# Reservations, Refunds, and Price Competition*

Staffan Ringbom†
Helsinki Center for Economics Research, Swedish School of Economics
Oz Shy‡
University of Haifa and
Helsinki Center for Economics Research, Swedish School of Economics

2nd April 2004

**Abstract**

We analyze the incentives of imperfectly-competitive service providers to utilize advance reservation systems for consumers who are heterogeneous with respect to their probability of showing up. We investigate how the refund option affects equilibrium prices, and characterize the conditions under which the refund option is utilized. Under weak competition, service providers utilize the same booking strategies. In contrast, intense competition might lead firms to segment the market by utilizing different booking strategies. Finally, if partial refunds are utilized, the equilibrium refunds equal exactly to the operation cost whereas the nonrefundable portion equals to capacity cost plus the profit markup.

**Keywords:** Advance booking, reservation systems, refunds, partial refunds, imperfect competition

**JEL Classification Numbers:** M2, L1

**(Draft** = refundduop35.tex 2004/04/02 09:46)

---

*We thank seminar participants at WZ-Berlin for most valuable comments on an earlier draft.

†HECER, Department of Economics, Swedish School of Economics, P.O. Box 479, 00101 Helsinki, Finland E-mail: `staffan.ringbom@shh.fi`

‡*Corresponding author*: Department of Economics, University of Haifa, 31905 Haifa, Israel. E-mail: `ozshy@econ.haifa.ac.il`

## 1. Introduction

Most of the literature on advance booking, reservations, and refunds analyzes a single-service provider. That is, the results are confined to a monopoly market structure. In this paper we analyze an imperfectly competitive service industry where service providers compete on consumers by utilizing advance reservation systems allowing for refunds, in addition to competing in prices.

Theories of Industrial Organization tend to associate the time of purchase with the time of delivery of goods and services. In practice, there are many markets for services and goods where buyers and sellers maintain contacts long before the service or the good are scheduled to be delivered. This pre-delivery contact is usually called *advance booking*, or simply a *reservation*. Reservation systems are observed in almost all privately-provided services. The most noticeable ones are transportation services such as the airline industry, railroad, car rentals and bus travel. Reservations are also utilized in small businesses such as restaurants, fancy barber shops, and law offices. However, refund policies tend to vary with the type of the service industry being investigated. For example whereas car rentals and hotels generally offer refundable bookings, entertainment places, such as movie theaters, offer nonrefundable tickets.

In the Economics literature there are a few papers analyzing the refundability option as a means for segmenting the market or the demand. Most studies, so far, focused on a single seller. Those studies that analyze industries with multiple sellers generally assume that prices are fixed, thus leaving firms to compete on capacity allocation only. Contributions by Gale and Holmes (1992, 1993) compare a monopolist's advance bookings with socially-optimal ones. Gale (1993) analyzes consumers who learn their preferences only after they are offered an advance purchase option. On this line, Miravete (1996) and more recently Courty and Li (2000) further investigate how consumers who learn their valuation over time can be screened via the introduction of refunds in their advance booking mechanism. Courty (2003) investigates resale and rationing strategies of a monopoly that can sell early to uninformed consumers or late to informed consumers. Dana (1998) also investigates market segmentation under advance booking made by price-taking firms. Finally, Ringbom and Shy (2003) study the effects of capacity constraints on the advance reservation strategy of a single service provider. Ringbom and Shy (2004) analyze partial refunds set by price-taking firms.

In contrast to the monopoly literature, we are aware of only two papers analyzing a duopoly market structure. Gale (1993) also analyzes a duopoly setting, with consumers who learn their preferences over time. Macskasi (2003) analyzes duopoly with product differentiation where each consumer gets an ex ante signal of her preferred location and only then learns the true location. The present paper adds to the duopoly literature by introducing two types of marginal costs: (i) Capacity costs that are borne by service providers in order to guarantee service to all customers who make reservations, and (ii) operation cost borne only if a customer shows up.

In the present paper, we formally introduce *competition* into an industry utilizing advance booking systems. We investigate an industry providing services like travel arrangements (airline, train, bus, hotel, car rental), or repair, maintenance, education, and so on. These industries are characterized by services that are *time dependent* and *non-storable*. This means that both buyers and sellers must commit to a certain predetermined time at which the service is set to be delivered. Therefore, service providers tend to utilize advance reservation systems as part of their business and marketing strategies. For this reason, our model allows for multiple strategies where service providers can choose their refund policy in addition to setting prices. We utilize this model to investigate how the degree of competition (manifested by the degree of service differentiation) affects equilibrium choices of refundability options that are offered to consumers.

The present study is organized as follows. Section 2 develops a model of a duopoly service industry utilizing advance booking systems. Section 3 solves for equilibrium prices and profits, when all services providers utilize the same booking strategy. That is, both either sell refundable or nonrefundable tickets. Section 4 computes equilibrium prices when service providers choose different booking strategies. Section 5 solves for the equilibrium booking strategies used by service providers. Section 6 extends the model to partial refunds.

Section 7 analyzes two possible extensions to our approach. Section 8 offers concluding comments.

## 2. A Model of Competition and Advance Booking

Consider a service industry with two imperfectly-competitive service providers, selling two differentiated services.

### 2.1 Service providers

There are two service providers, labeled $A$ and $B$. Let $p_A$ and $p_B$ be the prices they charge for providing the service. In addition to setting prices, each service provider utilizes an advance reservation procedure in which each provider must inform consumers whether the reservation price is refundable ($R$) or nonrefundable ($N$). By offering a refundable booking, the service provider collects the price only if the consumer shows up at the time of delivery. In contrast, offering a nonrefundable booking means that the consumers pay at the time when the reservation is made. In this case the revenue is independent of whether the consumers show up or not at the pre-agreed delivery time.

Service providers bear two types of per-customer costs. Let $k \geq 0$ denote the service provider's cost of making a reservation for one customer. Note that this cost could be significant if the provider does not have any alternative use (no salvage value) for an unused capacity. Alternatively, it may not exist if capacity has an immediate alternative use upon no shows of consumers. In addition, service providers bear a per-customer cost of operation which we denote by $c \geq 0$. The difference between the *reservation cost* and the *operation cost* is that the latter is borne only if the customer shows up for the service, whereas the reservation cost is borne regardless of whether the customer shows up. Finally, we assume that both service providers have a sufficient amount of capacity to accommodate all reservations without having to overbook consumers.

### 2.2 Consumers

Consumers are differentiated in two dimensions: Location preference and probability of showing up to collect a reserved service. We assume that consumers are uniformly distributed on the unit square indexed by $(\sigma, x)$. The index $x$ $(0 \le x \le 1)$ measures the distance (disutility) from service provider $A$, whereas $(1-x)$ measures the distance from $B$. Thus, $x$ serves as the standard Hotelling index of differentiation. The index $\sigma$ $(0 \le \sigma \le 1)$ measures the probability that this customer will show up for the delivery of the prebooked service.

Let $\beta$ denote a consumer's basic utility from actually consuming this service. We assume that the utility of a consumer indexed by $(\sigma, x)$ is given by[1]

$$U(\sigma, x) \stackrel{\text{def.}}{=} \begin{cases} \sigma(\beta - p_A) - \tau x & \text{Book with } A \text{ \& ticket is refundable} \\ \sigma\beta - p_A - \tau x & \text{Book with } A \text{ \& ticket is nonrefundable} \\ \sigma(\beta - p_B) - \tau(1-x) & \text{Book with } B \text{ \& ticket is refundable} \\ \sigma\beta - p_B - \tau(1-x) & \text{Book with } B \text{ \& ticket is nonrefundable} \end{cases} \tag{1}$$

where the parameter $\tau$ measures the degree of service differentiation, thus captures the degree of competition between the two service providers. That is, competition becomes more intense when $\tau$ takes lower values. The utility function (1) reveals that the benefit $\beta$ is collected only if the consumer shows up (with probability $\sigma$). The major difference between refundable and nonrefundable bookings is that a refundable ticket is paid only if the consumers shows up (with probability $\sigma$), whereas a nonrefundable ticket is paid upfront (with probability 1).

