(1a) [5 points]

$$t_A = R_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O \end{cases} \quad \text{and} \quad t_B = R_B(t_A) = \begin{cases} O & \text{if } t_A = N \\ N & \text{if } t_A = O. \end{cases}$$

Firm A does not have a dominant action because its profitable action is N if $t_B = N$ but changes to O if $t_B = O$. Hence, there does not exist an equilibrium in dominant actions.

(1b) [5 points] The above best-response functions imply that there does not exist a Nash equilibrium because

$$R_A(N) = N \Longrightarrow R_B(N) = O \Longrightarrow R_A(O) = O \Longrightarrow R_B(O) = N \Longrightarrow R_A(N) = N \cdots$$

Thus, there does not exist an outcome $\langle t_A, t_B \rangle$ which satisfies both best-response functions.

(1c) [5 points] The outcome $\langle N, N \rangle$ is not Pareto optimal because it is Pareto dominated by $\langle O, O \rangle$. That is, $\pi_A(N, N) = 100 < 200 = \pi_A(O, O)$ and $\pi_B(N, N) = 50 < 60 = \pi_B(O, O)$.

The outcome $\langle O, O \rangle$ is Pareto optimal because it is not Pareto dominated by any other outcome. That is, $\pi_A(O, O) = 200 > 50 = \pi_A(O, N)$ but $\pi_B(O, O) = 60 < 100 = \pi_B(O, N)$. Similarly, $\pi_A(O, O) = 200 > 40 = \pi_A(N, O)$ but $\pi_B(O, O) = 60 < 200 = \pi_B(N, O)$.

(1d) [5 points] This is a case of excess momentum because both firms choose the new technology $\langle N, N \rangle$, whereas the outcome in which both choose the old technology $\langle O, O \rangle$ Pareto dominates it.

(2a) [15 points] Firm A maximizes p_A subject to

$$\pi_B^U = (p_B^U - 4)200 \geq (p_A - 3 - 4)(200 + 200).$$

Firm B maximizes p_B subject to

$$\pi_A^U = (p_A^U - 1)200 \geq (p_B - 3 - 1)(200 + 200).$$

Solving two equations with two variables yields $p_A^U = \$9$ and $p_B^U = \$8$. The UPE profit levels are therefore

$$\pi_A^U = (9 - 1)200 = \$1600 \quad \text{and} \quad \pi_B^U = (8 - 4)200 = \$800.$$

(2b) [5 points]

$$U_A^U = 9 - p_A^U = 9 - 9 = 0, \quad U_B^U = 9 - p_B^U = 9 - 8 = 1, \text{ hence } CS^U = 200U_A^U + 200U_B^U = 200.$$

Social welfare is therefore

$$W^U = CS^U + \pi_A^U + \pi_B^U = 200 + 1600 + 800 = 2600.$$

(3) [20 points] Firm A maximizes p_A subject to

$$\pi_B^U = 120 p_B^U \geq (120 + 120)(p_A - 50).$$

Firm B maximizes p_B subject to

$$\pi_A^U = 120 p_A^U \geq (120 + 120)[p_B - 50 + 0.5(240 - 120)].$$

Solving two equations with two variables yields $p_A^U = \$60$ and $p_B^U = \$20$. The UPE profit levels are therefore $\pi_A^U = 120 \cdot 60 = \$7200$ and $\pi_B^U = 120 \cdot 20 = \$2400$.

Service provider A charges a higher price and earns a higher profit because it provides a "better" service in the sense that A's customers have access to A's and B's customers whereas B's customers don't have access to A's customers.

(3b) [5 points]

$U_A^U = \frac{1}{2}(120 + 120) - 60 = 60, \quad U_B^U = \frac{1}{2} 120 - 20 = 40$ hence $CS^U = 120 \cdot 60 + 120 \cdot 40 = 12,000.$

Thus, social welfare is given by $W^U = CS^U + \pi_A^U + \pi_B^U = 12000 + 7200 + 2400 = 21,600.$

(4a) [16 points] Since only complete systems are sold, let p_{AA} and p_{BB} denote system prices. Then, in an UPE, the producer of AA maximizes p_{AA} subject to:

$$\pi_{BB} = 100 p_{BB} \geq 200(p_{AA} - 2)$$

Similarly, the producer of BB maximizes p_{BB} subject to:

$$\pi_{AA} = 100 p_{AA} \geq 200(p_{BB} - 0)$$

Note that type AB consumers are indifferent between system $X_A Y_A$ and $X_B Y_B$. Solving 2 equations with 2 variables yields

$$p_{AA} = \frac{8}{3} \approx 2.66, \quad p_{BB} = \frac{4}{3} \approx 1.33, \quad \pi_{AA} = \frac{800}{3} \approx 266.66, \quad \text{and} \quad \pi_{BB} = \frac{400}{3} \approx 133.33$$

(4b) [4 points] Firm B sets a lower price for system $X_B Y_B$ relative to the price firm A sets for system $X_A Y_A$ in order to attract type AB consumers to buy system $X_B Y_B$ and not system $X_A Y_A$. Note that type AB consumers are indifferent between the two systems if $p_{AA} = p_{BB}$.

(5a) [8 points] When software is unprotected, type I consumers will use the software but will not buy it. Therefore, type O will buy the software (rather than pirate it) if

$$400 + 2q - p \geq 2q, \quad \text{hence if} \quad p \leq 400.$$

Therefore, TaxMe^{TM} sells 100 packages for a price of $p = 400$ and earns a profit of $\pi^u = 100 \cdot 400 = 40,000$.

(5b) [8 points] If TaxMe^{TM} sets $p = 600$ all 300 consumers purchase this software. Notice that under this price type I consumers buy this software because $2 \cdot 300 - 600 \geq 0$. type O consumers will also buy this software because $400 + 2 \cdot 300 - 600 \geq 0$. The resulting profit is $\pi^p = 300 \cdot 600 = 180,000$.

(5c) [4 points] When software is not protected, $U_O^{np} = 400 + 2 \cdot 300 - 400 = 600$. Notice that the number of users is 300 whereas the firm sells only to 100 consumers.

Next, when software is protected, $U_O^p = 400 + 2 \cdot 300 - 600 = 400 < 600$. Therefore, support-oriented consumers are better off when software is not protected.

THE END