(1) Consider a system composed of two components labeled X and Y. There are two firms producing two different systems (different brands), at zero production cost. Firm A produces components X_A and Y_A, and firm B produces X_B and Y_B. In this market there are 100 consumers labeled AB, and 100 consumers labeled BA. The Utility function of a consumer i, j where $i, j = A, B$ is

$$U_{i,j} = \begin{cases} 10 - \left(p_i^X + p_j^Y\right) & \text{buys system } X_iY_j \\ 10 - \left(p_j^X + p_j^Y\right) - 2 & \text{buys system } X_jY_j \\ 10 - \left(p_i^X + p_i^Y\right) - 2 & \text{buys system } X_iY_i \\ 10 - \left(p_j^X + p_i^Y\right) - 4 & \text{buys system } X_jY_i \end{cases}$$

(1a) [10 pts.] Calculate the undercut-proof equilibrium prices and the profit of each firm assuming that the components produced by different firms are incompatible. *Hint:* First make sure that you know to define price-undercutting.

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Since only complete systems are sold, let p_{AA} and p_{BB} denote system prices. Then, in an UPE, where consumer AB buys system AA, and consumer BA buys system BB, the producer of AA maximizes p_{AA} subject to:

$$\pi_{BB} = 100p_{BB} \geq 200(p_{AA} - 0)$$

Similarly, the producer of BB maximizes p_{BB} subject to:

$$\pi_{AA} = 100p_{AA} \geq 200(p_{BB} - 0)$$

Solving 2 equations with 2 variables yield

$$p_{AA} = p_{BB} = 0 \quad \text{and} \quad \pi_{AA} = \pi_{BB} = 0$$

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(1b) [5 pts.] Calculate the aggregate consumer surplus and social welfare.

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$U_{AA} = U_{BB} = 10 - 0 - 2 = 8$. Next, aggregate consumer surplus is

$$CS = 100U_{AA} + 100U_{BB} = 800 + 800 = 1600.$$

Next, Social welfare is given by

$$W = CS + \pi_{AA} + \pi_{BB} = 1600 + 0 + 0 = 1600.$$

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(1c) [10 pts.] Calculate the undercut-proof equilibrium prices and the firms' profit levels assuming that the components produced by different firms are compatible.

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In an UPE, the producer of X_A maximizes p_A^X subject to:

$$\pi_B^X = 100p_B^X \geq 200(p_A^X - 2)$$

Similarly, the producer of X_B maximizes p_B^X subject to:

$$\pi_A^X = 100p_A^X \geq 200(p_B^X - 2)$$

Solving 2 equations with 2 variables yield

$$p_A^X = p_B^X = 4 \quad \text{and} \quad \pi_A^X = \pi_B^X = 400$$

Replacing component X with Y yields:

$$p_A^Y = p_B^Y = 4 \quad \text{and} \quad \pi_A^Y = \pi_B^Y = 400$$

Therefore,

$$\pi_A = \pi_A^X + \pi_A^Y = 400 + 400 = 800 = \pi_B^X + \pi_B^Y = \pi_B.$$

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(1d) [5 pts.] Calculate the aggregate consumer surplus and social welfare.

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The equilibrium utility level of each consumer is $U_A = U_B = 10 - 4 - 4 = 2$. Therefore,

$$CS = 100U_A + 100U_B = 400.$$

Social welfare is given by

$$W = CS + \pi_{AA} + \pi_{BB} = 400 + 800 + 800 = 2000.$$

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(2) Consider a market for a popular software ACROPOP™. There are 100 (one-hundred) identical users, each with a utility function given by

$$U \stackrel{\text{def}}{=} \begin{cases} \beta + q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases}$$

where $\beta > 0$ measures the value of service provided by the software firm to its buyers, and q denotes the number of users of this software (which includes the number of buyers and the number of pirates, if piracy takes place). Suppose that the software is costless to produce. Also, assume that ACROPOP™ provides support only to those consumers who buy the software.

(2a) [10 pts.] Suppose that ACROPOP™ is *not* protected, so piracy is an option for every consumer. Calculate the software seller's profit-maximizing price. Prove your answer.

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The software is not protected. In order to induce consumers to buy the software (instead of pirate it) the price should be sufficiently low to satisfy:

$$\beta + q - p \geq q \Longrightarrow p \leq \beta$$

Hence, $p^{NP} = \beta$ and $\pi^{NP} = 100\beta$.

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(2b) [10 pts.] Suppose now that ACROPOP$^{\text{TM}}$ can invest a fixed (one time) amount of $\phi = 12,000$ to protect against piracy, so piracy becomes impossible. Calculate the software seller's profit-maximizing price and the profit level if the publisher invests in this anti-piracy measure. Prove your answer.

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Since piracy is not an option, the monopoly publisher sets the highest price subject to the constraint that the consumer buys the software. That is,

$$\beta + q - p = \beta + 100 - p \geq 0 \Longrightarrow p \leq 100 + \beta$$

Hence, $p^p = 100 + \beta$, and the total profit is:

$$\pi^p = 10,000 + 100\beta - \phi = 100\beta - 2000 < \pi^{NP}$$

which means that it is not profitable to invest in protecting this software.

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(3) Consider the library-pricing model analyzed in class. Suppose that there are $\eta = 1200$ potential readers and $\lambda = 50$ libraries (i.e., $1200/50 = 24$ readers per library). The utility function of each potential reader is given by

$$U \stackrel{\text{def}}{=} \begin{cases} 23 - p^b & \text{if she buys and owns the book} \\ 23 - 2p_i^r & \text{if she borrows (rents) from library } i \\ 0 & \text{if she does not read the book.} \end{cases}$$

There is one publisher who can sell either to individual readers, or to libraries but not to both. Each copy of the book costs $\mu = 12$ to produce. Answer the following questions.