## 3. Prices and Profits Under Symmetric Booking Strategies

The next two subsections analyze price equilibria when service providers utilize the same booking strategy. Either both choose to provide full refunds for no shows; or both choose to make only nonrefundable bookings.

[1]Since the main contribution of the paper is the introduction of price competition into an advance booking model, we deliberately ruled out any reservation utility. This ensures that the entire market is always served.

### 3.1 Both providers offer refundable bookings

The utility function (1) implies that the consumers who are indifferent between making reservations with $A$ and $B$ are implicitly solved from $\sigma(\beta - p_A) - \tau x = \sigma(\beta - p_B) - \tau(1 - x)$. Hence,

$$\hat{x}^R(\sigma) = \frac{1}{2} + \frac{\sigma(p_B - p_A)}{2\tau}. \tag{2}$$

Figure 1 illustrates how consumers are divided between making reservations with $A$ and $B$, according to (2).

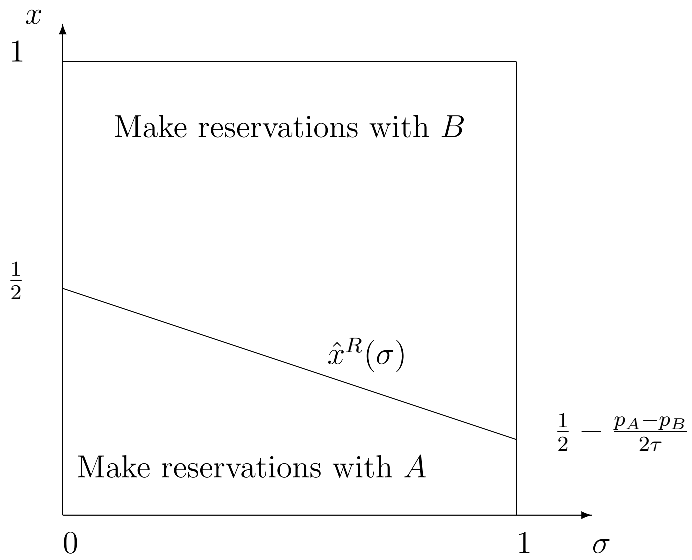


**Figure 1:** Reservations allocation between service providers $A$ and $B$. *Note:* The figure assumes out-of-equilibrium prices $p_A > p_B$.

Figure 1 illustrates under the arbitrary assumption $p_A > p_B$, that consumers of type $\sigma = 0$ are equally divided between the service provides, since these consumers will never show up and hence will collect the (full) refund with probability 1. In contrast, for consumer types with $\sigma > 0$, more of them will prefer to book with $B$ for the simple reason that $B$ is cheaper. Clearly, "most" high $\sigma$ types prefer the cheaper provider since they tend to show up (and pay) with a higher probability.

The main feature of the present model is that the number of reservations is always higher than the expected number of actual show-ups. Denote by $q_A$ and $q_B$ the number of reservations made at each provider, and by $s_A$ and $s_B$ the *expected* number of show-ups for the service at the preagreed delivery time. In view of (2) and Figure 1, the number of reservations are given by

$$q_A = \int_0^1 \hat{x}^R(\sigma) d\sigma = \int_0^1 \left[\frac{1}{2} + \frac{\sigma(p_B - p_A)}{2\tau}\right] d\sigma \tag{3a}$$

$$q_B = \int_0^1 \left[1 - \hat{x}^R(\sigma)\right] d\sigma = \int_0^1 \left[\frac{1}{2} + \frac{\sigma(p_A - p_B)}{2\tau}\right] d\sigma. \tag{3b}$$

Therefore, the expected number of show-ups are given by

$$s_A = \int_0^1 \sigma \hat{x}^R(\sigma) d\sigma = \int_0^1 \sigma \left[\frac{1}{2} + \frac{\sigma(p_B - p_A)}{2\tau}\right] d\sigma \tag{4a}$$

$$s_B = \int_0^1 \sigma \left[1 - \hat{x}^R(\sigma)\right] d\sigma = \int_0^1 \sigma \left[\frac{1}{2} + \frac{\sigma(p_A - p_B)}{2\tau}\right] d\sigma. \tag{4b}$$

Service provider $i$ $(i = A, B)$ takes $p_j$ $(j \neq i)$ as given, and chooses $p_i$ to solve[2]

$$\max_{p_i} \pi_i(p_i, p_j) = (p_i - c)s_i - kq_i = \frac{1}{12\tau}\left[(p_i - c)(3\tau - 2p_i + 2p_j) + 3k(p_j - p_i - 2\tau)\right] \tag{5}$$

Thus, the profit of service provider $i$ is composed of expected operation profits $(p_i - c)s_i$ which is conditional on $s_i$ show-ups, minus the cost of making $q_i$ reservations. The profit maximization problem (5) is concave in $p_i$, yielding a unique best response function given by $p_i(p_j) = (2c + 3k + 2p_j + 3\tau)/4$. Therefore, the unique equilibrium prices and profit levels when both providers allow for full refunds are

$$p_A^R = p_B^R = c + \frac{3(k+\tau)}{2} \quad \text{and} \quad \pi_A^R = \pi_B^R = \frac{3\tau - k}{8}. \tag{6}$$

[2]The profit functions assumed here are based on the assumption that service providers are not allowed to overbook. In other words, each service provider must invest and be prepared to provide service for the entire $q_i$ people who make reservations. Section 8 (Conclusion) suggests some alternative profit functions in which overbooking is allowed.

Substituting the equilibrium prices (6) into the participation rates (3a)–(4b) yields the number of reservations made with each provider and the expected show-ups

$$q_A^R = q_B^R = \frac{1}{2} \quad \text{and} \quad s_A^R = s_B^R = \frac{1}{4}. \tag{7}$$

### 3.2 Both providers offer nonrefundable bookings

The utility function (1) implies that the consumers who are indifferent between making reservations with $A$ and $B$ are implicitly solved from $\sigma\beta - p_A - \tau x = \sigma\beta - p_B - \tau(1-x)$. Hence,

$$\hat{x}^N = \frac{1}{2} + \frac{p_B - p_A}{2\tau}, \tag{8}$$

which is independent of $\sigma$ and therefore forms a horizontal line in Figure 1. Hence, the number of reservations made at each provider are

$$q_A = \hat{x}^N = \frac{1}{2} + \frac{p_B - p_A}{2\tau} \quad \text{and} \quad q_B = 1 - \hat{x}^N = \frac{1}{2} + \frac{p_A - p_B}{2\tau}. \tag{9}$$

The expected number of show-ups are then given by

$$s_A = \int_0^1 \sigma \left[\frac{1}{2} + \frac{p_B - p_A}{2\tau}\right] d\sigma \quad \text{and} \quad s_B = \int_0^1 \sigma \left[\frac{1}{2} + \frac{\sigma(p_A - p_B)}{2\tau}\right] d\sigma \tag{10}$$

Service provider $i$ $(i = A, B)$ takes $p_j$ $(j \neq i)$ as given, and chooses $p_i$ to solve

$$\max_{p_i} \pi_i(p_i, p_j) = (p_i - k)q_i - cs_i = \frac{1}{4\tau}\left[(2p_i - 2k - c)(p_j - p_i + \tau)\right] \tag{11}$$

yielding a best-response function given by $p_i = [2(p_j + k + \tau) + c]/4$. Therefore, the equilibrium prices and profit levels when both providers use nonrefundable booking are given by

$$p_A^N = p_B^N = \frac{c + 2(k+\tau)}{2} \quad \text{and} \quad \pi_A^N = \pi_B^N = \frac{\tau}{2}. \tag{12}$$

Substituting the equilibrium prices (12) into the participation rates (9) and (10) yields the number of reservation made with each provider and the expected show-ups

$$q_A^N = q_B^N = \frac{1}{2} \quad \text{and} \quad s_A^N = s_B^N = \frac{1}{4}. \tag{13}$$

# 4. Prices and Profits Under Asymmetric Booking Strategies

In this section we compute equilibrium prices given that service-provider $A$ utilizes refundable bookings, whereas $B$ utilizes nonrefundable bookings. We will distinguish between two types of market segmentation cases given in the following definition.

