ECON 2674: FINAL EXAMINATION — April 23, 2002

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Last Name (Please PRINT): ....

First Name (PRINT): ....

Your I.D. Number: ....

INSTRUCTIONS (please read!)

1. Please make sure that you have 10 pages, including this page. Complaints about missing pages will not be accepted.
2. Please answer all the questions. You are <u>not</u> allowed to use any course material. Calculators are permitted.
3. Maximum Time Allowed: 3 hours.
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution. Please do *not* use any "formulas" developed in class. You need to drive them by yourself.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip it and go to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is 'visible' and that your sentences are properly organized and fit into the designated blank space. If you think that your handwriting is poor, please print each word!
10. Good Luck !

(1) Consider a system composed of two components labeled X and Y. There are two firms producing two different systems (different brands), at zero production cost. Firm A produces components X_A and Y_A, and firm B produces X_B and Y_B. In this market there are 100 consumers labeled AB, and 100 consumers labeled BA. The Utility function of a consumer i, j where $i, j = A, B$ is

$$U_{i,j} = \begin{cases} \beta - \left(p_i^X + p_j^Y\right) & \text{buys system } X_i Y_j \\ \beta - \left(p_j^X + p_j^Y\right) - \delta & \text{buys system } X_j Y_j \\ \beta - \left(p_i^X + p_i^Y\right) - \delta & \text{buys system } X_i Y_i \\ \beta - \left(p_j^X + p_i^Y\right) - 2\delta & \text{buys system } X_j Y_i \end{cases}$$

(1a) [8 pts.] Calculate the undercut-proof equilibrium prices assuming that the components produced by different firms are incompatible. *Hint:* First make sure that you know to define price-undercutting.

(1b) [2 pts.] Calculate the equilibrium profit level of each firm.

(1c) [8 pts.] Calculate the undercut-proof equilibrium prices assuming that the components produced by different firms are compatible.

(1d) [2 pts.] Calculate the equilibrium profit level of each firm.

(2) [10 pts.] You are given the following information about a market with two hardware brands labeled A and B:

(a) There are 100 A-oriented consumers, and 200 B-oriented consumers.

(b) Each consumer type has a utility function given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} q_A - p_A & \text{buy } A \\ q_B - p_B - \delta & \text{buy } B, \end{cases} \quad U_B \stackrel{\text{def}}{=} \begin{cases} q_A - p_A - \delta & \text{buy } A \\ q_B - p_B & \text{buy } B, \end{cases}$$

where δ is the differentiation (switching cost) parameter.

(c) Production is costless and in an undercut-proof equilibrium, brands' prices are

$$p_A = \frac{1485}{7} \quad \text{and} \quad p_B = \frac{1188}{7}.$$

Calculate the differentiation (switching-cost) parameter δ.

(3) Consider a market for a popular software DOORS™. There are 100 support-oriented (type-O) consumers, and 100 support-independent (type-I) consumers, with utility functions given by

$$U^O \stackrel{\text{def}}{=} \begin{cases} 2q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases} \quad \text{and } U^I \stackrel{\text{def}}{=} \begin{cases} q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases}$$

where q denotes the number of users of this software (which includes the number of buyers and the number of pirates, if piracy takes place). Suppose that the software is costless to produce and costless to protect. Also, assume that DOORS™ provides support only to those consumers who buy the software.

(3a) [10 pts.] Suppose that DOORS™ is *not* protected, so piracy is an option for every consumer. Calculate the software seller's profit-maximizing price. Prove your answer.

(3b) [5 pts.] Suppose that DOORS™ is protected, so piracy is impossible. Calculate the software seller's profit-maximizing price. Prove your answer.

(4) Consider an economy with three types of consumers who wish to connect to a certain telecommunication service (e.g., obtaining a phone service). There are 20 type H consumers who place high value for connecting to this service, 10 type M consumers who place a lower value for this connection, and 10 type L consumers who place the lowest value on this service.

Let p denote the connection fee to this service, and q the actual number of consumers connecting to this service. Then, the utility function of each type

$$U_H \stackrel{\text{def}}{=} \begin{cases} 5q - p & \text{connected} \\ 0 & \text{disconnected} \end{cases} \quad \text{and } U_M \stackrel{\text{def}}{=} \begin{cases} 2q - p & \text{connected} \\ 0 & \text{disconnected.} \end{cases} \quad \text{and } U_L \stackrel{\text{def}}{=} \begin{cases} q - p & \text{connected} \\ 0 & \text{disconnected.} \end{cases}$$

(4a) [10 pts.] In the space below, draw the market demand function for connecting to this telecommunication service. Label the axes and prove and explain the graph.

