(1) Consider a duopoly (two-firm) computer industry producing two brands named *Artichoke* (Brand A), and *Banana* (Brand B). Assume that each computer costs \$2 to produce. Let p_A denote the price charged by Artichoke, and p_B the price charged by Banana.

There are 100 consumers who are brand A-oriented consumers, and 100 consumers who are brand B-oriented consumers. Let q_A be the number of consumers who purchase brand A, and q_B the number of consumers who purchase brand B. Formally, the utility of A-oriented and B-oriented consumers are given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} 0.5q_A - p_A & \text{buys } A \text{ ; } A \text{ is incompatible} \\ 0.5q_B - p_B - 300 & \text{buys } B \text{ ; } B \text{ is incompatible} \\ 0.5(q_A + q_B) - p_A & \text{buys } A \text{ ; } A \text{ is } B\text{-compatible} \\ 0.5(q_A + q_B) - p_B - 300 & \text{buys } B \text{ ; } B \text{ is } A\text{-compatible,} \end{cases} \tag{1}$$

and

$$U_B \stackrel{\text{def}}{=} \begin{cases} 0.5q_A - p_A - 300 & \text{buys } A \text{ ; } A \text{ is incompatible} \\ 0.5q_B - p_B & \text{buys } B \text{ ; } B \text{ is incompatible} \\ 0.5(q_A + q_B) - p_A - 300 & \text{buys } A \text{ ; } A \text{ is } B\text{-compatible} \\ 0.5(q_A + q_B) - p_B & \text{buys } B \text{ ; } B \text{ is } A\text{-compatible.} \end{cases}$$

(1a) [8 pts.] Calculate the undercut-proof equilibrium prices assuming that the computer brands are incompatible. *Hint:* First make sure that you know to define price-undercutting considering the fact that each unit costs \$2 to produce.

Firm B maximizes p_B subject to the constraint

$$\pi_A = (p_A - 2) \times 100 \ge (p_B - 2 - 300 + 0.5 \times 100) \times 200$$

Firm A maximizes p_A subject to the constraint

$$\pi_B = (p_B - 2) \times 100 \ge (p_A - 2 - 300 + 0.5 \times 100) \times 200$$

Solving 2 equations with two variables yields $\boxed{p_A^I = p_B^I = 502}$.

(1b) [2 pts.] Calculate the equilibrium profit level of each firm when the brands are incompatible.

$$\pi_A^I = (p_A^I - 2) \times 100 = 50,000 \quad \text{and} \quad \pi_B^I = (p_B^I - 2) \times 100 = 50,000$$

(1c) [8 pts.] Calculate the undercut-proof equilibrium prices assuming that the computer brands are compatible.

When the machines are compatible, undercutting does not affect the network component in consumers' utility (since each consumer can interact with 200 consumers regardless of which brand he buys). Hence, Firm B maximizes p_B subject to the constraint

$$\pi_A = (p_A - 2) \times 100 \ge (p_B - 2 - 300) \times 200$$

Firm A maximizes p_A subject to the constraint

$$\pi_B = (p_B - 2) \times 100 \ge (p_A - 2 - 300) \times 200$$

Solving 2 equations with two variables yields $\boxed{p_A^I = p_B^I = 602}$.

(1d) [2 pts.] Calculate the equilibrium profit level of each firm when the brands are compatible.

$$\pi_A^I = (p_A^I - 2) \times 100 = 60,000 \quad \text{and} \quad \pi_B^I = (p_B^I - 2) \times 100 = 60,000$$

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(2) Consider a world with two countries labeled N (for North) and S (for South). Country N has η_N consumers who wish to place at most one international phone call to country S. Country S has η_S consumers who wish to place at most one phone call to country N. Assume that $\eta_N > \eta_S$.

Let p_k denote the price of a phone call from country k as charged by the country k's carrier, $k = N, S$. Each potential consumer has a valuation of $\beta > 0$ for placing this phone call, meaning that the utility function of a consumer in country k is given by

$$U_k \stackrel{\text{def}}{=} \begin{cases} \beta - p_k & \text{if makes an international call} \\ 0 & \text{if does not make an international call.} \end{cases}$$

Let a denote the international access charge (settlement rate), which is the payment each carrier makes to the foreign carrier for carrying the phone call to its final destination in the foreign country. Then the profit of each national phone company is composed of profit generated from sales of international phone calls and the collection of access fees from incoming international phone calls. Thus,

$$\pi_N \stackrel{\text{def}}{=} (p_N - a)\eta_N + a\eta_S, \quad \text{and} \quad \pi_S \stackrel{\text{def}}{=} (p_S - a)\eta_S + a\eta_N.$$

Suppose now that the phone industry in country N is fully competitive, hence the price of an international phone call from N to S is $p_N = a$, where a is the negotiated access charge. Also, suppose that the phone industry in country S is a monopoly, hence the price of a phone call from country S to N is $p_S = \beta$. Answer the following questions.

