Instructor's Manual for

The Economics of Network Industries

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Contents

To the Instructor

Before planning the course, I urge the instructor to read carefully the Preface of the book that suggests different ways of organizing courses.

The goals of this manual are:

- To provide the readers (instructors, students, and researchers) with my solutions for all the problems listed at the end of each chapter.
- To convey to the instructor my views of what the important concepts in each topic are.
- To suggest which topics to choose for different types of classes and levels of students.

Also, you will find errata files for the book on my webpage

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Finally, please alert me to any errors or incorrect presentations that you detect in the book and in this manual. (e-mail addresses are given below).

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Chapter 2

The Hardware Industry

This is the most important chapter of this book as it introduces the student to a wide variety of network terminology via several definitions (e.g., coordination, perfect foresight, network externalities, compatibility, etc.). This chapter also introduces all the equilibrium concepts used in this book (for monopoly and duopoly market structures). I, therefore, recommend teaching the entire chapter. Before teaching the chapter, the instructor must have completed teaching Appendices A, B, and C. Note that Appendix C *cannot* be skipped as it defines our main equilibrium concept.

Answers to Exercises

1. (a) Under incompatibility, we solve for prices satisfying

$$\begin{aligned}\pi_B = (p_B - \mu)\eta &= (p_A - \delta + \alpha\eta - \mu)2\eta \\ \pi_A = (p_A - \mu)\eta &= (p_B - \delta + \alpha\eta - \mu)2\eta\end{aligned}$$

yielding

$$p_A = p_B = 2(\delta - \alpha\eta) + \mu \quad \text{and} \quad \pi_A^U = \pi_B^U = 2\eta(\delta - \alpha\eta).$$

Notice that the production costs are borne completely by consumers, so profit remains the same as in (2.23).

(b) Under compatibility, we solve for prices satisfying

$$\begin{aligned}\pi_B = (p_B - \mu)\eta &= (p_A - \delta - \mu)2\eta \\ \pi_A = (p_A - \mu)\eta &= (p_B - \delta - \mu)2\eta,\end{aligned}$$

yielding $p_A = p_B = 2\delta + \mu$, and $\pi_A = \pi_B = 2\eta\delta$.

(c) Both firms make a higher profit under compatibility. In fact, since costs are borne by consumers, the profit levels are the same as in Section 2.2.3 with zero production costs.

2. The UPE prices must satisfy

$$\begin{aligned}\pi_B = p_B\eta &= (p_A - \delta + 2\eta)2\eta \\ \pi_A = p_A\eta &= (p_B - \delta + 3\eta)2\eta,\end{aligned}$$

yielding

$$p_A = \frac{2(3\delta - 7\eta)}{3}, \quad \text{and} \quad p_B = \frac{2(3\delta - 8\eta)}{3},$$

and profit levels given by

$$\pi_A = \frac{2\eta(3\delta - 7\eta)}{3}, \quad \text{and} \quad \pi_B = \frac{2\eta(3\delta - 8\eta)}{3}.$$

3. (a) UPE prices must satisfy

$$\begin{aligned}100p_A &= 300(p_B - \delta + 100)\\ 200p_B &= 300(p_A - \delta + 200),\end{aligned}$$

yielding

$$p_A^U = \frac{15\delta - 2400}{7} = \frac{15(\delta - 160)}{7} \text{ and } p_B^U = \frac{12\delta - 1500}{7} = \frac{12(\delta - 125)}{7}.$$

Hence, the UPE profit levels under incompatibility are:

$$\pi_A^U = \frac{1500(\delta - 160)}{7} \quad \text{and} \quad \pi_B^U = \frac{2400(\delta - 125)}{7}.$$

(b)

$$p_A^U > p_B^U \iff 12\delta - 1500 < 15\delta - 2400 \iff \delta > 300,$$

which is assumed in the question. The firm with the higher market share charges the lower price since it is more vulnerable to being undercut.

(c) Under compatibility, undercutting does not enhance the network size of the undercutting firm. Hence, UPE prices must satisfy

$$\begin{aligned}100p_A &= 300(p_B - \delta)\\ 200p_B &= 300(p_A - \delta),\end{aligned}$$

yielding

$$p_A^U = \frac{15\delta}{7}, \quad p_B^U = \frac{12\delta}{7} \quad \text{and} \quad \pi_A^U = \frac{1500\delta}{7}, \quad \pi_B^U = \frac{2400\delta}{7}.$$

(d) Each firm earns a higher profit when the machines are compatible than when the machines are incompatible.

4. (a) By way of contradiction, suppose that there exists an UPE where 60 consumers buy A and 60 consumers buy B. Then, "equilibrium" prices must satisfy

$$\begin{aligned}\pi_B = 60p_B &= 120(p_A - \delta + 2 \times 60)\\ \pi_A = 60p_A &= 120(p_B - \delta + 2 \times 60),\end{aligned}$$

yielding $p_A = p_B = -40 < 0$, a contradiction to the assumption that firms are profit maximizing (loss minimizing in this case).

(b) We look for an equilibrium where firm A sells to *all* 120 consumers. In an UPE, firm A raises p_A subject to the constraint that firm B will not find it profitable to undercut. That is,

$$\pi_B = 0 \geq \max\left\{120(p_A - 100)\,,\ 60(p_A + 2 \cdot 60 - 2 \cdot 120 + 100)\right\}.$$

The first term is strong undercutting where firm B attracts all the 120 consumers to buy brand B (thus, no change in network size). The second term is mild undercutting where firm B attract only the 60 brand B oriented consumers (thus, a decline of 60 in the network size). The extra $+100$ is added because B-oriented consumers gain a utility of $+100$ when they switch from brand A to their ideal brand, brand B.

The first term yields $p_A = 100$, whereas the second $p_A = 20$. Hence, the UPE price is $p_A = 20$.

5. In an UPE, firm A will not undercut if

$$100p_A = 100 \times 50 \geq 200(p_B - \delta + 100\alpha) = 200(50 - \delta + 100\alpha).$$

Hence, $\delta = 25 + 100\alpha$.

6. (a) With 4 types of consumers, let us construct an UPE where firm A sells to consumers AA, AB, and BA and firm B sells to consumer BB, if both conditions are simultaneously satisfied:

$$\begin{aligned} \pi_B = p_{BB} \times 1 &= \max\{3p_{AA}, 4(p_{AA} - 2\delta)\} \\ \pi_A = p_{AA} \times 3 &= 4(p_{BB} - 2\delta), \end{aligned}$$

There are 2 candidate equilibria ("mild" and "strong" undercutting).

Solving the system of equations for the case of "mild" undercutting yield the equilibrium prices and profit levels

$$p_{AA} = \frac{8\delta}{9}, \quad p_{BB} = \frac{8\delta}{3}, \quad \pi_{AA} = 3p_{AA} = \frac{8\delta}{3}, \quad \text{and} \quad \pi_{BB} = 1 \times p_{BB} = \frac{8\delta}{3}.$$

Note that this is an equilibrium if

$$3p_{AA} \geq 4(p_{AA} - 2\delta) \quad \text{or} \quad p_{AA} \leq 8\delta,$$

which happens to be the case.

Solving the system of equations for the case of "strong" undercutting yield the equilibrium prices and profit levels

$$p_{AA} = \frac{40\delta}{13}, \quad p_{BB} = \frac{56\delta}{13}, \quad \pi_{AA} = 3p_{AA} = \frac{120\delta}{13}, \quad \text{and} \quad \pi_{BB} = 1 \times p_{BB} = \frac{56\delta}{3}.$$

This is *not* an equilibrium because

$$3p_{AA} \leq 4(p_{AA} - 2\delta) \quad \text{or} \quad p_{AA} \geq 8\delta,$$

which is clearly *not* the case.

