(1) Consider a system composed of two components labeled X and Y. There are two firms producing two different systems (different brands), at zero production cost. Firm A produces components X_A and Y_A, and firm B produces X_B and Y_B. In this market there are 100 consumers labeled AA, and 100 consumers labeled BB. The Utility function of a consumer i, j where $i, j = A, B$ is

$$U_{i,j} = \begin{cases} 10 - \left(p_i^X + p_j^Y\right) & \text{buys system } X_iY_j \\ 10 - \left(p_j^X + p_j^Y\right) - 2 & \text{buys system } X_jY_j \\ 10 - \left(p_i^X + p_i^Y\right) - 2 & \text{buys system } X_iY_i \\ 10 - \left(p_j^X + p_i^Y\right) - 3 & \text{buys system } X_jY_i \end{cases}$$

(1a) [10 pts.] Calculate the undercut-proof equilibrium prices and the profit of each firm assuming that the components produced by different firms are <u>incompatible</u>. *Hint:* First make sure that you know to define price-undercutting.

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Since only complete systems are sold, let p_{AA} and p_{BB} denote system prices. Then, in an UPE, the producer of AA maximizes p_{AA} subject to:

$$\pi_{BB} = 100p_{BB} \geq 200(p_{AA} - 3)$$

Similarly, the producer of BB maximizes p_{BB} subject to:

$$\pi_{AA} = 100p_{AA} \geq 200(p_{BB} - 3)$$

Solving 2 equations with 2 variables yield

$$p_{AA} = p_{BB} = 6 \quad \text{and} \quad \pi_{AA} = \pi_{BB} = 600$$

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(1b) [5 pts.] Calculate the aggregate consumer surplus and social welfare.

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$U_{AA} = U_{BB} = 10 - 6 = 4$. Next, aggregate consumer surplus is

$$CS = 100U_{AA} + 100U_{BB} = 400 + 400 = 800.$$

Next, Social welfare is given by

$$W = CS + \pi_{AA} + \pi_{BB} = 800 + 600 + 600 = 2000.$$

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(1c) [10 pts.] Calculate the undercut-proof equilibrium prices and firms' profit levels assuming that the components produced by different firms are <u>compatible</u>.

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In an UPE, the producer of X_A maximizes p_A^X subject to:

$$\pi_B^X = 100p_B^X \geq 200(p_A^X - 2)$$

Similarly, the producer of X_B maximizes p_B^X subject to:

$$\pi_A^X = 100p_A^X \geq 200(p_B^X - 2)$$

Solving 2 equations with 2 variables yield

$$p_A^X = p_B^X = 4 \quad \text{and} \quad \pi_A^X = \pi_B^X = 400$$

Replacing component X with Y yields:

$$p_A^Y = p_B^Y = 4 \quad \text{and} \quad \pi_A^Y = \pi_B^Y = 400$$

Therefore,

$$\pi_A = \pi_A^X + \pi_A^Y = 400 + 400 = 800 = \pi_B^X + \pi_B^Y = \pi_B.$$

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(1d) [5 pts.] Calculate the aggregate consumer surplus and social welfare.

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The equilibrium utility level of each consumer is $U_A = U_B = 10 - 4 - 4 = 2$. Therefore,

$$CS = 100U_A + 100U_B = 400.$$

Social welfare is given by

$$W = CS + \pi_{AA} + \pi_{BB} = 400 + 800 + 800 = 2000.$$

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(2) Consider a market for a popular software ACROPOP™. There are 100 (one-hundred) support-oriented (type-O) consumers, and 200 (two-hundred) support-independent (type-I) consumers, with utility functions given by

$$U^O \stackrel{\text{def}}{=} \begin{cases} 400 + q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases} \quad \text{and } U^I \stackrel{\text{def}}{=} \begin{cases} q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases}$$

where q denotes the number of users of this software (which includes the number of buyers and the number of pirates, whenever piracy prevails). Suppose that the software is costless to produce and costless to protect. Also, assume that the publisher of ACROPOP™ is a monopoly who provides support only to those consumers who buy the software.

(2a) [10 pts.] Suppose that ACROPOP™ is *not* protected, so piracy is an option for every consumer. Calculate the software publisher's profit-maximizing price. Prove your answer.

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Clearly, type I users pirate the software since $q - p < q = U^I$. Therefore, if there are buyers, they must be of type O.

In order to induce type O users to buy the software, the price must be low enough to satisfy:

$$400 + q - p \geq q \quad \text{or} \quad p \leq 400.$$

Therefore, the price of software and the publisher's profit when the software is not protected are: $p^{NP} = 400$, and $\pi^{NP} = 100 \times 400 = 40,000$.