(2a) [5 pts.] Let a_N be the access charge that maximizes π_N, and let a_S be the access charge that maximizes π_S. Calculate a_N and a_S. Show your calculations.

$$\begin{aligned} \pi_N &= (a - a)\eta_N + a\eta_S = a\eta_S \Longrightarrow a_N = \beta \\ \pi_S &= (\beta - a)\eta_S + a\eta_N = \beta\eta_S + a(\eta_N - \eta_S) \Longrightarrow a_S = \beta \end{aligned}$$

(2b) [5 pts.] Using the bargaining rule $\hat{a} = (a_N + a_S)/2$, calculate the net flow of money transferred from company N to company S. That is, calculate $T_{\vec{NS}}$. Show your calculations!

$$\hat{a} = \frac{\beta + \beta}{2} = \beta \Longrightarrow T_{\vec{NS}} = \beta\eta_N - \beta\eta_S = \beta(\eta_N - \eta_S) > 0.$$

(2c) [10 pts.] Answer questions (2a) and (2b) assuming that the phone industry in country N and in country S are both competitive. Show your calculations.

$$\begin{aligned} \pi_N &= (a - a)\eta_N + a\eta_S = a\eta_S \Longrightarrow a_N = \beta \\ \pi_S &= (a - a)\eta_S + a\eta_N = a\eta_N \Longrightarrow a_S = \beta \end{aligned}$$

Hence,

$$\hat{a} = (\beta + \beta)/2 = \beta \Longrightarrow T_{\vec{NS}} = \beta\eta_N - \beta\eta_S = \beta(\eta_N - \eta_S) > 0.$$

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(3) Suppose that there are 4 possible TV program types indexed by $i = 1, 2, 3, 4$. For example, type 1 could be a talk show, type 2 could be the news, type 3 could be a fashion show, and type 4 could be a sports program. Each type of program i is watched by η_i viewers. Suppose that $\eta_1 = 400$, $\eta_2 = 150$, $\eta_3 = 100$, and $\eta_4 = 80$.

Assume that (i) Programs are to be aired in prime time only; hence each broadcasting station can air at most one program type. (ii) If several stations choose to air the same program type, then the program's viewers are evenly split among the stations.

There are three broadcasting stations indexed by $j = A, B, C$. Production is costless. Each station earns a profit of \$1 on each viewer, so each station attempts to maximize the number of viewers. We denote by $p_j \in 1, 2, 3, 4$ the action (program type) chosen by station j.

(3a) [10 pts.] Find which type of program will be broadcasted by each station in a Nash equilibrium. You must PROVE your answer using the definition of a Nash equilibrium.

$\langle p_A, p_B, p_C \rangle = \langle 1, 1, 2 \rangle$ is a NE (among several others). Proof:

$$\begin{aligned}
\pi_A(1,1,2) = 200 &\geq 75 = \pi_A(2,1,2)\\
\pi_A(1,1,2) = 200 &\geq 100 = \pi_A(3,1,2)\\
\pi_A(1,1,2) = 200 &\geq 80 = \pi_A(4,1,2)\\
\pi_B(1,1,2) = 200 &\geq 75 = \pi_B(1,2,2)\\
\pi_B(1,1,2) = 200 &\geq 100 = \pi_B(1,3,2)\\
\pi_B(1,1,2) = 200 &\geq 80 = \pi_B(1,4,2)\\
\pi_C(1,1,2) = 150 &\geq 133 = \pi_C(1,1,1)\\
\pi_C(1,1,2) = 150 &\geq 100 = \pi_C(1,1,3)\\
\pi_C(1,1,2) = 150 &\geq 80 = \pi_C(1,1,4)
\end{aligned}$$

(3b) [10 pts.] Suppose that each viewer gains a utility of $U_i = \beta$ if the program of his choice is aired, and $U_i = 0$ if the program of his choice is not aired. Define the social welfare function W by the sum of viewers' utilities. Find the allocation of programs among the three networks that would maximize social welfare. Prove your answer!

The sum of viewers' utilities is maximized when

$$\langle p_A, p_B, p_C \rangle = \langle 1, 2, 3 \rangle$$

in which case

$$W(1, 2, 3) = 400\beta + 150\beta + 100\beta + 80 \times 0 = 650\beta.$$

Remark 1: In general, social welfare is defined as the sum of viewers' utilities and stations' profit. Here, we simplified by ignoring the profit part.

Remark 2: You can observe the market failure since at the NE of the previous section,

$$W(1, 1, 2) = 400\beta + 150\beta + 100 \times 0 + 80 \times 0 = 550\beta < 650\beta.$$

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(4) Consider a monopoly cable-TV operator providing a service to three consumers by transmitting three channels: CNN, BBC, and SHOP(ping). Assume that the monopoly's production (transmission) is costless. The Table below shows the valuation (maximum willingness to pay) of each consumer for each channel.

Consumer	CNN	BBC	SHOP
1	5	1	2
2	5	1	5
3	1	5	2

Table 1: Consumers' valuation of three channels.

(4a) [10 pts.] Calculate the profit-maximizing prices assuming that the monopoly must sell each channel separately.

See solution to Exercise # 5 on p.160 (Ch.6).

(4b) [5 pts.] Calculate the profit-maximizing price assuming that the monopoly sells all the channels in a single package.

See solution to Exercise # 5 on p.160 (Ch.6).

(4c) [5 pts.] Suppose now that the monopoly can package channels any way it wants to. Which combination of packages maximize the monopoly's profit?

See solution to Exercise # 5 on p.160 (Ch.6).

(5) Consider a technology-adoption game played by two users (or firms) displayed in following table.

		User *B*			
		New Technology		Old Technology	
User *A*	New	3	3	1	0
	Old	0	1	2	2

(5a) [10 pts.] Which technology will be adopted by each user in Nash equilibrium. That is, find the Nash equilibrium(ia) for this game (if they exist). Prove your answer!

See solution to Exercise # 1 on p.97 (Ch.4).

(5b) [10 pts.] Does the outcome (New, New) constitute a case of *excess momentum*? Explain your answer using the definition of the term.

See solution to Exercise # 1 on p.97 (Ch.4).

THE END