(b) Under component compatibility we look for an UPE where firm A sells component X to consumers AA and AB, and firm B sells component X to consumers BB and BA. Hence, UPE prices must satisfy

$$\begin{aligned} p_B^X \times 2 &= (p_A^X - \delta)4 \\ p_A^X \times 2 &= (p_B^X - \delta)4. \end{aligned}$$

For the Y-component market, UPE prices satisfy

$$\begin{aligned} p_B^Y \times 2 &= (p_A^Y - \delta)4 \\ p_A^Y \times 2 &= (p_B^Y - \delta)4. \end{aligned}$$

Therefore, equilibrium prices and profit levels given by

$$p_A^X = p_B^X = p_A^Y = p_B^Y = 2\delta, \quad \text{and} \quad \pi_A^C = \pi_B^C = 8\delta.$$

Chapter 3

The Software Industry

Answers to Exercises

1. Substituting these prices into (3.3) yield

$$s|_{p=\omega/4} = \frac{3\eta\omega}{4\phi}, \quad s|_{p=\omega/2} = \frac{\eta\omega}{\phi}, \quad \text{and} \quad s|_{p=\omega} = 0.$$

This follows from the fact that consumers' budget is fixed and is to be allocated between hardware and software. So, a rise in the price of hardware, reduces software expenditure, and hence the variety of available software.

2. (a) The maximum price the monopoly can charge is $p = \alpha s/\gamma$. Equation (3.3) implies that software variety is

$$s = \frac{\eta(\omega - p)}{\phi} = \frac{\eta(\gamma\omega - \alpha s)}{\gamma\phi}.$$

Solving for s, and then substituting to find the equilibrium price yield

$$s = \frac{\eta\gamma\omega}{\alpha\eta + \gamma\phi}, \quad \text{and} \quad p^m = \frac{\alpha\eta\omega}{\alpha\eta + \gamma\phi}.$$

(b) An increase in γ reduces the monopoly price because consumers' willingness to pay for hardware has been reduced. As a result, the equilibrium variety of software s increases.

3. (a) If firm A wishes to undercut the price of B it has to set

$$p'_A = p_B - \delta + \alpha(400 - 600) = p_B - \delta - 200\alpha.$$

That is, since variety is fixed, undercutting does not increase variety of software supporting each machine.

(b) If firm B wishes to undercut the price of A it has to set

$$p'_B = p_A - \delta + \alpha(600 - 400) = p_A - \delta + 200\alpha.$$

(c) We need to solve

$$\begin{aligned} \eta p_B &= 2\eta(p_A - \delta + 200\alpha) \\ \eta p_A &= 2\eta(p_B - \delta - 200\alpha) \end{aligned}$$

yielding

$$p^I_A = \frac{2(3\delta - 200\alpha)}{3}, \quad \text{and} \quad p^I_B = \frac{2(3\delta + 200\alpha)}{3},$$

where superscript I stands for "incompatibility." Profit levels are then

$$\pi_A^I = \frac{2\eta(3\delta - 200\alpha)}{3} < \pi_B^I = \frac{2\eta(3\delta + 200\alpha)}{3}.$$

Clearly, the firm supported by a larger variety of software makes a higher profit since its consumers are willing to pay a higher price of the machine supported by more software.

(d) If firm A wishes to undercut the price of B it has to set

$$p'_A = p_B - \delta + \alpha(1000 - 1000) = p_B - \delta.$$

This is because under compatiblity all users can run 1000 packages on any machine.

(e) Due to compatibility, the variety of software available to any consumer is 1000. We need to solve

$$\begin{aligned} \eta p_B &= 2\eta[p_A - \delta + \alpha(1000 - 1000)] \\ \eta p_A &= 2\eta[p_B - \delta + \alpha(1000 - 1000)] \end{aligned}$$

yielding $p_A^C = p_B^C = 2\delta$ and $\pi_A^C = \pi_B^C = 2\eta\delta$, where superscript C stands of "compatibility."

(f) Firm A makes a higher profit under compatibility for two reasons: First, its consumers have access to 1000 software packages (compared with 400 under incompatibility). Second, compatibility eliminates its strategic disadvantage stemming from a lower number of supporting packages under incompatibility.

In contrast, firm B make a higher profit under incompatibility as it can exploit its strategic advantage of having a larger variety of supporting software.

4. (a) Type I consumers pirate the software (their utility function implies that there is no reason for them to buy this software). Hence, if type O consumers buy the software, the total number of users is $q = 100 + 200 = 300$.

Now, the producer maximizes its price subject to the constraint that piracy is not beneficial for type O consumers. That is,

$$3 \times 300 - p \geq 300.$$

Hence, $p^n = 600$, and $\pi^n = 100 \times 600 = 60,000$ since only 100 consumers buy this software.

(b) In principle, we should consider two cases:

i. Type I users buy the software. In this case, $q = 300$, so the maximum price the producer can charge so that type I buy this software is $p = 300$.

ii. Type I users don't buy the software. Therefore, since software is protected, $q = 100$. In view of the utility function of type O consumers, the maximum price the producer can charge is $p = 3 \times 100 = 300$.

However, notice that at the price $p = 300$ all consumers buy the software. Hence, $p^p = 300$, $q^p = 300$ (which is also the number of buyers) and therefore $\pi^p = 300 \times 300 = 90,000$.

(c) Protection policy yields a higher profit that non-protection since

$$\pi^p = 90,000 > 60,000 = \pi^n.$$

Chapter 4

Technology Advance and Standardization

This chapter is divided into two parts: Technology advance and international standardization. There is no connection between the topics, except that both deal with aspects of technology.

The topics analyzed in this chapter should be viewed as an extension (or a continuation) of Chapter 2. The sections on technology advance is self-contained. Some undergraduate students (especially, those who are not familiar with dynamic models) may find Section 4.2 too difficult. In this case, I suggest studying only the static model (Section 4.1).

The section on international standardization relies on knowing UPE with network externalities in the same way as practiced in Chapter 2.

1. (a) There are two Nash equilibria: (N, N) and (O, O). *Proof.*

$$\begin{aligned} \pi_A(N,N) = 3 &> 0 = \pi_A(O,N) \\ \pi_B(N,N) = 3 &> 0 = \pi_B(N,O) \\ \pi_A(O,O) = 2 &> 1 = \pi_A(N,O) \\ \pi_B(O,O) = 2 &> 1 = \pi_B(O,N). \end{aligned}$$

■

   (b) The outcome (N, N) does *not* constitute excess momentum since $\pi_A(N,N) = 3 > 2 = \pi_A(O,O)$ and $\pi_B(N,N) = 3 > 2 = \pi_B(O,O)$. That is, there is no market failure here since the new technology is more profitable to all firms.

2. (a) The adoption condition (4.6) is now given by

$$\lambda t_{g+1} + t_{g+1}\eta \geq \lambda t_g + (t_{g+1} - 1)\eta + t_{g+1}\eta.$$

   (b) Solving the above condition for t_{g+1} yields

$$t_{g+1} = \left\lceil \frac{\lambda t_g - \eta}{\lambda - \eta} \right\rceil.$$

   (c) Substituting the parameters into the above equation yields

$$t_2 = \frac{2 \times 2 - 1}{2 - 1} = 3, \quad \text{hence} \quad t_3 = \frac{2 \times 3 - 1}{2 - 1} = 5, \quad \text{hence} \quad t_4 = \frac{2 \times 5 - 1}{2 - 1} = 9.$$

3. (a) In equilibrium, each firm sells to η consumers. In an UPE, Firm i maximizes p_i subject to

$$\eta p_j \geq 2\eta[p_i - \delta + \alpha(2\eta - \eta)].$$

   In symmetry, the UPE prices and profit levels are

$$p_1^{\mathsf{MR}} = p_2^{\mathsf{MR}} = 2(\delta - \alpha\eta) \quad \text{and} \quad \pi_1^{\mathsf{MR}} = \pi_2^{\mathsf{MR}} = \eta p_i^{\mathsf{MR}} = 2\eta(\delta - \alpha\eta).$$

   Notice that we must assume that $\delta > \alpha\eta$ for this equilibrium to exist.