Definition 1

*We say that the asymmetric booking strategies lead to*

*(a)* ***weak market segmentation*** *if for each consumer type $\sigma$, some consumers purchase refundable tickets (from $A$), and some purchase nonrefundable tickets (from $B$);*

*(b)* ***strong market segmentation*** *if there are some consumer types $\sigma$, who all purchase refundable tickets (from $A$), and some other types who purchase only nonrefundable tickets (from $B$).*

Figure 2 illustrates the consumer allocations under the different market segmentation categories. In Figure 2(left), refundable and nonrefundable tickets are purchased by any con-

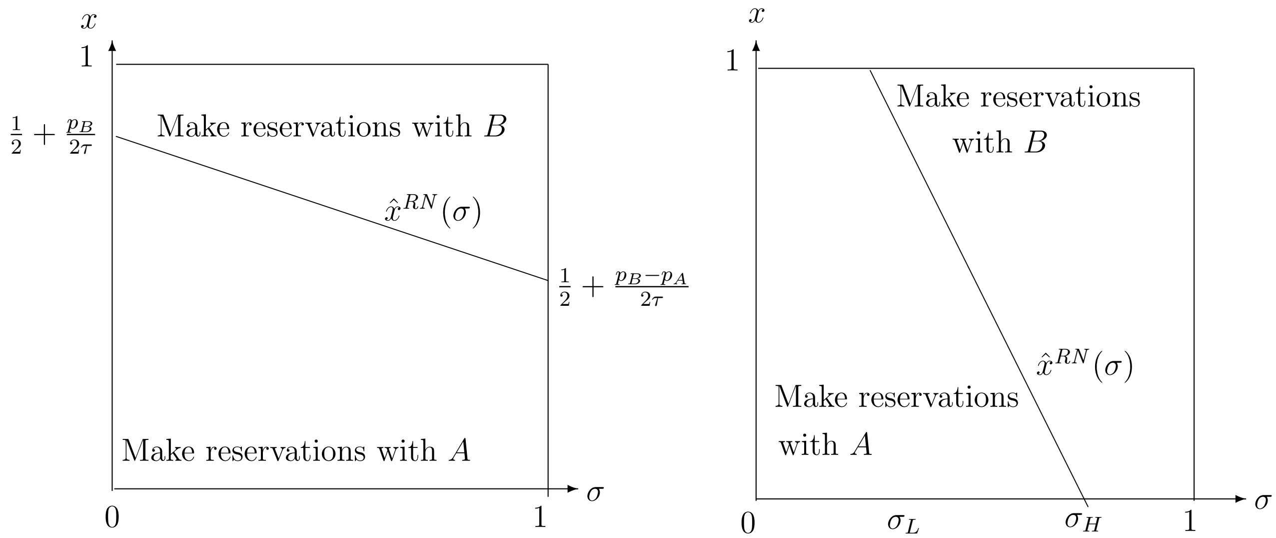


**Figure 2:** Reservations allocation between service providers $A$ (practicing refundable bookings) and $B$ (practicing nonrefundable bookings).
*Left*: Weak market segmentation. *Right*: Strong market segmentation.

sumer type. Thus, both service providers make reservation to the entire spectrum of probabilities of showing up. In contrast, in Figure 2(right), all consumer types indexed by $\sigma \in [0, \sigma_L]$ buy only refundable tickets; and all consumers indexed by $\sigma \in [\sigma_H, 1]$ buy only nonrefundable tickets.

As we show below, weak market segmentation occurs when the providers $A$ and $B$ are highly differentiated in the eyes of consumers ($\tau$ takes high values), hence competition is limited. In contrast, strong market segmentation occurs when competition is intense ($\tau$ takes low values).[3]

### 4.1 Weak market segmentation

We now compute equilibrium prices under weak market segmentation. As we show below Figure 2(left) occurs when service providers are sufficiently differentiated. Thus, for this section we assume (and later verify) that

$$\tau > 3c + 5.5k. \tag{14}$$

The utility function (1) implies that the consumers who are indifferent between making reservations with $A$ and $B$ are implicitly solved from $\sigma(\beta - p_A) - \tau x = \sigma\beta - p_B - \tau(1-x)$. Hence,

$$\hat{x}^{RN}(\sigma) = \frac{1}{2} + \frac{p_B - \sigma p_A}{2\tau}, \tag{15}$$

which forms a downward sloping linear division line as in Figure 1, but with a higher intercept given by $1/2 + p_B/(2\tau)$. Hence, the number of reservations made at each provider is

$$q_A = \int_0^1 \left[\frac{1}{2} + \frac{p_B - \sigma p_A}{2\tau}\right] d\sigma \quad \text{and} \quad q_B = \int_0^1 \left[\frac{1}{2} + \frac{\sigma p_A - p_B}{2\tau}\right] d\sigma. \tag{16}$$

The expected number of show-ups are then given by

$$s_A = \int_0^1 \sigma \left[\frac{1}{2} + \frac{p_B - \sigma p_A}{2\tau}\right] d\sigma \quad \text{and} \quad s_B = \int_0^1 \sigma \left[\frac{1}{2} + \frac{\sigma p_A - p_B}{2\tau}\right] d\sigma. \tag{17}$$

[3] In order to reduce the amount of writing, Figure 2 ignores a third possibility associated with intermediate values of $\tau$, where $x^{RN}(\sigma)$ has one intersection with a vertical axis and another with the horizontal axis.

Each service provider chooses her price to solve

$$\max_{p_A} \pi_A(p_A, p_B) = (p_A - c)s_A - kq_A \quad \text{and} \quad \max_{p_B} \pi_B(p_A, p_B) = (p_B - k)q_B - cs_B \tag{18}$$

yielding best-response functions given by $p_A = [3(p_B+k+\tau)+2c]/4$ and $p_B = [p_A+2(k+\tau)+c]/4$. Therefore, the equilibrium prices and profit levels when service providers use different booking strategies are

$$p_A^R = \frac{11c + 18(k+\tau)}{13} \quad \text{and} \quad p_B^N = \frac{6c + 11(k+\tau)}{13}. \tag{19}$$

Substituting (19) into (15) implies that $x^{RN}(0) = (6c + 11k + 24\tau)/26\tau$ and $x^{RN}(1) = (6\tau - 7k - 5c)/26\tau$. Now, it can be easily verified that condition (14) implies that $x^{RN}(0) < 1$ and $x^{RN}(1) > 0$, hence according to Figure 2(left) the market is indeed weakly segmented.