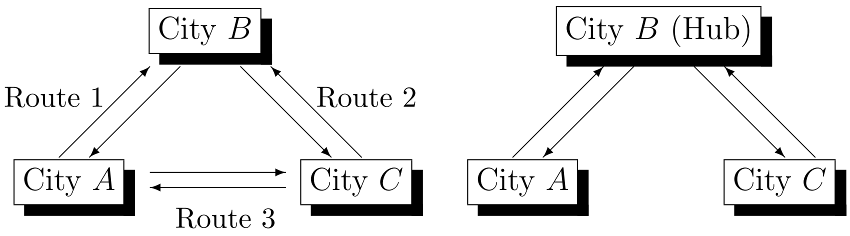
(4b) [5 pts.] Suppose now that it costs the telephone company $\mu = 10$ to connect each consumer to this service. Calculate the connection price that maximizes the profit of this monopoly phone company.

(5) Consider the broadcasted news scheduling model with three broadcasting stations labeled A, B, and C. There are η viewers whose ideal watching time is 17:00, and η viewers whose ideal watching time is 18:00. Let t_A denote the broadcasting time of station A, t_B the broadcasting time of station B, and t_C the broadcasting time of station C.
Assume that each station can air its news broadcast at one and only one time period. Also assume that each station earns exactly \$1 per viewer (as determined by rating surveys conducted during the broadcasting hours).

(5a) [5 pts.] List all the Nash equilibria in broadcasting time. (You do <u>not</u> have to provide a formal proof).

(5b) [5 pts.] Answer the previous question assuming that there are 3η viewers whose idea watching time is 17:00, and η viewers whose ideal watching time is 18:00.

(6) A single airline companies serves 3 cities as illustrated in the following figure.



On each route i, $i = 1, 2, 3$, there are η_i passengers. The cost of operating a flight on each route i is given by the function $c(q_i) = 1000 + \sqrt{q_i}$, where q_i is the actual number of passengers flying on route i. That is, each route requires a fixed cost of 1000 independent of the number of passengers plus variable cost which depends on the number of passengers q_i.

The airline considers two alternative networks of operations (displayed in the above figure): A fully-connected (FC), and a Hub-and-Spoke (with a hub in city B).

(6a) [5 pts.] Suppose $\eta_1 = \eta_2 = \eta_3 = \eta > 0$. Calculate which network of operation (FC or HS) minimized the airline's cost. Show your calculation!

(6b) [10 pts.] Suppose that the airline has already decided to operate a Hub-and-Spoke network. Also suppose that $\eta_1 = 100$ and $\eta_2 = \eta_3 = 800$. Does the airline minimize cost by locating the hub at city B? Prove your answer!

(7) In an Island named *Bilingwa* off the coast of Mexico there are 100 inhabitants. 60 are native English speakers, whereas 40 are native Spanish speakers. Let n_{ES} denote the number of native English speakers who learn to speak Spanish. Similarly, let n_{SE} denote the number of native Spanish speakers who learn English. The utility of each residents increases with the number of residents to whom he is able to communicate with. We define the utility function of each native English and each native Spanish speakers, respectively, by

$$U_E = \begin{cases} \frac{60 + n_{SE}}{10} & \text{does not learn Spanish} \\ \frac{60 + 40}{10} - 3 & \text{learns Spanish} \end{cases} \qquad U_S = \begin{cases} \frac{40 + n_{ES}}{10} & \text{does not learn English} \\ \frac{40 + 60}{10} - 7 & \text{learns English} \end{cases}$$

These utility functions reveal that it is "easier" (less costly) for a native English speaker to learn Spanish, than for a native Spanish speaker to learn English (cost of 3 compared with 7).

(7a) [10 pts.] Find the number of native English speakers who learn Spanish, n_{ES}, and the number of native Spanish speakers who learn English, n_{SE} in a language-acquisition equilibrium. Is the equilibrium you found unique? Prove your results!

(7b) [5 pts.] Find the socially-optimal levels of n_{ES} and n_{SE}. Prove your answer!

Scratch Paper

This page will NOT be read by the instructor !

THE END