(b) Profit levels are calculated in the previous section. As for utility, since each consumer buys his ideal brand, and since η consumers buy each brand

$$U_1^B = U_2^A = \alpha\eta - p_i^{\mathsf{MR}} = \alpha\eta - 2\eta(\delta - \alpha\eta) = 3\alpha\eta - 2\delta.$$

(c) The welfare of country A (similarly, of country B) is the sum of residents' utilities and the profit of the domestic firm. Therefore,

$$W^A = \eta U_2^A + \pi_1 = \eta(3\alpha\eta - 2\delta) + 2\eta(\delta - \alpha\eta) = \alpha\eta^2.$$

(d) Each firms is a monopoly in its country and sells its brand to domestic consumers who do not value its brand as the ideal brand. Therefore,

$$p_i^{\mathsf{NR}} = \alpha\eta - \delta, \quad \text{and} \quad \pi_i^{\mathsf{NR}} = \eta p^{\mathsf{MR}} = \eta(\alpha\eta - \delta).$$

Notice that we must assume that $\delta < \alpha\eta$ for this equilibrium to exist.

(e) Each consumer gains zero utility since the domestic monopoly extract the entire surplus. Hence, social welfare of each country equal the profit of the domestic firm given by

$$W^A = W^B = \eta(\alpha\eta - \delta).$$

(f) This comparison cannot be performed here since the equilibrium in part (a) was computed under the assumption that $\delta > \alpha\eta$, whereas the equilibrium in part (d) was computed under the assumption that $\delta < \alpha\eta$.

Chapter 5

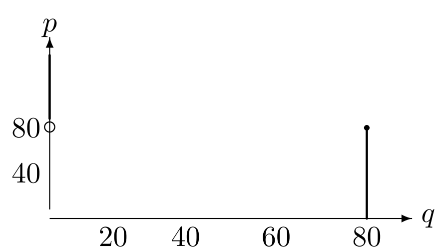
Telecommunication

The core of this chapter is the demand and supply for telecommunication services. Section 5.1 is the basic representation of the construction of the demand for telecommunication services in the presence of network externalities, as well as solutions for the monopoly service provision as well as the social planner's. If your students are trained to use some calculus, I recommend teaching them Section 5.2 which is based on Rohlfs' original paper, extended to include entry. The entry analysis (more precisely the construction of the residual demand) may be somewhat more difficult for undergraduate students as it requires inversion of the inverse-demand function.

I recommend teaching Section 5.3 since it corresponds to current issues arising with the ongoing regulation of telephony industries throughout the world. In case that the instructor is short of time, I recommend focusing on Subsection 5.3.1 and Subsection 5.3.3.

Answers to Exercises

1. The aggregate demand is depicted in Figure 5.1. If only type H subscribe, for $p > 2 \times 20$, type H consumers won't subscribe since $U_H = 40 - p < 0$.



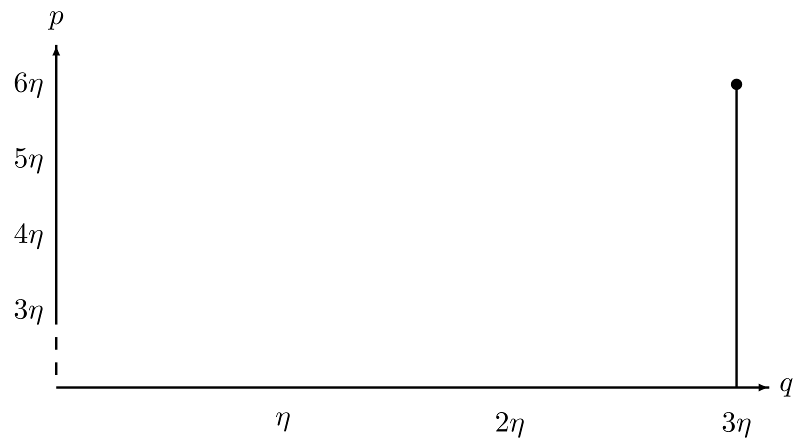
Solution-Figure 5.1: Aggregate demand with two consumer types.

For $p > 80$ type L consumers will not subscribe even if all type H consumers subscribe since $U_L = 20 + 60 - p < 0$. Now, the last inequality is reversed if $p \leq 80$, and hence at this price range all the $20 + 60$ consumers subscribe.

2. (a) If only η consumers connect, the maximum connection fee type 3 consumer is willing to pay is $p = 4\eta$. If only 2η consumers are connected, the maximum fee type 2 consumer is willing to pay is $p = 3 \times 2\eta = 6\eta$. Finally, if all the 3η consumers are connected, the maximum fee type 3 is willing to pay is $p = 2 \times 3\eta = 6\eta$. The aggregate demand is depicted in Figure 5.2

   (b) For a type 3 consumer, given that q are connected, her willingness to pay is $p = 4q$. Hence, at a given price of $p = 2\eta$, the *critical mass* is $q^{\text{cm}} = p/4 = 2\eta/4 = \eta/2$.

   (c) With no production costs, Figure 5.2 reveals that price times quantity is maximized when $p = 6\eta$ and $q = 3\eta$, hence $\pi = 18\eta^2$.



Solution-Figure 5.2: Aggregate demand with three consumer types.

3. (a) The inverse demand facing the monopolist is now

$$p = (2 - \hat{x})\eta\hat{x}.$$

(b) The monopoly's profit is

$$\pi = (2 - \hat{x})\eta\hat{x}^2,$$

yielding first- and second-order conditions given by

$$0 = \frac{\partial \pi}{\partial \hat{x}} = \hat{x}(4 - 3\hat{x}), \quad \text{and} \quad 0 > \frac{\partial^2 \pi}{\partial \hat{x}^2} = 2(2 - 3\hat{x}),$$

which holds for all $\hat{x} > 2/3$.

(c) Solving for the larger root of the first-order condition, then substituting it into the inverse-demand function and the profit function yield

$$\hat{x} = \frac{4}{3}, \quad p = \frac{8}{9}, \quad \text{and} \quad \pi = \frac{32}{27}.$$

4. (a) $p_D = 20$ yields a higher profit than $p = 80$ if

$$20(\eta_L + \eta_H) \geq 80\eta_H \quad \text{if} \quad \eta_H \leq \frac{\eta_L}{3}.$$

(b) $p_D = 20$ yields a higher profit than $p = 80$ if

$$(20 - a_{\vec{DC}})(\eta_L + \eta_H) \geq (80 - a_{\vec{DC}})\eta_H \quad \text{if} \quad a_{\vec{DC}} \leq \frac{20(\eta_L - 3\eta_H)}{\eta_L}.$$

Hence,

$$p_D = \begin{cases} 20 & \text{if } a_{\vec{DC}} \leq \frac{20(\eta_L - 3\eta_H)}{\eta_L} \\ 80 & \text{if } \frac{20(\eta_L - 3\eta_H)}{\eta_L} < a_{\vec{DC}} \leq 80 \\ a_{\vec{DC}} & \text{if } a_{\vec{DC}} > 80. \end{cases}$$

(c) Substituting $\eta_H = 100$ and $\eta_L = 500$ into the above condition we have that the "threshold" access charge is

$$\bar{a} = \frac{20(500 - 3 \times 100)}{500} = 8.$$

Therefore, the demand for phone calls facing carrier D as a function of the access charge set by carrier C is

$$q_D = \begin{cases} 600 & \text{if } a_{\vec{DC}} \leq 8 \\ 100 & \text{if } a_{\vec{DC}} > 8. \end{cases}$$

Hence, carrier C has two access pricing options yielding the profit levels:

$$\pi'_C = \begin{cases} 600 \times 8 & \text{if } a_{\vec{DC}} = 8 \\ 100 \times 80 & \text{if } a_{\vec{DC}} = 80, \end{cases}$$

implying that the profit-maximizing access charge set by carrier C is $a_{\vec{DC}} = 80$.

