251.31.$$

Total cost of operating hub-and-spoke network (city B as the hub) is

$$TC_B^{HS} = 50 + 40 + \sqrt{1800 + 1800} + \sqrt{1800 + 700} = \$200.$$

Therefore, a hub-and-spoke network is less costly to operate.

(2b) [10 points] Total cost of operating hub-and-spoke network (city A as the hub) is

$$TC_A^{HS} = 50 + \sqrt{q_2 + q_1} + 50 + \sqrt{q_2 + q_3} = 100 + \sqrt{1800 + 700} + \sqrt{1800 + 700} = \$200.$$

Total cost of operating hub-and-spoke network (city B as the hub) is

$$TC_B^{HS} = 50 + \sqrt{q_3 + q_1} + 50 + \sqrt{q_3 + q_2} = 90 + \sqrt{1800 + 1800} + \sqrt{1800 + 700} = \$200.$$

Total cost of operating hub-and-spoke network (city C as the hub) is

$$TC_C^{HS} = 40 + \sqrt{q_1 + q_2} + 50 + \sqrt{q_1 + q_3} = 90 + \sqrt{1800 + 1800} + \sqrt{1800 + 700} = \$200.$$

Hence, the choice of a hub city does not have any effect on total cost.

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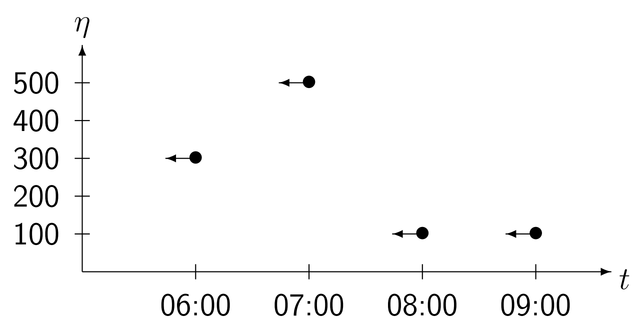
(3) See the solution to Exercise 2 on page 131. Set $\eta = 300$.

(4a) [5 points] The distribution of viewers' ideal watching time is plotted on the figure below.



The arrows indicate that viewers can postpone their watching but are unable advance it due to work obligations. The broadcasting time for network i as a function of broadcasting time of network j is

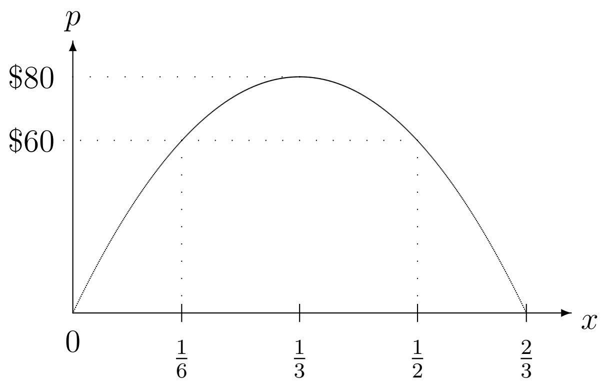
$$t_i = BR_i(t_j) = \begin{cases} 7 & \text{if } t_j = 6 \text{ (hence, } \pi_i = 700\rho) \\ 7 & \text{if } t_j = 7 \text{ (hence, } \pi_i = 350\rho) \\ 6 & \text{if } t_j = 8 \text{ (hence, } \pi_i = 800\rho) \\ 6 & \text{if } t_j = 9 \text{ (hence, } \pi_i = 900\rho). \end{cases}$$

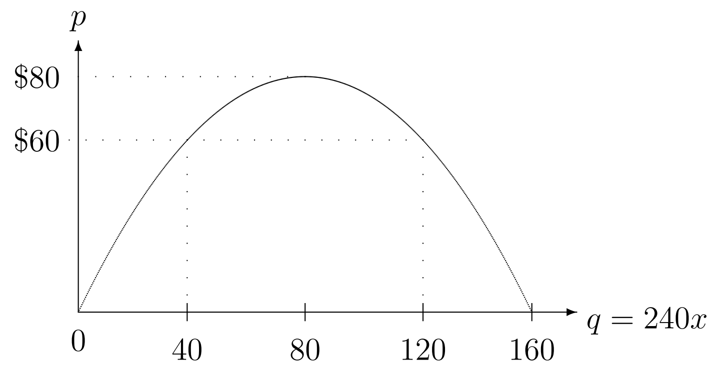
(4b) [5 points] The unique Nash equilibrium is $\langle t_A, t_B \rangle = \langle 7, 7 \rangle$. The networks split the $100+100+500$ viewers whose ideal watching time are: 07:00, 08:00, and 09:00 (other viewers cannot watch). Therefore, $\pi_A(7,7) = \pi_B(7,7) = 350\rho$.

