Last Name (Please PRINT): .....

First Name (PRINT): .....

Your I.D. Number: .....

INSTRUCTIONS (please read!)

1. Please make sure that you have 8 pages, including this page. Complaints about missing pages will not be accepted.
2. Please answer all the questions. You are not allowed to use any course material. Calculators are permitted.
3. Maximum Time Allowed: 1 hour and 30 minutes.
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution. Please do *not* use any "formulas" developed in class. You need to drive them by yourself.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip it and go to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is 'visible' and that your sentences are properly organized and fit into the designated blank space. If you think that your handwriting is poor, please print each word!
10. Good Luck !

..... *Instructor's use only* .....

Question no.	Maximum	Actual Grade
1	30	
2	20	
3	15	
4	15	
5	20	

(1) Consider a system composed of two components labeled X and Y. There are two firms producing two different systems (different brands), at zero production cost. Firm A produces components X_A and Y_A, and firm B produces X_B and Y_B. In this market there are 100 consumers labeled AB, and 100 consumers labeled BA. The Utility function of a consumer i, j where $i, j = A, B$ is

$$U_{i,j} = \begin{cases} 10 - \left(p_i^X + p_j^Y\right) & \text{buys system } X_iY_j \\ 10 - \left(p_j^X + p_j^Y\right) - 2 & \text{buys system } X_jY_j \\ 10 - \left(p_i^X + p_i^Y\right) - 2 & \text{buys system } X_iY_i \\ 10 - \left(p_j^X + p_i^Y\right) - 4 & \text{buys system } X_jY_i \end{cases}$$

(1a) [10 pts.] Calculate the undercut-proof equilibrium prices and the profit of each firm assuming that the components produced by different firms are incompatible. *Hint:* First make sure that you know to define price-undercutting.

(1b) [5 pts.] Calculate the aggregate consumer surplus and social welfare.

(1c) [10 pts.] Calculate the undercut-proof equilibrium prices and the firms' profit levels assuming that the components produced by different firms are compatible.

(1d) [5 pts.] Calculate the aggregate consumer surplus and social welfare.

(2) Consider a market for a popular software ACROPOP™. There are 100 (one-hundred) identical users, each with a utility function given by

$$U \stackrel{\text{def}}{=} \begin{cases} \beta + q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases}$$

where $\beta > 0$ measures the value of service provided by the software firm to its buyers, and q denotes the number of users of this software (which includes the number of buyers and the number of pirates, if piracy takes place). Suppose that the software is costless to produce. Also, assume that ACROPOP™ provides support only to those consumers who buy the software.

(2a) [10 pts.] Suppose that ACROPOP™ is *not* protected, so piracy is an option for every consumer. Calculate the software seller's profit-maximizing price. Prove your answer.

(2b) [10 pts.] Suppose now that ACROPOP™ can invest a fixed (one time) amount of $\phi = 12,000$ to protect against piracy, so piracy becomes impossible. Calculate the software seller's profit-maximizing price and the profit level if the publisher invests in this anti-piracy measure. Prove your answer.

(3) Consider the library-pricing model analyzed in class. Suppose that there are $\eta = 1200$ potential readers and $\lambda = 50$ libraries (i.e., $1200/50 = 24$ readers per library). The utility function of each potential reader is given by

$$U \stackrel{\text{def}}{=} \begin{cases} 23 - p^b & \text{if she buys and owns the book} \\ 23 - 2p_i^r & \text{if she borrows (rents) from library } i \\ 0 & \text{if she does not read the book.} \end{cases}$$

There is one publisher who can sell either to individual readers, or to libraries but not to both. Each copy of the book costs $\mu = 12$ to produce. Answer the following questions.

(3a) [5 pts.] Calculate the publisher's profit-maximizing price and her profit level, assuming that the publisher sells directly to individual readers only.

(3b) [10 pts.] Calculate the publisher's profit-maximizing library price and her profit level assuming that the publisher sells one copy to each library only.

(4) [15 pts.] Consider a city with only two local banks labeled bank A and bank B. Each person is allowed to maintain only one bank account (either with bank A, or B, but not both). Let δ_A denote the cost of switching a bank account from bank A to bank B. That is, the total cost (including inconvenience) of closing an account with bank A and opening a fully-operative account with bank B. Similarly, let δ_B denote the cost of switching a bank account from bank B to bank A.

You are now given the following information:

(a) Bank A maintains 100 (one hundred) accounts, whereas bank B maintains 200 (two hundred) accounts.

(b) Bank A levies a fee of $f_A = 30$ per account and bank B levies a fee of $f_B = 30$ per account.

(c) The utility functions of a bank A and a bank B account holder, respectively, are given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} -f_A & \text{staying with bank } A \\ -f_B - \delta_A & \text{switching to bank } B, \end{cases} \quad \text{and} \quad U_B \stackrel{\text{def}}{=} \begin{cases} -f_A - \delta_B & \text{switching to bank } A \\ -f_B & \text{staying with bank } B \end{cases}$$

Suppose that banks do not bear any costs and that their fees are set in an undercut-proof equilibrium. Using the above data, calculate the switching-cost parameters δ_A and δ_B. Show your calculations!

(5) In an Island named *Bilingwa* off the coast of Mexico there are 100 inhabitants. 60 are native English speakers, whereas 40 are native Spanish speakers. Let n_{ES} denote the number of native English speakers who learn to speak Spanish. Similarly, let n_{SE} denote the number of native Spanish speakers who learn English. The utility of each resident increases with the number of residents to whom he is able to communicate with. We define the utility function of each native English and each native Spanish speakers, respectively, by

$$U_E = \begin{cases} \frac{60 + n_{SE}}{10} & \text{does not learn Spanish} \\ \frac{60 + 40}{10} - 5 & \text{learns Spanish} \end{cases} \qquad U_S = \begin{cases} \frac{40 + n_{ES}}{10} & \text{does not learn English} \\ \frac{40 + 60}{10} - 7 & \text{learns English} \end{cases}$$

These utility functions reveal that it is "easier" (less costly) for a native English speaker to learn Spanish, than for a native Spanish speaker to learn English (cost of 5 compared with 7).

(5a) [10 pts.] Find the number of native English speakers who learn Spanish, n_{ES}, and the number of native Spanish speakers who learn English, n_{SE} in a language-acquisition equilibrium. Prove your results!

(5b) [10 pts.] Find the socially-optimal levels of n_{ES} and n_{SE}. Prove your answer!

Scratch Paper

This page will NOT be read by the instructor !

THE END