(d) Yes, there is a market failure since with zero production costs socially-optimal outcome implies that all consumers are served, whereas in this equilibrium only high-income consumers are served.

The regulator can implement the socially optimal outcome by setting a ceiling $\bar{a} = 8$ on all access charges.

5. (a)

$$\pi_N = (p_N - a)\eta_H + a\eta_S = a\eta_S, \text{ and } \pi_S = (p_S - a)\eta_S + a\eta_N = \beta\eta_S + a(\eta_N - \eta_S).$$

Hence, $a = \beta$ maximizes the profit of each company.

(b) $\hat{a} = (\beta + \beta)/2 = \beta$. There is no difference in company S's profit function. However, company N now prefers maximal access charges since access charges constitute the sole source of revenue under competition.

(c) The amount transferred is:

$$T_{\vec{NS}} = -T_{\vec{SN}} = \beta(\eta_N - \eta_S).$$

6. (a)

$$\pi_N = (p_N - a)\eta_H + a\eta_S = \beta\eta_N - a(\eta_N - \eta_S), \text{ and } \pi_S = (p_S - a)\eta_S + a\eta_N = a\eta_N.$$

Hence, the profit of company N is maximized when $a = 0$, whereas the profit of company S is maximized when $a = \beta$.

(b) $\hat{a} = (\beta + 0)/2 = \beta/2$. Company N now prefers minimal access charges since due to a shortage of incoming phone calls net transfers are negative. So, company N makes maximum profit when there are no access charges so the sole source of revenue is monopoly profit from the sale of international calls.

(c) The amount transferred is:

$$T_{\vec{NS}} = -T_{\vec{SN}} = \beta(\eta_N - \eta_S)/2.$$

Chapter 6

Broadcasting

Section 6.1 constitutes the core of this chapter as it analyzes competition among broadcasting stations. Instructors who are short in time should, in my opinion choose to teach this section. Section 6.2 is geared to those instructors who concentrate their course on telecommunication and broadcasting policy, in which case spectrum allocation becomes a key topic from a regulatory point of view. Section 6.3 should be given the lowest priority because digital convergence is of interest mostly to specialists.

Answers to Exercises

1. (a) Station i's best response function is:

$$R_i(t_j) = \begin{cases} \{17, 18\} & \text{if } t_j = 17 \\ \{17, 18\} & \text{if } t_j = 18 \\ 18 & \text{if } t_j = 19. \end{cases}$$

Therefore, here are 4 Nash equilibria: $\langle t_A, t_B \rangle = \langle 17, 17 \rangle$ in which case each station earns $\pi_A = \pi_B = 2\rho\eta$. $\langle t_A, t_B \rangle = \langle 18, 18 \rangle$ in which case each station earns $\pi_A = \pi_B = 2\rho\eta$. $\langle t_A, t_B \rangle = \langle 17, 18 \rangle$ in which case each station also earns $\pi_A = \pi_B = 2\rho\eta$. $\langle t_A, t_B \rangle = \langle 18, 17 \rangle$ in which case each station also earns $\pi_A = \pi_B = 2\rho\eta$.

(b) Station i's best response function is now given by:

$$R_i(t_j) = \begin{cases} 17 & \text{if } t_j = 17 \\ 17 & \text{if } t_j = 18 \\ 18 & \text{if } t_j = 19. \end{cases}$$

There exists only one equilibrium given by $\langle t_A, t_B \rangle = \langle 17, 17 \rangle$ in which case each station earns $\pi_A = \pi_B = 5\rho\eta/2$.

2. Station i's best-response function is:

$$R_i(t_j, t_k) = \begin{cases} 18 & \text{if } t_j = t_k = 17 \\ 17 & \text{if } t_j = 17 \text{ and } t_k = 18 \\ 17 & \text{if } t_j = 18 \text{ and } t_k = 17 \\ 17 & \text{if } t_j = 17 \text{ and } t_k = 19 \\ 17 & \text{if } t_j = 19 \text{ and } t_k = 17 \\ 17 & \text{if } t_j = 18 \text{ and } t_k = 19 \\ 17 & \text{if } t_j = 19 \text{ and } t_k = 18 \\ 17 & \text{if } t_j = t_k = 18 \\ 18 & \text{if } t_j = t_k = 19. \end{cases}$$

Therefore, here are three equilibria: $\langle t_A, t_B, t_C \rangle = \langle 17, 17, 18 \rangle$ in which case stations' profit levels are $\pi_A = \pi_B = 3\rho\eta/2$, and $\pi_C = 2\rho\eta$. The remaining two equilibria are $\langle t_A, t_B, t_C \rangle = \langle 17, 18, 17 \rangle$ and $\langle t_A, t_B, t_C \rangle = \langle 18, 17, 17 \rangle$.

3. (a) $\langle t_A, t_B, t_C \rangle = \langle 5, 5, 6 \rangle$ is a NE.
   Proof.

$$\begin{aligned}
\pi_A(5,5,6) = 300 &> (200+100)/2 = \pi_A(6,5,6) \\
\pi_A(5,5,6) = 300 &> 200 = \pi_A(7,5,6) \\
\pi_B(5,5,6) = 300 &> (200+100)/2 = \pi_B(5,6,6) \\
\pi_B(5,5,6) = 300 &> 200 = \pi_B(5,7,6) \\
\pi_C(5,5,6) = 300 &\geq 900/3 = \pi_C(5,5,5) \\
\pi_C(5,5,6) = 300 &> 100/3 + 200 = \pi_C(5,5,7),
\end{aligned}$$

   Remark: Although you were asked to characterize only one equilibrium, you should be able to prove that $\langle 6,5,5 \rangle$, $\langle 5,6,5 \rangle$, and $\langle 5,5,5 \rangle$ are also NE.

   (b) We define social welfare by

$$W \stackrel{\text{def}}{=} 600U_5 + 100U_6 + 200U_7 + \pi_A + \pi_B + \pi_C.$$

   Then,

$$W(5,5,6) = 600\beta + 100\beta + 200(\beta - \delta) + 300 + 300 + 300 = 900(1+\beta) - 200\delta.$$

   Now, look at $\langle t_A, t_B, t_C \rangle = \langle 5, 6, 7 \rangle$.

$$W(5,6,7) = 900(1+\beta) > W(5,5,6).$$

   Hence, $\langle t_A, t_B, t_C \rangle = \langle 5, 5, 6 \rangle$ is not socially optimal.