Substituting the equilibrium prices (19) into the profit functions (18) yields

$$\pi_A^R = \frac{8c^2 - 3c(35k + 48\tau) - 6(35k^2 + 96k\tau - 108\tau^2)}{2028\tau} \tag{20a}$$

$$\pi_B^N = \frac{6(2k - 11\tau)^2 - 70c^2 - 3c(35k + 61\tau)}{2028\tau} \tag{20b}$$

Equation (19) implies the following proposition.

**Proposition 1**

*In a price competition between the service providers, the provider utilizing refundable bookings charges a higher price than the provider utilizing nonrefundable booking. Formally,* $p_A^R > p_B^N$.

We now turn to investigating consumers' participation rates. Substituting the equilibrium prices (19) into (16), the number of reservations made with $A$ and $B$ are

$$q_A^R = \frac{c + 2(2k + 15\tau)}{52\tau} \quad \text{and} \quad q_B^N = \frac{2(11\tau - 2k) - c}{52\tau}. \tag{21}$$

Substituting (19) into (17), the expected number of show-ups are given by

$$s_A^R = \frac{3(12\tau - k) - 4c}{156\tau} \quad \text{and} \quad s_B^N = \frac{4c + 3(k + 14\tau)}{156\tau}. \tag{22}$$

We can therefore state the following proposition.

**Proposition 2**

(a) *The service provider utilizing the refund booking strategy makes more reservations compared to the provider utilizing a nonrefundable booking strategy.*
*Formally,* $q_A^R > q_B^N$. *However,*

(b) *the expected number of show-ups is lower for the provider utilizing the refundable booking strategy. Formally,* $s_A^R < s_B^N$.

Proposition 2(a) is illustrated in Figure 2. Figure 2 demonstrates that despite the high price, the refund option makes provider $A$ more attractive to most consumers. Proposition 2(b) demonstrates that despite the higher number of reservations, service provider $A$ has a lower expected number of show-ups. This stems from the fact that $A$ is more attractive to "most" consumers with a low probability of showing up.

We conclude our analysis of the asymmetric equilibrium with a comparison between the profit made by firm $A$ (20a) which sells refundable tickets, and firm $B$ (20b) which sells only nonrefundable tickets. Since a general comparison is tedious, we confine it to two extreme cases: First, when there are only *operation* costs, that is, $k = 0$ and $c \geq 0$. Second, when there are only *capacity* costs so $c = 0$ and $k \geq 0$.

**Proposition 3**

*Suppose that service provider $A$ sells only refundable tickets,*
*whereas service provider $B$ sells only nonrefundable tickets.*

(a) *Also, suppose that there is no capacity cost ($k = 0$). Then, selling refundable tickets is more profitable than selling nonrefundable tickets. Formally,* $\pi_A^R > \pi_B^N$.

(b) *Suppose that there is no cost of operation ($c = 0$). Then, selling nonrefundable tickets is more profitable. Formally,* $\pi_A^R < \pi_B^N$.

The intuition behind Proposition 3(b) is rather simple. Capacity costs are always paid when the reservations are made. Thus, the service provider who sells only refundable tickets bears a loss for every no-show, whereas selling nonrefundable tickets insures the service provider against already-spent capacity costs. In contrast, Proposition 3(a) shows that when there are no capacity costs, selling refundable tickets is more profitable, since the cost is incurred only if the customer shows up and pays for the service.

### 4.2 Strong market segmentation

We now compute equilibrium prices under strong market segmentation. As we show below strong market segmentation occurs when service providers are not very differentiated ($\tau$ takes "low" values), so competition is intense. As it turns out, we cannot explicitly solve for price equilibria for the general case where $k > 0$. Therefore we compute the price equilibrium when $k = 0$, and discuss some qualitative properties of the general case.

The points $\sigma_L$ and $\sigma_H$ in Figure 2(right) are found by extracting $\sigma$ from $x^{RN}(\sigma_L) = 0$ and $x^{RN}(\sigma_H) = 1$, respectively, where the function $x^{RN}(\sigma)$ is given in (15). Hence,

$$\sigma_L = \frac{p_B - \tau}{p_A} \quad \text{and} \quad \sigma_H = \frac{p_B + \tau}{p_A}. \tag{23}$$

Figure 2(right) implies that strong market segmentation occurs when $0 < \sigma_L \leq \sigma_H < 1$; hence this equilibrium must satisfy $0 \leq \tau < \min\{p_B, p_A - p_B\}$. The number of reservations made with each provider is given by

$$q_A = \sigma_L + \int_{\sigma_L}^{\sigma_H} \left[\frac{1}{2} + \frac{p_B - \sigma p_A}{2\tau}\right] d\sigma \quad \text{and} \quad q_B = (1 - \sigma_H) + \int_{\sigma_L}^{\sigma_H} \left[\frac{1}{2} + \frac{\sigma p_A - p_B}{2\tau}\right] d\sigma, \tag{24}$$

yielding $q_A = p_B/p_A$, and $q_B = (p_A - p_B)/p_A$. The expected number of show-ups are then given by

$$s_A = \int_0^{\sigma_L} \sigma d\sigma + \int_{\sigma_L}^{\sigma_H} \sigma \left[\frac{1}{2} + \frac{p_B - \sigma p_A}{2\tau}\right] d\sigma \quad \text{and} \quad s_B = \int_{\sigma_H}^{1} \sigma d\sigma + \int_{\sigma_L}^{\sigma_H} \sigma \left[\frac{1}{2} + \frac{\sigma p_A - p_B}{2\tau}\right] d\sigma, \tag{25}$$

yielding $s_A = (3p_B^2 + \tau^2)/(6p_A^2)$ and $s_B = (3p_A^2 - 3p_B^2 - \tau^2)/(6p_A^2)$. Now we have sufficient information to formulate the profit functions:

$$
\begin{aligned}
\pi_A^R &= (p_A^R - c)s_A - kq_A \\
&= \frac{\left(3p_B^2 + \tau^2 - 6kp_B^N\right)}{6p_A^R} - c \cdot \frac{(3p_B^2 + \tau^2)}{6(p_A^R)^2}, \text{ and} && \text{(26a)} \\
\pi_B^N &= -cs_B + (p_B^N - k)q_B \\
&= (p_B^N - k)\left(1 - \frac{p_B^N}{p_A^R}\right) - \frac{c}{2} + c \cdot \frac{(3(p_B^N)^2 + \tau^2 - 6kp_B)}{6(p_A^R)^2}. && \text{(26b)}
\end{aligned}
$$

Therefore, the optimal responses of the firms $A$ and $B$ should satisfy the necessary conditions:

$$
\begin{aligned}
6(p_A^R)^3 \frac{\partial \pi_A^R}{\partial p_A^R} &= -\left(3p_B^2 + \tau^2 - 6kp_B^N\right) p_A^R + 2c \cdot (3p_B^2 + \tau^2) = 0, \text{ and} && \text{(27a)} \\
(p_A^R)^2 \frac{\partial \pi_B^N}{\partial p_B^N} &= (p_A^R)^2 - 2p_B^N p_A^R + kp_A^R + c \cdot p_B^N - c \cdot k = 0. && \text{(27b)}
\end{aligned}
$$

The price best response functions are:

$$
\begin{aligned}
p_A^R &= 2c \cdot \left(1 + \frac{6kp_B^N}{3p_B^2 + \tau^2 - 6kp_B^N}\right) \text{ and} && \text{(28a)} \\
p_B^N &= \frac{(p_A^R)^2 + kp_A^R - c \cdot k}{2p_A^R - c}. && \text{(28b)}
\end{aligned}
$$

We conclude from the first order condition (27a) that an asymmetric equilibrium under strong market segmentation exists only if $c > 0$.[4]

The condition $\partial \pi_A^R / \partial p_A^R = \partial \pi_B^N / \partial p_B^N = 0$ has a closed form solution only if $k = 0$. For this case, substituting for $q_A$, $q_B$, $s_A$, and $s_B$ into the first order conditions (27a, 27b), yields the equilibrium prices and profit levels

$$
p_A^R = 2c, \quad p_B^N = \frac{4c}{3}, \quad \pi_A^R = \frac{16c^2 + 3\tau^2}{72c}, \quad \text{and} \quad \pi_B^N = \frac{4c^2 + \tau^2}{24c}. \tag{29}
$$

Therefore, $p_A^R > p_B^N$ and $\pi_A^R > \pi_B^N$, which confirm Propositions 1 and 3 also for the case of strong market segmentation. Substituting these equilibrium prices into (23) and (29) implies that

$$
\sigma_L = \frac{4c - 3\tau}{6c}, \quad \text{and} \quad \sigma_H = \frac{4c + 3\tau}{6c}. \tag{30}
$$

[4] If $c = 0$, $\langle N, N \rangle$ is the unique equilibrium.