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(2b) [10 pts.] Suppose that ACROPOP™ is protected, so piracy is impossible. Calculate the software publisher's profit-maximizing price. Prove your answer.

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If the publisher charges as "low" price (in order to attract type$I$ to buy the software, instead of not using it at all), then the price must satisfy:

$$U^I = q - p = 100 + 200 - p \geq 0 \quad \text{or} \quad p \leq 300$$

in which case, type O users also buy the software and hence $\pi = 300 \times 300 = 90,000$.

If the publisher charges a "high" price (thereby, potentially, excluding type I users) the maximum price that type O will be willing to pay must satisfy:

$$U^O = 400 + q - p = 400 + 100 - p \geq 0 \quad \text{or} \quad p \leq 500.$$

At this price, type I do not buy the software since if they buy, $U^I = 100+200-500 < 0$. Therefore, under this price the publisher earns $\pi = 100 \times 500 = 50,000$.

Clearly, this publisher maximizes profits by protecting the software and charging a price of $p^p = 300$ and earning a profit of $\pi^p = 90,000$.

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(3) Consider a monopoly cable-TV operator providing a service to 3 types of consumers by transmitting 2 channels: CNN and BBC. Assume that the monopoly's production (transmission) is costless and that no royalties are paid to the content providers . The Table below shows the valuation (maximum willingness to pay) of each consumer type for each channel, and the number of consumers of each type.

Consumer type	# consumers	CNN	BBC
1	1000	4	1
2	9000	5	5
3	1000	1	4

(3a) [10 pts.] Calculate the profit-maximizing prices assuming that the monopoly must sell each channel separately.

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$$p_C = 5 \implies q_C = 9000 \implies \pi_C = 9000 \times 5 = 45,000$$
$$p_C = 4 \implies q_C = 10000 \implies \pi_C = 10000 \times 4 = 40,000$$
$$p_C = 1 \implies q_C = 11000 \implies \pi_C = 11000 \times 1 = 11,000$$

Therefore, the profit-maximizing price for CNN is $p_C = 5$ resulting in a profit of $\pi_C = 45,000$.

Similarly, the profit-maximizing price for BBC is $p_B = 5$ resulting in a profit of $\pi_B = 45,000$. Altogether, the profit of this Cable-TV provider is $\pi = \pi_C + \pi_B = 90,000$.

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(3b) [5 pts.] Calculate the profit-maximizing price assuming that the monopoly sells all the channels in a single package.

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$$\begin{aligned} p_{CB} = 5 &\implies q_{CB} = 11000 \implies \pi_{CB} = 11000 \times 5 = 55,000 \\ p_{CB} = 10 &\implies q_{CB} = 9000 \implies \pi_{CB} = 9000 \times 10 = 90,000 \end{aligned}$$

Therefore, the profit-maximizing price of the package is $p_{CB} = 5$ and the associated profit level is $\pi_{CB} = 90,000$. Notice that in this case pure tying in not profit-increasing.

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(4) Consider two airline companies: airline α and airline β, who are the only airlines providing a service connecting city A with city B. Suppose that the frequency of flights provided by airline α and airline β are $f_\alpha = 6$ and $f_\beta = 3$, respectively. That is, airline α provides 6 flights per day, whereas airline β provides only 3 flights per day. There are η consumers who are oriented towards airline α, and η who are oriented towards airline β. Suppose now that passengers' utility functions are given by:

$$U_\alpha \stackrel{\text{def}}{=} \begin{cases} f_\alpha - p_\alpha & \text{flies } \alpha \\ f_\beta - 4 - p_\beta & \text{flies } \beta, \end{cases} \quad \text{and} \quad U_\beta \stackrel{\text{def}}{=} \begin{cases} f_\alpha - 4 - p_\alpha & \text{flies } \alpha \\ f_\beta - p_\beta & \text{flies } \beta. \end{cases}$$

Assume that the airline firms do not bear any type of cost. Answer the following questions.

(4a) [10 pts.] Calculate the UPE airfare charged by each airline and the associated profit levels assuming that there are no agreements between the two airline firms.

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Airline α maximizes the airfare p_α subject to:

$$\pi_\beta = \eta p_\beta \geq 2\eta(p_\alpha - 4 + f_\beta - f_\alpha) = 2\eta(p_\alpha - 4 + 3 - 6) = 2\eta(p_\alpha - 7).$$

Airline β maximizes the airfare p_β subject to:

$$\pi_\alpha = \eta p_\alpha \geq 2\eta(p_\beta - 4 + f_\alpha - f_\beta) = 2\eta(p_\beta - 4 + 6 - 3) = 2\eta(p_\beta - 1).$$

Solving 2 equations with 2 variables, the equilibrium airfares and airlines' profit levels are given by

$$p_\alpha = 10, \quad p_\beta = 6, \quad \text{and} \quad \pi_\alpha = 10\eta, \quad \pi_\beta = 6\eta.$$

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(4b) [10 pts.] Calculate the UPE airfares and the associated profit levels assuming that the two airline firms are engaged in a code-sharing agreement. Assume that airline α continues to maintain $f_\alpha = 6$ flights per day and airline β continues to maintain $f_\beta = 3$ flights per day even after the agreement is signed.