4. (a) No ! *Proof:*

$$\pi_C(1,2,2) = \frac{\rho\eta_2}{2} < \frac{\rho\eta_1}{2} = \pi_C(1,2,1).$$

   (b)

$$\begin{aligned}
\pi_A(1,2,3) = \rho\eta_1 &> \rho\eta_2/2 = \pi_A(2,2,3) \text{ [since } \eta_1 > \eta_2\text{].} \\
\pi_A(1,2,3) = \rho\eta_1 &> \rho\eta_3/2 = \pi_A(3,2,3) \text{ [since } \eta_1 > \eta_3\text{].} \\
\pi_B(1,2,3) = \rho\eta_2 &> \rho\eta_1/2 = \pi_A(1,1,3) \text{ [since } \eta_1/2 < \eta_3 < \eta_2\text{].} \\
\pi_B(1,2,3) = \rho\eta_2 &> \rho\eta_3/2 = \pi_A(1,3,3) \text{ [since } \eta_2 > \eta_3\text{].} \\
\pi_C(1,2,3) = \rho\eta_3 &> \rho\eta_1/2 = \pi_A(1,2,1) \text{ [since } \eta_3 > \eta_1/2\text{].} \\
\pi_C(1,2,3) = \rho\eta_3 &> \rho\eta_2/2 = \pi_A(1,2,2) \text{ [since } \eta_3 > \eta_1/2 > \eta_2/2\text{].}
\end{aligned}$$

5. (a) When channels are sold separately, the profit maximizing prices are: $p_C = 5$, $p_B = 5$, and $p_S = 2$. Total profit is then $\pi = 2 \times 5 + 5 + 3 \times 2 = 21$.
   (b) When all 3 channels are sold in a single package, $p = 8$, so $\pi = 8 \times 3 = 24$.
   (c) There is no way to improve over selling all channels in a single package.

Chapter 7

Markets for Information

Answers to Exercises

1. Aggregate total surplus is given by

$$\text{TS} \approx \frac{3/4}{1-3/4} = 3.$$

   Uncaptured surplus is given by

$$\text{UCP} = \text{TS} = 3.$$

   Note: Printings 1–4 (at least) contain errors on p.169–170 as that analysis contradicted our assumption that consumers don't pay for copies, see errata files. Under this mistaken analysis, the answer to (b) should be: Uncaptured surplus is given by

$$\text{UCP} = \text{TS} - \rho \approx \frac{9}{4}.$$

2. (a) Selling only to individual readers: $p^b = \beta$, hence $\pi^b = 1200(\beta - \mu)$.

   (b) Selling one copy to each of the 50 libraries: Each borrower will not pay more that $\beta/2$ for borrowing the book. Hence, the maximum price the publisher can charge library i is

$$p_i^r = \frac{\beta}{2}\frac{\eta}{\lambda} = \frac{\beta}{2}\frac{1200}{50} = 12\beta.$$

   Hence, with 50 libraries,

$$\pi^r = 50(12\beta - \mu).$$

   (c)

$$\pi^r \geq \pi^b \quad \text{if and only if} \quad \mu \geq \frac{12}{23}\beta.$$

Chapter 8

Banks and Money

Switching costs (Section 8.1) is a key topic in network industries and should be taught in my opinion. Switching costs characterize all the industries analyzed in this book, and there is was no particular reason why I chose to introduce it in the context of banks and not in the context of the hardware industry (say, in Chapter 2). The real reason why I deferred this analysis to Chapter 8 was that Chapter 2 became "too long." Therefore, I urge the instructor to emphasize in class that the analysis of switching costs applies not only to banks, but to all other industries discussed in this book, and to many other industries that don't necessarily fall into the category of network industries.

Section 8.2 resembles a little bit the software industry, since when a bank makes its ATM available to customers of the competing bank for cash withdrawals, it actually enhances the value of the competing bank. Instructors who use this book for an industrial organization course may skip Section 8.3 which utilizes network theory in explaining the coexistence of several payment media.

Answers to Exercises

1. Following exactly the same formulas, we have

$$\begin{aligned}
\delta_1 &= f_1 - \frac{\eta_4 f_4}{\eta_1 + \eta_4} = 600 - \frac{849,955}{5,744,741 + 849,955} 525 \approx 532, \\
\delta_2 &= f_2 - \frac{\eta_4 f_4}{\eta_2 + \eta_4} = 600 - \frac{849,955}{4,695,078 + 849,955} 525 \approx 344, \\
\delta_3 &= f_3 - \frac{\eta_4 f_4}{\eta_3 + \eta_4} = 600 - \frac{849,955}{3,937,119 + 849,955} 525 \approx 531, \\
\delta_4 &= f_4 - \frac{\eta_1 f_1}{\eta_1 + \eta_4} = 525 - \frac{5,744,741}{5,744,741 + 849,955} 600 \approx 2.33.
\end{aligned}$$

Hence, the completed Table 8.1 is

Data	Bank 1	Bank 2	Bank 3	Bank 4
# Accounts (η_i)	5,744,741	4,695,078	3,937,119	849,955
Average Balance	4952	4459	2756	4722
Over Lifetime (f_i)	600	425	625	525
Switching Costs	532	344	531	2.33
SC/Avg.Bal.(%)	10%	7.7%	19%	0.57%

Solution-Table 8.1: Switching costs in The Finnish banking industry 1996.

2. (a) Bank B sets the maximal f_B such that

$$\pi_A = \eta f_A - 2\mu a \geq 2\eta[f_B - \delta + \alpha(2a - a) -] - 2\mu a.$$

Bank A sets the maximal f_A such that

$$\pi_B = \eta f_B - \mu a \geq 2\eta[f_A - \delta + \alpha(a - 2a)] - \mu a.$$

Solving the two equations assuming equality

$$f_A = 2\delta + \frac{2\alpha a}{3} \quad \text{and} \quad f_B = 2\delta - \frac{2\alpha a}{3}.$$

Hence,

$$\pi_A = \eta 2\delta + \frac{2\alpha\eta a}{3} - 2\mu a \quad \text{and} \quad \pi_B = \eta 2\delta - \frac{2\alpha\eta a}{3} - \mu a.$$

(b) An increase ATMs' maintenance cost parameter μ is 100% absorbed by the banks, and therefore does not affect the customers.

(c) Bank B sets the maximal f_B such that

$$\pi_A = \eta f_A - 2\mu a \geq 2\eta[f_B - \delta] - 2\mu a.$$

Bank A sets the maximal f_A such that

$$\pi_B = \eta f_B - \mu a \geq 2\eta[f_A - \delta] - \mu a.$$

Solving the two equations assuming equality

$$f_A = 2\delta \quad \text{and} \quad f_B = 2\delta.$$

Hence,

$$\pi_A = \eta 2\delta - 2\mu a \quad \text{and} \quad \pi_B = \eta 2\delta - \mu a.$$

(d) An increase ATMs' maintenance cost parameter μ is 100% absorbed by the banks, and therefore does not affect the customers.

3. Figures are not drawn.

(a) Currency is less costly than checks for merchant p if

$$\tau^M + \lambda^M p \leq \tau^{M,ck} \quad \text{or} \quad p \leq \frac{\tau^{M,ck} - \tau^M}{\lambda^M}.$$

(b) Currency is less costly than checks for buyers p if

$$\tau^B + \lambda^B p \leq \tau^{B,ck} \quad \text{or} \quad p \leq \frac{\tau^{B,ck} - \tau^B}{\lambda^B}.$$

(c) Since currency is the legal tender, each party can "force" on the other party to transact with currency. Hence, currency will be used for all transaction satisfying

$$p \leq \min\left\{\frac{\tau^{M,ck} - \tau^M}{\lambda^M}, \frac{\tau^{B,ck} - \tau^B}{\lambda^B}\right\}.$$

(d) Currency is less costly from a social welfare point of view than checks for all transactions satisfying

$$\tau^M + \tau^B + (\lambda^M + \lambda^B)p \leq \tau^{M,ck} + \tau^{B,ck}, \quad \text{or} \quad p \leq \frac{\tau^{M,ck} + \tau^{B,ck} - \tau^M - \tau^B}{\lambda^M + \lambda^B}.$$

(e) Since

$$\min\left\{\frac{\tau^{M,ck} - \tau^M}{\lambda^M}, \frac{\tau^{B,ck} - \tau^B}{\lambda^B}\right\} < \frac{\tau^{M,ck} + \tau^{B,ck} - \tau^M - \tau^B}{\lambda^M + \lambda^B},$$

checks are over used (in the middle transaction range). This typical market failure stems from the fact that each party does not take into account the cost that checks inflict on the other party.