Strong market segmentation occurs when $0 < \sigma_L < \sigma_H < 1$, that is $\tau < 2c/3$, when $k = 0$. Finally, substituting these equilibrium prices into (24) and (25) yields

$$q_A^R = \frac{2}{3}, \quad q_B^N = \frac{1}{3}, \quad s_A^R = \frac{16c^2 + 3\tau^2}{72c^2}, \quad \text{and} \quad s_B^N = \frac{20c^2 - 3\tau^2}{72c^2}. \tag{31}$$

Therefore, $q_A^R > q_B^N$ and $s_A^R < s_B^N$ as in Proposition 2, since $\tau < 2c/3 < \sqrt{6}c/3$. Finally, observe that $s_A^R + s_B^N = 1/2$ meaning that the aggregate expected number of show-ups consists of half of the population.

## 5. Equilibrium Booking Strategies

In this section we solve for the equilibrium booking strategies utilized by service providers. Our interpretation for this two-strategy market game is that both service providers first announce whether they allow for refunds, and only then both providers set their prices according to the computations described in Section 3 and Section 4. Thus, this two-stage game portrays a market for a service where the refund policy is a *commitment* on the part of service providers, which cannot be revoked in the short run. Here, the booking strategy also serves as a basic factor for the reputation of firms.[5]

We also compute the semicollusion equilibrium strategies. We define *semicollusion* as a market structure where at stage I, both service providers jointly agree on which booking strategy is utilized by each firm. At stage II, both firms compete noncooperatively in prices. A semicollusion equilibrium is an agreement on booking strategies that maximizes joint industry profit.

### 5.1 Equilibrium under weak market segmentation

Recall from (14) that weak market segmentation occurs under low competition, formally when $\tau > 3c + 5.5k$. Substituting $k = 0$ into (6), (12), (20a), and (20b), Table 1 displays the profit levels of each service provider under each outcome of this pure-strategy game when there are only *operation* costs.

[5]Section 6 relaxes the commitment aspect of our model by allowing for simultaneous decisions on pricing and booking strategies.

Service Provider $B$:

| | | $R$ | | $N$ | |
|---|---|---|---|---|---|
| Provider $A$: | $R$ | $\frac{3\tau}{8}$ | $\frac{3\tau}{8}$ | $\frac{2(81\tau^2-18c\tau+c^2)}{507\tau}$ | $\frac{726\tau^2-183c\tau-70c^2}{2028\tau}$ |
| | $N$ | $\frac{726\tau^2-183c\tau-70c^2}{2028\tau}$ | $\frac{2(81\tau^2-18c\tau+c^2)}{507\tau}$ | $\frac{\tau}{2}$ | $\frac{\tau}{2}$ |

**Table 1:** Profit levels under all refund policy outcomes ($k = 0$).

Table 2 displays the profit levels of each service provider under each outcome of this pure-strategy game when there are only *capacity* costs, that is $c = 0$.

Service Provider $B$:

| | | $R$ | | $N$ | |
|---|---|---|---|---|---|
| Provider $A$: | $R$ | $\frac{3\tau-k}{8}$ | $\frac{3\tau-k}{8}$ | $\frac{108\tau^2-96k\tau-35k^2}{338\tau}$ | $\frac{(11\tau-2k)^2}{338\tau}$ |
| | $N$ | $\frac{(11\tau-2k)^2}{338\tau}$ | $\frac{108\tau^2-96k\tau-35k^2}{338\tau}$ | $\frac{\tau}{2}$ | $\frac{\tau}{2}$ |

**Table 2:** Profit levels under all refund policy outcomes ($c = 0$).

Tables 1 and 2 imply the following proposition which is proved in Appendix A.

**Proposition 4**

(a) *There are exactly two equilibria: Both providers choose refundable bookings or both choose nonrefundable bookings.*

(b) *The unique semicollusion equilibrium is that both service providers sell only nonrefundable tickets.*

Proposition 4 rules out asymmetric equilibria where one provider utilizes refundable bookings whereas the other only nonrefundable bookings. This result is attributed to the assumed high value of $\tau$ which weakens the competition between the service providers. In Section 5.2 below, Proposition 5 indeed demonstrates that under low values of $\tau$ service providers may be able reduce price competition by segmenting the market by utilizing different refund policies.

Another immediate implication of Proposition 4 is that an industry (coordination) failure occurs when both providers sell refundable tickets. Therefore,

**Corollary 1**

*Industries with observed refundable bookings are not colluding on their booking strategy.*

### 5.2 Equilibrium under strong market segmentation

Similar to Section 4.2, we confine our investigation to the case where $k = 0$. Recall from Section 4.2 that strong market segmentation occurs under intense competition. Formally, $\tau < 2c/3$ when $k = 0$. Table 3 displays the profit levels of each service provider under each outcome of this pure-strategy game when there are only *operation* costs.

| | | Service Provider $B$: $R$ | | Service Provider $B$: $N$ | |
|---|---|---|---|---|---|
| Provider $A$: | $R$ | $\frac{3\tau}{8}$ | $\frac{3\tau}{8}$ | $\frac{16c^2+3\tau^2}{72c}$ | $\frac{4c^2+\tau^2}{24c}$ |
| | $N$ | $\frac{4c^2+\tau^2}{24c}$ | $\frac{16c^2+3\tau^2}{72c}$ | $\frac{\tau}{2}$ | $\frac{\tau}{2}$ |

**Table 3:** Profit levels under all refund policy outcomes, ($k = 0$).

We now define the following three constants, which are also illustrated in Figure 3.

$$\phi_L \stackrel{\text{def.}}{=} 6 - \frac{\sqrt{282}}{3} \approx 0.402, \quad \phi_M \stackrel{\text{def.}}{=} 6 - \frac{2\sqrt{69}}{3} \approx 0.462, \quad \phi_H \stackrel{\text{def.}}{=} \frac{9 - \sqrt{65}}{2} \approx 0.469. \quad (32)$$

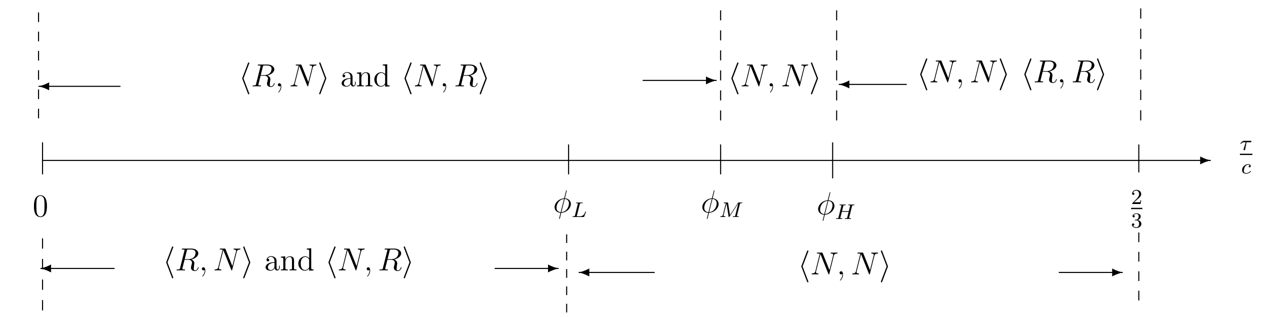


**Figure 3:** Strong market segmentation:
*Above*: Equilibrium outcomes. *Below*: Semicollusion outcomes.

We summarize our analysis of strong market segmentation (intense competition) with the following proposition which is proved in Appendix B, and illustrated in Figure 3.