(3a) [5 pts.] Calculate the publisher's profit-maximizing price and her profit level, assuming that the publisher sells directly to individual readers only.

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The publisher's profit-maximizing price is $p^b = 23$. Hence, the profit when selling to individual readers only is $\pi^b = (23 - 12)1200 = 13,200$.

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(3b) [10 pts.] Calculate the publisher's profit-maximizing library price and her profit level assuming that the publisher sells one copy to each library only.

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The maximum rental price each library i can charge each reader is $p_i^r = 23/2$. Therefore, the maximum price in which each library is willing to pay for one copy of the book is:

$$p_i = \left(\frac{\eta}{\lambda}\right) p_i^r = \left(\frac{1200}{50}\right)\left(\frac{23}{2}\right) = 276.$$

Now, the publisher produces 50 copies (one for each library), and therefore earns a profit of:

$$\pi = (p_i - \mu)50 = (276 - 12)50 = 13,200.$$

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(4) [15 pts.] Consider a city with only two local banks labeled bank A and bank B. Each person is allowed to maintain only one bank account (either with bank A, or B, but not both). Let δ_A denote the cost of switching a bank account from bank A to bank B. That is, the total cost (including inconvenience) of closing an account with bank A and opening a fully-operative account with bank B. Similarly, let δ_B denote the cost of switching a bank account from bank B to bank A.

You are now given the following information:

(a) Bank A maintains 100 (one hundred) accounts, whereas bank B maintains 200 (two hundred) accounts.

(b) Bank A levies a fee of $f_A = 30$ per account and bank B levies a fee of $f_B = 30$ per account.

(c) The utility functions of a bank A and a bank B account holder, respectively, are given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} -f_A & \text{staying with bank } A \\ -f_B - \delta_A & \text{switching to bank } B, \end{cases} \quad \text{and} \quad U_B \stackrel{\text{def}}{=} \begin{cases} -f_A - \delta_B & \text{switching to bank } A \\ -f_B & \text{staying with bank } B \end{cases}$$

Suppose that banks do not bear any costs and that their fees are set in an undercut-proof equilibrium. Using the above data, calculate the switching-cost parameters δ_A and δ_B. Show your calculations!

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In an UPE, bank A maximizes f_A subject to:

$$\pi_B = 200 f_B \geq (200 + 100)(f_A - \delta_A)$$

Hence,

$$200 \times 30 = 300(30 - \delta_A) \Longrightarrow \delta_A = 10.$$

In an UPE, bank B maximizes f_B subject to:

$$\pi_A = 100 f_A \geq (100 + 200)(f_B - \delta_B)$$

Hence,

$$100 \times 30 = 300(30 - \delta_B) \Longrightarrow \delta_B = 20.$$

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(5) In an Island named *Bilingwa* off the coast of Mexico there are 100 inhabitants. 60 are native English speakers, whereas 40 are native Spanish speakers. Let n_{ES} denote the number of native English speakers who learn to speak Spanish. Similarly, let n_{SE} denote the number of native Spanish speakers who learn English. The utility of each resident increases with the number of residents to whom he is able to communicate with. We define the utility function of each native English and each native Spanish speakers, respectively, by

$$U_E = \begin{cases} \frac{60 + n_{SE}}{10} & \text{does not learn Spanish} \\ \frac{60 + 40}{10} - 5 & \text{learns Spanish} \end{cases} \qquad U_S = \begin{cases} \frac{40 + n_{ES}}{10} & \text{does not learn English} \\ \frac{40 + 60}{10} - 7 & \text{learns English} \end{cases}$$

These utility functions reveal that it is "easier" (less costly) for a native English speaker to learn Spanish, than for a native Spanish speaker to learn English (cost of 5 compared with 7).

(5a) [10 pts.] Find the number of native English speakers who learn Spanish, n_{ES}, and the number of native Spanish speakers who learn English, n_{SE} in a language-acquisition equilibrium. Prove your results!

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$\langle n_{ES}, n_{SE}\rangle = \langle 0, 0\rangle$ (i.e., no one learns any language) is an equilibrium. *Proof*:
Given $n_{SE} = 0$,

$$U_E(0,0) = \frac{60+0}{10} = 6 > 5 = \frac{60+40}{10} - 5 = U_E(60,0)$$

hence, not learning Spanish yields a higher utility to English native speakers.
Given $n_{ES} = 0$,

$$U_S(0,0) = \frac{40+0}{10} = 4 > 3 = \frac{40+60}{10} - 7 = U_S(0,40)$$

hence, not learning English yields a higher utility to Spanish native speakers.

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(5b) [10 pts.] Find the socially-optimal levels of n_{ES} and n_{SE}. Prove your answer!

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Social welfare is defined by: $W = 60U_E + 40U_S$. Now,

$$\begin{aligned} W(0,0) &= 60\frac{60}{10} + 40\frac{40}{10} = 520 \\ W(60,0) &= 60\left(\frac{60+40}{10} - 5\right) + 40\frac{40+60}{10} = 700 \\ W(0,40) &= 60\frac{100}{10} + 40\left(\frac{100}{10} - 7\right) = 720 \\ W(60,40) &= 60\left(\frac{100}{10} - 5\right) + 40\left(\frac{100}{10} - 7\right) = 420 \end{aligned}$$

Therefore, the socially-optimal learning outcome is $\langle n_{ES}, n_{SE}\rangle = \langle 0, 40\rangle$ meaning that only the native Spanish speaking learn English.

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THE END