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Under a code-sharing agreement, passengers of all airlines are exposed to the same frequency of flights given by $f = f_\alpha + f_\beta$.

Airline α maximizes the airfare p_α subject to:

$$\pi_\beta = \eta p_\beta \geq 2\eta(p_\alpha - 4 + f - f) = 2\eta(p_\alpha - 4).$$

Airline β maximizes the airfare p_β subject to:

$$\pi_\alpha = \eta p_\alpha \geq 2\eta(p_\beta - 4 + f - f) = 2\eta(p_\beta - 4).$$

Solving 2 equations with 2 variables, the equilibrium airfares and airlines' profit levels are given by

$$p_\alpha = 8, \quad \text{and} \quad \pi_\alpha = \pi_\beta = 8\eta.$$

Clearly airline β benefits from the code-sharing agreement since it provides a lower frequency of flights. Airline α loses from this agreement since it can no longer charge an extra premium for providing a higher frequency of flights.

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(5) Consider a program-type competition among 4 independent broadcasting channels: Channels A, B, C, and D. Each channel maximizes the number of viewers times 1¢ (which he receives as a revenue from advertising per viewer). Also assume that that each channel can broadcast only one program type: A talk-show, a news program, or a movie.

There are 3 types of TV viewers: There are 800 viewers who would like to watch only *talk-shows* (T). Similarly, there are 400 viewers who like to watch *news* programs (N) only. Finally, there are 200 viewers who watch *movies* (M) only.

(5a) [10 pts.] Calculate which program will be broadcasted by each channel in a Nash equilibrium. Prove your answer. *Remark*: You do NOT need to demonstrate all NE. You are being asked to prove an existence of one equilibrium).

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The outcome $\langle p_A, p_B, p_C, p_D \rangle = \langle T, T, T, N \rangle$ is a NE. *Proof*:

$$\begin{aligned}
\pi_A(T,T,T,N) = 800/3 &> 200 = \pi_A(N,T,T,N)\\
\pi_A(T,T,T,N) = 800/3 &> 200 = \pi_A(M,T,T,N)\\
\pi_B(T,T,T,N) = 800/3 &> 200 = \pi_B(T,N,T,N)\\
\pi_B(T,T,T,N) = 800/3 &> 200 = \pi_B(T,M,T,N)\\
\pi_C(T,T,T,N) = 800/3 &> 200 = \pi_C(T,T,N,N)\\
\pi_C(T,T,T,N) = 800/3 &> 200 = \pi_C(T,T,M,N)\\
\pi_D(T,T,T,N) = 400 &> 200 = \pi_D(T,T,T,M)\\
\pi_D(T,T,T,N) = 400 &> 200 = \pi_D(T,T,T,T)
\end{aligned}$$

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(5b) [5 pts.] Suppose that the utility function of each viewer is given by

$$U = \begin{cases} 5 & \text{if she watches her favorite program} \\ 0 & \text{if she does not watch her favorite program} \end{cases}$$

Define a social welfare function and calculate the amount of social welfare in a Nash equilibrium you found in (5a). Explain whether social welfare is maximized at this equilibrium or whether there is a *market failure*. Prove your answer!

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In the above NE,

$$CS = 800U_T + 400U_N + 200U_M = 800 \times 5 + 400 \times 5 + 200 \times 0 = 6000.$$

Total industry profit is:

$$T\pi = \pi_A + \pi_B + \pi_C + \pi_D = \frac{800}{3} + \frac{800}{3} + \frac{800}{3} + 400 = 1200.$$

Finally, the equilibrium level of social welfare is:

$$W(T, T, T, N) = CS + T\pi = 6000 + 1200 = 7200.$$

We now prove that the outcome $\langle T, T, T, N \rangle$ does not maximize social welfare. Consider now the outcome $\langle T, T, M, N \rangle$.

$$CS = 800U_T + 400U_N + 200U_M = 800 \times 5 + 400 \times 5 + 200 \times 5 = 7000.$$

Also,

$$T\pi = \pi_A + \pi_B + \pi_C + \pi_D = \frac{800}{2} + \frac{800}{2} + 200 + 400 = 1400.$$

Therefore,

$$W(T, T, M, N) = CS + T\pi = 7000 + 1400 = 8400 > 7200 = W(T, T, T, N).$$

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THE END