**Proposition 5**

(a) *Intense competition generates only asymmetric equilibria where one service provider sells refundable tickets whereas the other only nonrefundable tickets. Formally, if* $\tau/c < \phi_M$ *then* $\langle R, N\rangle$ *and* $\langle N, R\rangle$ *are the only equilibria.*

(b) *If* $\tau/c > \phi_M$*, asymmetric equilibria do not exist. For intermediate levels of competition,* $\phi_M \le \tau/c \le \phi_H$ *the unique equilibrium is both firms selling only nonrefundable tickets. For weaker competition,* $\phi_H \le \tau/c < 2/3$*, both symmetric outcomes* $\langle N, N\rangle$ *and* $\langle R, R\rangle$ *are equilibria.*

(c) *Under intense competition where* $\tau/c < \phi_L$*, firms can semicollude by offering different booking systems. Formally,* $\langle R, N\rangle$ *and* $\langle N, R\rangle$ *constitute semicollusion.*

Proposition 5(a) implies that strong market segmentation is a necessary condition for realizing asymmetric booking policies of service providers in the same industry. In order to weaken price competition, service providers segment the market by specializing in different booking systems. Under more intense competition, Proposition 5(c) states that the asymmetric booking strategies maximize industry joint profits. In contrast, if competition is not intense, Proposition 5(b) shows that the equilibrium outcomes are the same as under the weak market segmentation case as summarized in Proposition 4.

## 6. Partial Refunds

Our analysis so far assumed that service providers can either fully refund a ticket or not refund it at all. In this section we extend our model to include partial refunds. In addition, we remove the commitment aspect of choosing a booking strategy before setting prices, by considering simultaneous choices of both strategies in a Nash equilibrium.[6] Let $r_i$ denote the refund rate of service provider $i$, $i = A, B$. That is, $r_i$ is the fraction of the ticket price refunded to a consumer who does not show up to pick up the service. Then, we modify the utility function (1) to take the form

[6] In fact, due to the change from a discrete booking strategy space to a continuous one, simultaneous decisions Nash equilibrium outcomes can be easily formulated.

$$U(\sigma, x) \stackrel{\text{def.}}{=} \begin{cases} \sigma\beta - p_A + (1-\sigma)r_A p_A - \tau x & \text{Book with } A \\ \sigma\beta - p_B + (1-\sigma)r_B p_B - \tau(1-x) & \text{Book with } B. \end{cases} \tag{33}$$

Hence, if the customer doesn't show up she is refunded with the amount $r_i p_i$.

The utility function (33) implies that the consumers who are indifferent between making reservations with $A$ and $B$ are implicitly solved from $\sigma\beta - (1 - r_A + \sigma)p_A - \tau x = \sigma\beta - (1 - r_B + \sigma)p_B - \tau(1-x)$. Hence,

$$\hat{x}(\sigma) = \frac{1}{2} + \frac{(1 - r_B + \sigma r_B)p_B - (1 - r_A + \sigma r_A)p_A}{2\tau}. \tag{34}$$

Then, the number of reservations are given by

$$q_A = 1 - q_B = \int_0^1 \hat{x}(\sigma)d\sigma = \frac{1}{2} + \frac{(2 - r_B)p_B - (2 - r_A)p_A}{4\tau}. \tag{35}$$

Therefore, the expected number of show-ups are given by

$$s_A = \int_0^1 \sigma\hat{x}(\sigma)d\sigma = \frac{1}{4} + \frac{(3 - r_B)p_B - (3 - r_A)p_A}{12\tau}, \tag{36a}$$

$$s_B = \int_0^1 \sigma\left[1 - \hat{x}(\sigma)\right]d\sigma = \frac{1 - 2s_A}{2}. \tag{36b}$$

Therefore the expected number of no-shows is

$$q_i - s_i = \frac{1}{4} + \frac{(3 - 2r_j)p_j - (3 - 2r_i)p_i}{12\tau} \tag{37}$$

Service provider $i$ ($i = A, B$) takes $p_j$ and $r_j$ ($j \neq i$) as given, and chooses $p_i$ and $r_j$ to solve

$$\begin{aligned} \max_{p_i, r_i} \pi_i(p_i, r_i, p_j, r_j) &= p_i s_i + p_i(1 - r_i)(q_i - s_i) - c s_i - k q_i \\ &= 3[p_i(1 - r_i) - k]\frac{2\tau + (2 - r_j)p_j - (2 - r_i)p_i}{12\tau} \\ &+ [p_i r_i - c]\frac{3\tau + (3 - r_j)p_j - (3 - r_i)p_i}{12\tau} \end{aligned} \tag{38}$$

Comparing the profit function (38) with (5) and (11) reveals that in the presence of partial refunds the revenue of service providers becomes weighted by the sum of revenue generated from consumers who show up, $p_i s_i$, plus the nonrefundable portion $1 - r_i$ collected from the $q_i - s_i$ consumers who do not show up.

Then the necessary conditions for profit maximum are given by

$$\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= q_i - r_i q_i + r_i s_i + \frac{\partial q_i}{\partial p_i}\left[p_i(1-r_i) - k\right] + \frac{\partial s_i}{\partial p_i}\left[p_i r_i - c\right] \\
&= q_i - r_i q_i + r_i s_i - \frac{6-3r_i}{12\tau}\left[p_i(1-r_i) - k\right] - \frac{3-r_i}{12\tau}\left[p_i r_i - c\right] = 0, \text{ and} \qquad (39) \\
\frac{\partial \pi_i}{\partial r_i} &= -p_i(q_i - s_i) + \frac{\partial q_i}{\partial r_i}\left[p_i(1-r_i) - k\right] + \frac{\partial s_i}{\partial r_i}\left[p_i r_i - c\right] \\
&= p_i\left[-(q_i - s_i) + \frac{3}{12\tau}\left[p_i(1-r_i) - k\right] + \frac{1}{12\tau}\left[p_i r_i - c\right]\right] = 0. \qquad (40)
\end{aligned}$$

The profit function (38) is concave in $p_i$, concave in $r_i$, and locally concave at any extreme point (see Appendix C). Since the profit function is quasiconcave we have a unique equilibrium at $p_A = p_B$ and $r_A = r_B$. The equilibrium prices, the levels of partial refunds, and the profits are given by

$$p_i = c + k + \tau, \quad r_i = \frac{c}{c+k+\tau} \quad \text{and} \quad \pi_i = \frac{\tau}{2}, \; i = A, B. \qquad (41)$$

Finally, we observe the following relationships.

**Proposition 6**

(a) *When service providers compete in prices and partial refunds, they set the refunded portion of the price equals to the marginal operation costs. Formally,* $r_i p_i = c$.