Chapter 9

The Airline Industry

Answers to Exercises

1. (a) $TC^{\mathsf{FC}} = 3\phi + \eta + \eta + \eta = 3\phi + 3\eta$, and
$TC^{\mathsf{HS}} = 2\phi + (2\eta) + (2\eta) = 2\phi + 4\eta$. Therefore,

$$TC^{\mathsf{HS}} < TC^{\mathsf{FC}} \quad \text{if} \quad \eta < \phi.$$

(b) $TC^{\mathsf{FC}} = 3\phi + \sqrt{\eta} + \sqrt{\eta} + \sqrt{\eta} = 3\phi + 3\sqrt{\eta}$, and
$TC^{\mathsf{HS}} = 2\phi + \sqrt{2\eta} + \sqrt{2\eta} = 2\phi + 2\sqrt{2}\sqrt{\eta}$. Therefore,

$$TC^{\mathsf{HS}} < TC^{\mathsf{FC}} \quad \text{if} \quad \left(2\sqrt{2} - 3\right)\sqrt{\eta} < \phi,$$

which must hold since $\left(2\sqrt{2} - 3\right) < 0$.

2. (a) $p_i = \beta$, $i = 1, 2, 3$. Hence, $\pi^{\mathsf{FC}} = 6\eta\beta - 3\mu$.

(b) Since price discrimination is not possible all airfares are equalized, $p_1 = p_2 = p_3$. Hence,

$$p_i = \begin{cases} \beta & \text{if } \beta \le 6\delta \\ \beta - \delta & \text{if } \beta > 6\delta. \end{cases} \quad \text{and} \quad {\pi_i}^{\mathsf{HS}} = \begin{cases} 5\eta\beta - 2\mu & \text{if } \beta \le 6\delta \\ 6\eta(\beta - \delta) - 2\mu & \text{if } \beta > 6\delta. \end{cases}$$

(c) For the case where $\beta \le 6\delta$, $\pi^{\mathsf{FC}} > \pi^{\mathsf{HS}}$ if $\mu < \eta\beta$. For the case where $\beta > 6\delta$, $\pi^{\mathsf{FC}} > \pi^{\mathsf{HS}}$ if $\mu < 6\eta\delta$. Hence, a sufficient condition is that $\mu < \min\{\eta\beta, 6\eta\delta\}$.

(d) In an UPE, the entrant maximizes p^E subject to

$$5\eta p_i^I - 2\mu \ge 6\eta(p_3^E - \delta) - 2\mu.$$

Similarly, the incumbent maximizes p_i^I subject to

$$\eta p_3^E - \mu \ge 2\eta p_3^I - \mu.$$

Solving the two constraints for the case of equality yield

$$p_3^I = \frac{6\delta}{7}, \quad \text{and} \quad p_3^E = 2p_3^I = \frac{12\delta}{7}.$$

Clearly, $p_3^E > p_3^I$ since the entrant provides a nonstop service.

3. (a) With no code-sharing agreement, the UPE conditions are

$$\begin{aligned} \eta p_\alpha &\ge 2\eta(p_\beta - \delta + 3(f_\alpha - f_\beta)) \\ \eta p_\beta &\ge 2\eta(p_\alpha - \delta + 3(f_\beta - f_\alpha)). \end{aligned}$$

The unique UPE airfares are

$$p_\alpha = 2(\delta + f_\alpha - f_\beta) \quad \text{and} \quad p_\beta = 2(\delta + f_\beta - f_\alpha).$$

Hence, the profit levels are

$$\pi_\alpha = 2\eta(\delta + f_\alpha - f_\beta) \quad \text{and} \quad \pi_\beta = 2\eta(\delta + f_\beta - f_\alpha).$$

(b) Under code sharing, the UPE conditions are

$$\begin{aligned} \eta p_\alpha &\geq 2\eta(p_\beta - \delta) \\ \eta p_\beta &\geq 2\eta(p_\alpha - \delta). \end{aligned}$$

The unique UPE airfares are

$$p_\alpha = 2\delta \quad \text{and} \quad p_\beta = 2\delta.$$

Hence, the profit levels are

$$\pi_\alpha = 2\eta\delta \quad \text{and} \quad \pi_\beta = 2\eta\delta.$$

(c) A simple profit comparisons reveal that airline α loses from code sharing and airline β gains from code sharing. This happens since code sharing removes the advantage of airline α as the supplier of a higher frequency of flights.

Chapter 10

Social Interaction

The first section (Basic Definitions) can be skipped by those instructors who wish to have a purely non-calculus course, although very little calculus is used. Section 2 is the core of the chapter demonstrating the market effects of consumers whose preferences exhibit either conformity or vanity. The section on gifts is important as it preaches my philosophy that for the sake of economic efficiency no one should accept or give a gift.

Answers to Exercises

1. (a)
$$p'_A = p_B - 100 - \frac{300}{2}, \quad \text{and} \quad p'_B = p_A - 100 - \frac{100}{2}.$$
Thus, in order to attract B's customers, firm A must compensate them for the increase in the network size by 300. Similarly, in order to attract A's customers, firm B must compensate A's customers for the increase in the network size by 100.

   (b) In an UPE,
$$100p_B \geq 400\left(p_A - 100 - \frac{100}{2}\right) \quad \text{and} \quad 300p_A \geq 400\left(p_B - 100 - \frac{300}{2}\right).$$
Solving the two equations for the equality case yields
$$p_A = \frac{3400}{13} \quad \text{and} \quad p_B = \frac{5800}{13}.$$
Clearly, firm A charges the lower price since it entertains a larger network which hurts its consumers in this vanity case.

   (c)
$$\pi_A = 300p_A = \frac{1,020,000}{13} \quad \text{and} \quad \pi_B = 100p_b = \frac{580,000}{13}.$$
Thus, despite charging a lower price, firm A earns a higher profit since it has more customers.

2. (a) Each resident gives one gift. Therefore, the total number of gifts exchanged in Lake Gift is η.

   (b)
$$\mathsf{EV}_i^R = \frac{1}{\lambda}\beta + \frac{\lambda - 1}{\lambda}(\beta - \delta) = \beta - \frac{\lambda - 1}{\lambda}\delta.$$

   (c)
$$L(\eta, \lambda, \delta) \stackrel{\text{def}}{=} \eta\frac{\lambda - 1}{\lambda}\delta, \quad \text{for} \quad \eta, \lambda > 1.$$
Therefore, social loss increases linearly with the population size parameter, eta, since the total number of gifts exchanged in this town is η.

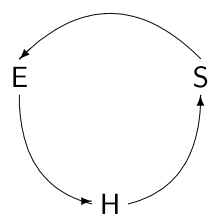
Chapter 11

Other networks

The topics in this chapter are not related, so I cannot advice you on what to emphasize. You should probably pick the topics you like for the class which is a value judgment.

Answers to Exercises

1. The language-acquisition equilibrium that we wish to examine is displayed in Figure 11.1.



Solution-Figure 11.1: A language-acquisition equilibrium with 3 languages.