(4c) [5 points] A single network would choose to broadcast the morning news at $06:00$, thereby earning

$$\pi_A(6) = 1000\rho > \pi_A(7) = 700\rho > \pi_A(8) = 200\rho > \pi_A(9) = 100\rho.$$

(5a) [5 points] The utility function implies that the market inverse demand function (as a function of the # types subscribed) is $p = (2 - 3x)240x$, which is drawn below on the left.





The figure on the right "stretches" the horizontal axis by 240 to obtain price as a function of aggregate number of subscriptions (instead of the number of types). To find the maximum solve

$$0 = \frac{dp}{dx} = \frac{d[(2-3x)240x]}{dx} = 480(1-3x) \Longrightarrow x = \frac{1}{3} \Longrightarrow p = (2 - 3 \cdot \frac{1}{3})240 \cdot \frac{1}{3} = \$80.$$

(5b) [5 points] Solving

$$p = 60 = (2-3x)240x \quad \text{yields} \quad x = \frac{1}{6} \quad \text{or} \quad x = \frac{1}{2}.$$

Hence, the critical mass is $q^{cm} = 240/6 = 40$ connections.

(5c) [5 points] The monopoly service provider chooses the number of connections q to solve

$$\max_q = \pi = [(2-3x)240x]240x = 57600(2-3x)x^2.$$

The first-order condition for a maximum is

$$0 = \frac{d\pi}{dx} = \frac{d[57600(2-3x)x^2]}{dx} = 57600x(4-9x) \Longrightarrow x = \frac{4}{9} \Longrightarrow q = \frac{4}{9}240 = \frac{320}{3} \approx 106.66.$$

Hence, the connection fee should be set to

$$p = \left(2 - 3\,\frac{4}{9}\right) 240\,\frac{4}{9} = \frac{640}{9} \approx \$71.11 \quad \text{and} \quad \pi = \frac{640}{9} \times \frac{320}{3} \approx \$7585.$$

(6a) [5 points] True. $n_{HE} = 40$ and $n_{BE} = 60$ is a Nash equilibrium. To prove, we show that no one can benefit from deviating from $n_{HE} = 40$ and $n_{BE} = 60$. Given $n_{HE} = 40$ and $n_{BE} = 60$, the utility of a Bengali native speaker is

$$U_B = \begin{cases} 60 & \text{does not study} \\ 100 - 50 = 50 & \text{studies Hindi} \\ 60 + 0 + 40 - 20 = 80 & \text{studies English} \end{cases}$$

Hence, Bengali speakers don't have an incentive to deviate and they will study English, $n_{BE} = 60$. Similarly, the utility of an Hindi native speaker is

$$U_H = \begin{cases} 40 & \text{does not study} \\ 100 - 50 = 50 & \text{studies Bengali} \\ 40 + 0 + 60 - 20 = 80 & \text{studies English} \end{cases}$$

Hence, Hindi speakers don't have an incentive to deviate and they will also study English, $n_{HE} = 40$.

(6b) [5 points] True. $n_{HE} = 40$ and $n_{BE} = 0$ is a Nash equilibrium. To prove, we show that no one can benefit from deviating from $n_{HE} = 40$ and $n_{BE} = 0$. Given $n_{HE} = 40$ and $n_{BE} = 0$, the utility of a Bengali native speaker is

$$U_B = \begin{cases} 60 + 40 = 100 & \text{does not study} \\ 100 - 50 = 50 & \text{studies Hindi} \\ 60 + 0 + 0 - 20 = 40 & \text{studies English} \end{cases}$$

Hence, Bengali speakers don't have an incentive to deviate and they will not study any new language, $n_{BE} = 0$. Basically, Bengali speakers "free ride" on the investment made by Hindi speakers. Similarly, the utility of an Hindi native speaker is

$$U_H = \begin{cases} 40 & \text{does not study} \\ 100 - 50 = 50 & \text{studies Bengali} \\ 40 + 60 + 0 - 20 = 80 & \text{studies English} \end{cases}$$

Hence, Hindi speakers don't have an incentive to deviate and they will all study Bengali, $n_{HB} = 40$.

(6c) [10 points]

I. $CS^{I} = 60(40 + 0 + 60 - 20) + 40(60 + 0 + 40 - 20) = 8000$.

II. $CS^{II} = 60(100 - 50) + 40(40 + 60) = 7000$.

III. $CS^{III} = 60(60 + 40) + 40(100 - 50) = 8000$.

IV. $CS^{IV} = 60 \cdot 60 + 40 \cdot 40 = 5200$.

Therefore, the two socially-optimal outcome is that either all Hindi native speakers learn Bengali, or instead that everybody learns English.

THE END