(b) *Intense competition leads to higher refund rates. Formally a decrease in* $\tau$ *increases* $r_i$.

Proposition 6 implies that consumers are refunded exactly by the cost they do *not* inflict on firms by not showing up, i.e., the cost of operation, $c$. Therefore, the nonrefundable portion equals capacity costs, $k$, and the profit markup, $\tau$ (which depends on the degree of competition). This means that service providers also make a profit from those consumers who do not show up.

## 7. Extensions

The following two subsections suggest some extensions for future research.

### 7.1 Dual booking strategies

Suppose now that service providers offer customers a menu of two price options at the time reservations are made. Formally, we assume that at the time reservations are made, service provider $i$ ($i = A, B$) offers customers to choose whether to pay a nonrefundable price $p_i^N$ or instead to pay $p_i^R$ and obtain a full-refund upon no-show.

Let $\hat{\sigma}_A$ and $\hat{\sigma}_B$ denote customers who make a reservation with $A$ and $B$, respectively, and are indifferent between choosing the nonrefundable option and the full-refund option. Formally, the utility function (1) implies that $\hat{\sigma}_i$ is implicitly solved from $\sigma(\beta p_i) - \tau x = \sigma\beta p_i - \tau x$. Hence,

$$\hat{\sigma}_A = \frac{p_A^N}{p_A^R} \quad \text{and} \quad \hat{\sigma}_B = \frac{p_B^N}{p_B^R}. \tag{42}$$

Note that $\hat{\sigma}_i < 1$ since in equilibrium the price of a nonrefundable ticket can never exceed the price of a refundable ticket.

Next, recall from (2) that $\hat{x}^R(\sigma)$ index consumers who book a refundable ticket and are indifferent between booking with $A$ and $B$. Similarly, $\hat{x}^N$ defined in (8) index consumers who book a nonrefundable ticket and are also indifferent between booking with $A$ and $B$. Finally, from (15) we can derive the index of consumers who are indifferent between making a nonrefundable booking with $A$ and a refundable booking with $B$. Therefore,

$$\hat{x}^R(\sigma) = \frac{1}{2} + \frac{\sigma(p_B - p_A)}{2\tau} \quad \hat{x}^N = \frac{1}{2} + \frac{p_B - p_A}{2\tau}, \quad \text{and} \quad \hat{x}^{NR}(\sigma) = \frac{1}{2} + \frac{\sigma p_B^R - p_A^N}{2\tau.} \tag{43}$$

Figure 4 illustrates, in the $\sigma \times x$ space of consumers, how consumers are divided among the four booking options (two by each provider).

Figure 4, (42) and (43) imply that the number of consumers who book with $A$ (according to the two booking classes) are

$$q_A^R = \int_0^{\hat{\sigma}_A} \hat{x}^R(\sigma)\, d\sigma \quad \text{and} \quad q_A^N = \int_{\hat{\sigma}_A}^{\hat{\sigma}_B} \hat{x}^{NR}(\sigma)\, d\sigma + \int_{\hat{\sigma}_B}^{1} \hat{x}^N\, d\sigma. \tag{44}$$

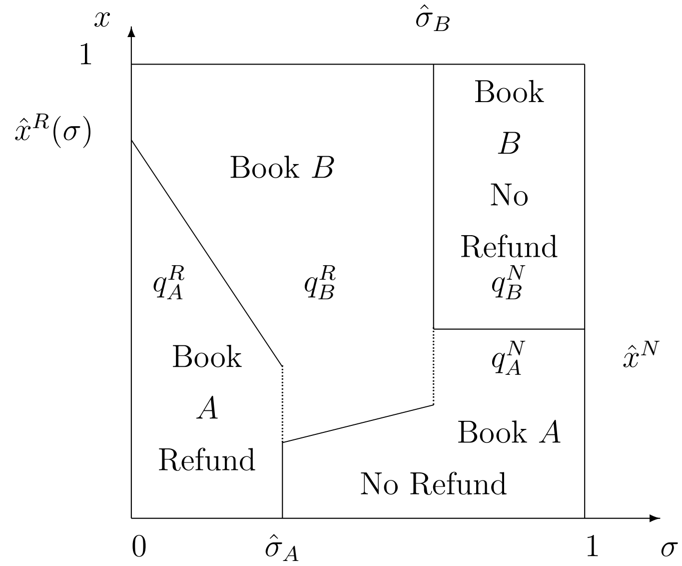


**Figure 4:** Customers' choices of booking options under dual booking strategy. *Note*: The figure ignores strong market segmentation.

It follows that the *expected* show-ups to $A$'s services are

$$s_A^R = \int_0^{\hat{\sigma}_A} \sigma \hat{x}^R(\sigma)\, d\sigma \quad \text{and} \quad s_A^N = \int_{\hat{\sigma}_A}^{\hat{\sigma}_B} \sigma \hat{x}^{NR}(\sigma)\, d\sigma + \int_{\hat{\sigma}_B}^{1} \sigma \hat{x}^N \, d\sigma. \tag{45}$$

Similarly, the number of reservations made with $B$ are

$$q_B^R = \int_0^{\hat{\sigma}_A} \left[1 - \hat{x}^R(\sigma)\right] \, d\sigma + \int_{\hat{\sigma}_A}^{\hat{\sigma}_B} \left[1 - \hat{x}^{NR}(\sigma)\right] \, d\sigma \quad \text{and} \quad q_B^N = \int_{\hat{\sigma}_B}^{1} \left[1 - \hat{x}^N\right] \, d\sigma. \tag{46}$$

It follows that the expected number of show-ups for $B$'s services are

$$s_B^R = \int_0^{\hat{\sigma}_B} \sigma \left[1 - \hat{x}^R(\sigma)\right] \, d\sigma + \int_{\hat{\sigma}_A}^{\hat{\sigma}_B} \sigma \left[1 - \hat{x}^{NR}(\sigma)\right] \, d\sigma \quad \text{and} \quad s_B^N = \int_{\hat{\sigma}_B}^{1} \sigma \left[1 - \hat{x}^N\right] \, d\sigma. \tag{47}$$

The problem of each service provider $i$ ($i = A, B$) is the choose a pair of prices $(p_i^R, p_i^N)$ to solve

$$\max_{p_i^R, p_i^N} \pi_i = p_i^R s_i^R + p_i^N q_i^N - c(s_i^R + s_i^N) - k\left(q_i^R + q_i^N\right). \tag{48}$$

The first two terms are the revenue generated from consumers who book refundable and nonrefundable tickets. The last two terms are the expected costs of delivering the service and the cost of making the reservations. Unfortunately, the profit functions (48) are not concave. Under strong market segmentation, the profit functions become even more complicated than (48). Since computations are tedious and long we refrain from solving these optimization problems.

### 7.2 Scrap value for underutilized capacity

Another missing aspect from the present formulation is our assumption that there is no scrap value for excess capacity, as measured by the difference $q_i - s_i$, $i = A, B$. Positive scrap value means that excess capacity can be sold in an event when the number of show-ups falls below the prepared capacity. Let $\rho$ denote the scrap value of excess capacity. In this case, (5) should be written as

$$\pi_i(p_i, p_j) = (p_i - c)s_i - kq_i + \rho\,(q_i - s_i)\,.$$

## 8. Conclusion

The novelty of the present paper is the explicit introduction of competition into service industries utilizing advance bookings. Most studies, so far, focused on a single seller, or multiple sellers under fixed prices.