E-speakers prefer learning Hebrew over Spanish if

$$\alpha(\eta_E + \eta_H + n_{SE} + n_{SH}) - \phi \geq \alpha(\eta_E + \eta_S + n_{HS} + n_{HE}) - \phi,$$

hence if $\alpha\eta - \phi \geq \alpha\eta - \phi$ which always holds.

E-speakers prefer learning Hebrew over not learning a second language if

$$\alpha\eta - \phi \geq \alpha(\eta_E + n_{HE} + n_{SE})$$

hence if

$$\alpha\eta \geq \alpha(\eta_E + \eta_S),$$

hence if $\alpha\eta_H \geq \phi$.

S-speakers prefer learning English over Hebrew if

$$\alpha(\eta_S + \eta_E + n_{HE} + n_{HS}) - \phi \geq \alpha(\eta_S + \eta_H + n_{ES} + n_{EH}) - \phi,$$

hence if $\alpha\eta - \phi \geq \alpha\eta - \phi$ which always holds.

S-speakers prefer learning English over not learning a second language if

$$\alpha\eta - \phi \geq \alpha(\eta_S + n_{ES} + n_{HS})$$

hence if

$$\alpha\eta \geq \alpha(\eta_E + \eta_S),$$

hence if $\alpha\eta_E \geq \phi$.

H-speakers prefer learning Spanish over English if

$$\alpha(\eta_H + \eta_S + n_{ES} + n_{EM}) - \phi \geq \alpha(\eta_H + \eta_E + n_{SE} + n_{SH}) - \phi,$$

hence if $\alpha\eta - \phi \geq \alpha\eta - \phi$ which always holds.

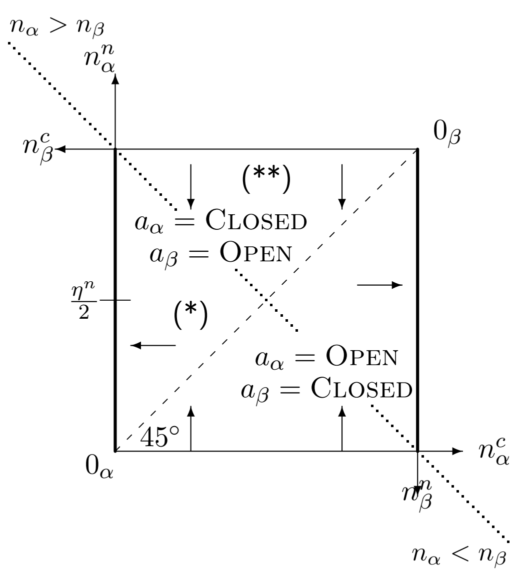
H-speakers prefer learning Spanish over not learning a second language if

$$\alpha\eta - \phi \geq \alpha(\eta_H + n_{EH} + n_{SH})$$

hence if $\alpha\eta_S \geq \phi$.

Altogether, for this equilibrium to exist, $\alpha\eta_E \geq \phi$, $\alpha\eta_S \geq \phi$, and $\alpha\eta_H \geq \phi$ must hold. However, since $\eta_E > \eta_S > \eta_H$, the three conditions become $\alpha\eta_H \geq \phi$.

2. Below the upward-sloping dashed diagonal, conformists constitute a majority in religion α and a minority in religion β. Therefore the area below this diagonal corresponds to believer allocations where α is open and β is closed. In addition, in the region marked by a (*), all conformists will convert to religion β since there are more believers affiliated with α than in β. In the region marked with (**) nonconformists affiliated with α convert to β since β is the smaller religion.



Solution-Figure 11.2: Conversion rules and religion equilibria when $\eta^c = \eta^n$. Thick side lines represent religion equilibria.

By symmetry the region above the diagonal can be analyzed as the region below the diagonal by replacing α with β, etc.. Altogether, the believer allocations that constitute religion equilibria are on the left and right sides of the allocation box.

3. (a)

$$\#\text{administrators} = \lambda(n)n = \left(1 - \frac{1}{n^2}\right)n = \frac{n^2-1}{n}.$$

(b) $[1-\lambda(n)]n = 1/n$. Hence, $\pi = \phi/n$.

(c) To determine efficiency,

$$\frac{\mathrm{d}\lambda(n)}{\mathrm{d}n} = \frac{2}{n^3} > 0.$$

Hence, inefficient.

(d)

$$w = \frac{\pi}{\lambda(n)n} = \frac{\frac{\phi}{n}}{\frac{n^2-1}{n}} = \frac{\phi}{n^2-1}.$$

This religion will be closed for new converts in order to protect the wages paid to the administrators.

4. (a)

$$\#\text{administrators} = \lambda(n)n = \left(\frac{1}{n^2}\right)n = \frac{1}{n}.$$

(b)

$$[1-\lambda(n)]n = \left(1-\frac{1}{n^2}\right)n = \frac{n^2-1}{n}.$$

Hence,

$$\pi = \frac{n^2-1}{n}\phi$$

(c) To determine efficiency,

$$\frac{\mathrm{d}\lambda(n)}{\mathrm{d}n} = -\frac{2}{n^3} < 0,$$

hence, efficient.

(d)

$$w = \frac{\pi}{\lambda(n)n} = \frac{\frac{n^2-1}{n}\phi}{\frac{1}{n}} = (n^2-1)\phi.$$

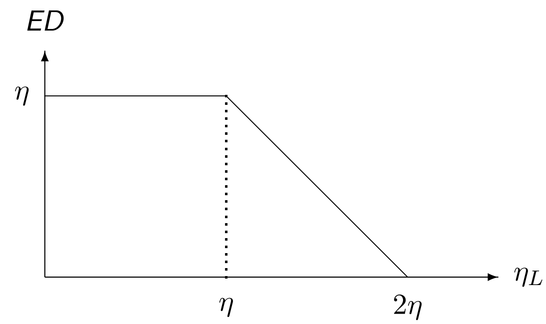
This religion will be open for new converts since the wage paid to each administrator increases with n.

5. (a) The excess demand for attorney is

$$ED \stackrel{\text{def}}{=} Q - \eta_L = \begin{cases} \eta & \text{if } \eta_L \le \eta \\ 2\eta - \eta_L & \text{if } \eta_L > \eta, \end{cases}$$

which is drawn in Figure 11.3. Excess demand is maximized at any η_L satisfying $0 \le \eta_L \le \eta$. Excess demand equal zero when the supply of attorneys is $\eta_L = 2\eta$.

(b) A plaintiff hires an attorney if $(1/2)120 - f^p \ge 0$, or $f^p \le 60$. A defendant hires an attorney if $-(1/2)120 - f^d \ge -120$, or $f^d \le 60$.



Solution-Figure 11.3: Excess demand for attorneys.

(c) Since $\eta_L < \eta$, part (a) implies that $ED = 100,000 + 5,000 - 5,000 = 100,000$. Hence, $f = 120$.

(d) Since $\eta_L > \eta$, part (a) implies that $ED = 2 \times 100,000 - 120,000 = 80,000$. Hence,

$$f = 60 + \frac{60}{120,000} 80,000 = 108.$$

Another method for solving parts (c) and (d) is to compute the fee as a function η_L,

$$f = \begin{cases} 120 & \text{if } 0 \le \eta_L \le \eta \\ 180 - \frac{60}{\eta}\eta_L & \text{if } \eta < \eta_L \le 3\eta \\ 0 & \text{if } \eta_L > 3\eta, \end{cases}$$

and then substituting for η and η_L.