From a policy point of view, our model demonstrates a clear tradeoff between the division of surplus amongst service providers and consumers when refunds become available. That is, under competition, a higher refund rate reduces firms' profits in favor of an increase in consumer surplus. This finding differs from the results obtained in the monopoly advance booking models where a single service provider can extract a higher surplus when it offers full refunds. Here, due to price competition, service providers cannot raise prices to the level where the entire surplus from the refundability option can be captured.

We demonstrate that under weak price competition (leading to weak market segmentation) aggregate industry profit is maximized when both service providers utilize nonrefundable bookings. This result may explain why refundable tickets are rarely observed in the various entertainment industries. For example, movie theaters, and sports events such as soccer, basketball, and hockey games rarely offer refunds on no-shows. We also show there does not exist an asymmetric equilibrium where one providers offers full refunds whereas the other no refunds. This result may explain why all firms in the same industry generally adopt the same booking strategy. That is, the use of refunds may vary across industries but not within the industry.

Under intense price competition, in a two-stage game where firms first commit on a booking strategy, service providers may end up segmenting the market by utilizing different booking policies.

When partial refunds are introduced, and when service providers can simultaneously decide on their booking strategies as well as setting prices, consumers are refunded exactly by the cost they have not inflicted on firms. Therefore, the nonrefundable portion equals capacity costs plus and the markup.

In competitive industries with small capacity costs, we expect a wide use of refundability options. We argue that this might be a reason why the car rental industry frequently rent cars without reservation costs. An unrented car can wait at the next customer, but an empty seat in a cinema cannot. Finally, we conjecture that another reason (not captured in the present model) why car-rental companies offer full refunds instead of no refunds stems from the fact that their customers are generally screened by providers of complementary services who themselves select only those customers with a high probability of showing up. For example, consumers who make car-rental reservations at major airports fly in by airline companies who also screen consumers via price discrimination mechanisms. Moreover, in this example, no shows in the car-rental industry are often caused by the airline companies (e.g., traffic delays) and not by customer behavior. Thus, no-shows in the car rental industry cannot be compared with no-shows in the entertainment industries for which only the consumers themselves are to be accounted for no-shows.

## Appendix A. Proof of Proposition 4

**Part (a):** First, suppose that $k = 0$. Then, (14) becomes $\tau > 3c$. Table 1 implies that $\pi_A(N, N) > \pi_A(R, N)$ if $\tau > 4c/75$ which clearly holds. Similarly,$\pi_B(N, N) > \pi_B(N, R)$. Next, $\pi_A(R, R) > \pi_A(N, R)$ if $\tau^2 - 2\tau^2 > -2(183c\tau + 70c^2)$ which always holds. Similarly, $\pi_B(R, R) > \pi_B(R, N)$.

Now, suppose that $c = 0$. Then, (14) becomes $\tau > 5.5k$. Table 2 implies that $\pi_A(N, N) > \pi_A(R, N)$ if $\tau > -k$ while always holds. Similarly,$\pi_B(N, N) > \pi_B(N, R)$. Next, $\pi_A(R, R) > \pi_A(N, R)$ if $\tau > 16k/23$ which follows from $\tau > 5.5c$. Similarly, $\pi_B(R, R) > \pi_B(R, N)$.

**Part (b):** We first examine industry joint profit under $k = 0$. Table 1 implies that $\pi_A(N, N) + \pi_B(N, N) > \pi_A(R, N) + \pi_B(N, R)$ if $\tau > (\sqrt{282}/3 + 6)$ which follows immediately from $\tau > 2c/3$.

Now, suppose that $c = 0$. Table 2 implies that $\pi_A(N, N) + \pi_B(N, N) > \pi_A(R, N) + \pi_B(N, R)$ if $\tau > -31k/109$, which clearly holds. $\square$

## Appendix B. Proof of Proposition 5

**Parts (a) and (b):** Table 3 and (32) imply that $\pi_A(N, N) \geq \pi_A(R, N)$ and $\pi_B(N, N) \geq \pi_B(N, R)$ if and only if $\tau \geq \phi_M$. Next, $\pi_A(R, R) \geq \pi_A(N, R)$ and $\pi_B(R, R) \geq \pi_B(R, N)$ if and only if $\tau \geq \phi_H$.

**Part (c):** Table 3 and (32) imply that $\pi_A(N, N) + \pi_B(N, N) \geq \pi_A(R, N) + \pi_B(N, R)$ if and only if $\tau \geq \phi_L$. $\square$

## Appendix C. Quasiconcavity of Parital Refund Profit Functions

The profit function $\pi_i(p_i, r_i, p_j, r_j) = p_i s_i + p_i(1 - r_i)(q_i - s_i) - cs_i - kq_i$(38) has the following second derivatives:

$$
\begin{aligned}
\frac{\partial^2 \pi_i}{(\partial p_i)^2} &= -\frac{2r_i(3-r_i)+6(2-r_i)(1-r_i)}{12\tau} < 0,\\
\frac{\partial^2 \pi_i}{(\partial p_i)^2} &= -\frac{4p_i^2}{12\tau} < 0, \text{ and}\\
\frac{\partial^2 \pi_i}{\partial p_i \partial r_i} &= \frac{4(3p_i-2r_i)-(3p_j-2r_j)-3(c+k+\tau)}{12\tau}.
\end{aligned}
$$

The only interior profit maximizing solution when $p_i > 0$ and $0 \leq r_i \leq 1$ is

$$
p_i = \frac{p_j + c + k + \tau}{2} \quad \text{and} \quad r_i = \frac{c + p_j r_j}{p_j + c + k + \tau}\,, i, j = A, B,\ i \neq j.
$$

Moreover, in any stationary point the profit function is locally concave as

$$
\det \nabla^2 \pi_i \Bigg|_{\frac{\partial \pi_i}{\partial p_i} = \frac{\partial \pi_i}{\partial r_i} = 0} = \frac{(c+k+\tau)^2 + p_j[p_j + 2(c+k+\tau)]}{48\tau^2} > 0\,, i, j = A, B,\ i \neq j.
$$

As $\partial^2\pi_i/(\partial p_i)^2 < 0$ and $\partial^2\pi_i/(\partial r_i)^2 < 0$, the profitfunctions $\pi_A$ and $\pi_B$ are quasiconcave. $\square$

## References


Courty, P. 2003. "Ticket Pricing Under Demand Uncertainty," *Journal of Law & Economics,* XLVI: 627–652.

Courty, P., and H. Li. 2000. "Sequential Screening," *Review of Economic Studies,* 67: 697–717.

Dana, J. 1998. "Advanced Purchase discounts and Price Discrimination in Competitive Markets," *Journal of Political Economy*, 106: 395–422.

Gale, I., and T. Holmes. 1992. "The Efficiency of Advance-Purchase Discounts in the Presence of Aggregate Demand Uncertainty," *International Journal of Industrial Organization,* 10: 413–437.

Gale, I., and T. Holmes. 1993. "Advance-Purchase Discounts and Monopoly Allocation of Capacity," *American Economic Review,* 83: 135–146.

Gale, I. 1993. "Price Dispersion in a Market with Advance-Purchases," *Review of Industrial Organization,* 8: 451–464.

Macskasi, Z. 2003. "Horizontal Differentiation with Consumer Uncertainty." Northwestern University.

Miravete, E. 1996. "Screening Consumers through Alternative Pricing Mechanisms," *Journal of Regulatory Economics*, 9: 111–132.

Ringbom, S., and O. Shy. 2003. "Advance Booking and Refunds Under Capacity Constraints." Available for downloading from `www.ozshy.com`.

Ringbom, S., and O. Shy. 2004. "Advance booking, cancellations, and partial refunds." *Economics Bulletin*, Vol. 13 no. 1 pp. 1-7.