Appendix A

Normal-Form Games

Answers to Exercises

1. (a)

$$R^1(a^2) = \begin{cases} \textsc{Low} & \text{if } a^2 = \textsc{Low} \\ \textsc{Low} & \text{if } a^2 = \textsc{High} \end{cases} \quad \text{and} \quad R^2(a^1) = \begin{cases} \textsc{Low} & \text{if } a^1 = \textsc{Low} \\ \textsc{Low} & \text{if } a^1 = \textsc{High}. \end{cases}$$

   $\langle \textsc{Low}, \textsc{Low} \rangle$ constitutes a unique Nash equilibrium since this outcome is the only one that lies on *both* best-response functions.

   (b)

$$R^1(a^2) = \begin{cases} \alpha & \text{if } a^2 = \alpha \\ \beta & \text{if } a^2 = \beta \end{cases} \quad \text{and} \quad R^2(a^1) = \begin{cases} \beta & \text{if } a^1 = \alpha \\ \alpha & \text{if } a^1 = \beta. \end{cases}$$

   There does *not* exist a Nash equilibrium for this game since there no outcome lies on *both* best-response functions.

2. (a) $a \geq e$ and $b \geq d$.

   (b) ($a > e$ and $c \geq g$) or ($a \geq e$ and $c > g$),
   and
   ($b > d$ and $f \geq h$) or ($b \geq d$ and $f > h$).

   (c) i. (α, α) Pareto dominates (α, β) if
   either ($a \geq c$ and $b > d$) or ($a > c$ and $b \geq d$).

   ii. (α, α) Pareto dominates (β, α) if
   either ($a \geq e$ and $b > f$) or ($a > e$ and $b \geq f$).

   iii. (α, α) Pareto dominates (β, β) if
   either ($a \geq g$ and $b > h$) or ($a > g$ and $b \geq h$).

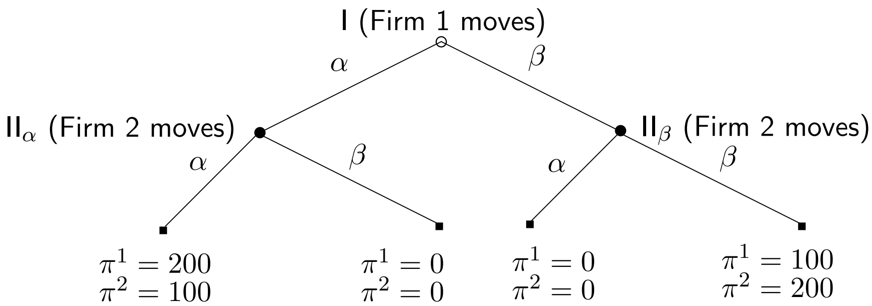
   (d) Either ($a > g$ and $b < h$) or ($a < g$ and $b > h$).

Appendix B

Extensive-Form Games

Answers to Exercises

1. (a) The dynamic standardization game is described by the following tree:



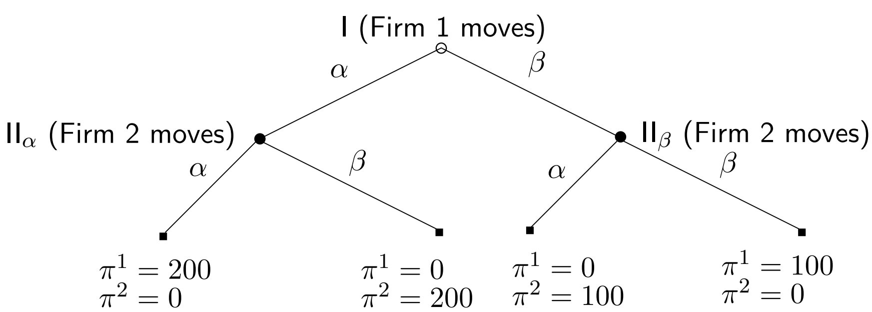
Solution-Figure B.1: Dynamic Battle of the Sexes Game

(b) The unique SPE is $(\alpha, (\alpha, \beta))$.
Proof. We first look at the two subgames. The NE for the subgame II_α is $a^2 = \alpha$. The NE for the subgame II_β is $a^2 = \beta$.
Next, we look at the game starting at I. If firm 1 plays $a^1 = \alpha$ it earns

$$\pi^1(\alpha, (\alpha, \beta)) = 200 > 100 = \pi^1(\beta, (\alpha, \beta)).$$

■

2. The game tree is:



Solution-Figure B.2: Multiple SPE

There are two SPE given by $(\alpha, (\beta, \alpha))$ and $(\beta, (\beta, \alpha))$.

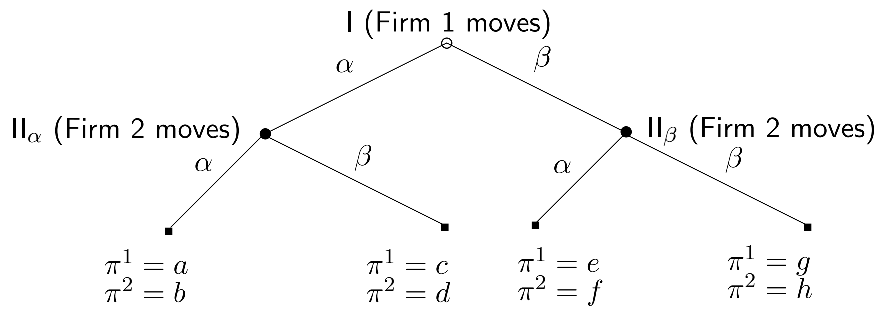
Proof. We first look at the two subgames. The NE for the subgame II_α is $a^2 = \beta$. The NE for the subgame II_β is $a^2 = \alpha$.

Next, we look at the game starting at I. If firm 1 plays $a^1 = \alpha$ it earns

$$\pi^1(\alpha, (\beta, \alpha)) = 0 = 0 = \pi^1(\beta, (\beta, \alpha)).$$

Hence, firm 1 is indifferent between its two actions. ∎

3. The dynamic standardization game is described by the following tree:



Solution-Figure B.3: Dynamic "a b c d e f g" game

The unique SPE is $(\beta, (\alpha, \beta))$.

Proof. We first look at the two subgames. The NE for the subgame II_α is $a^2 = \alpha$. The NE for the subgame II_β is $a^2 = \beta$.

Next, we look at the game starting at I. If firm 1 plays $a^1 = \alpha$ it earns

$$\pi^1(\beta, (\alpha, \beta)) = g > a = \pi^1(\alpha, (\alpha, \beta)).$$

∎

Appendix C

Undercut-Proof Equilibria

The equilibrium concept developed in this appendix is widely used in this book. The advantage of this concept is that it allows us to handle discrete game theoretic model without the use of calculus. Such models generally "suffer' from non-existence of Nash equilibria.

Answers to Exercises

1. (a) The UPE conditions are:

$$\begin{aligned}(p_A - \mu_A)\eta &= (p_B - \delta - \mu_A)2\eta \\ (p_B - \mu_B)\eta &= (p_A - \delta - \mu_B)2\eta.\end{aligned}$$

   Solving the two equations for the 2 prices yield the unique UPE:

$$p_A^U = 2\delta + \frac{\mu_A + 2\mu_B}{3} \quad \text{and} \quad p_B^U = 2\delta + \frac{2\mu_A + \mu_B}{3}.$$

   (b) The firm with lower cost (firm A) charges a higher price. To see this,

$$p_A^U - p_B^U = \frac{\mu_B - \mu_A}{3} \geq 0 \quad \text{if and only if} \quad \mu_B \geq \mu_A.$$

2. The UPE conditions are

$$\begin{aligned}5 \times p_A &= 15 \times (12 - \delta) \\ 10 \times 12 &= 15 \times (p_A - \delta),\end{aligned}$$

   yielding $\delta = 7$ and $p_A = 15$. Again, one can see that the firm with the larger market share charges a lower price.