Reservations and Refunds

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Abstract

We investigate economic and strategic incentives of service providers to engage in advance booking while allowing for a full-refund for those customers who cancel or do not show up at the time when the good or the service is provided.

We find that:

- a fully refundable booking strategy is more profitable than a non-refundable booking strategy for small marginal costs,
- a dual price booking strategy which separates the customers with respect to their showing up probability, improves social welfare and industry profits.

Background

- Reservation systems are observed in almost all service industries
  - airline industry, railroad, car rentals and bus travel.
  - restaurants, fancy barber shops, and law offices.
  - Advance booking of orders in bookstores.

- Different firms utilize different booking and refundability strategies. Airlines are the most sophisticated ones.

- As advance booking and reservations are commonly observed, both buyers and sellers must find them beneficial.

Refundable or non-refundable?

- Full refund $\Longrightarrow$ more no-shows.
- Full refund $\Longrightarrow$ more reservations $\Longrightarrow$higher revenue? $\Longrightarrow$higher profits?
- Well . . . it depends . . .
- Ryanair: no refunds (but a rebooking option). Other airlines: an assortment of refundability options.
- Car rental firms seldom enforced obligations ... *de facto* free option to rent a car.

Consequences of a refund strategy

- For buyers, advance booking $\Longrightarrow$ "assured service"
- Market segmentation in the refundability dimension – through customer self-selection
- Customers with **L**ow showing up probability $\Longrightarrow$buys with refund options
- Customers with **H**igh showing up probability $\Longrightarrow$buys without refund options, as they are less willing to pay for this feature.

Earlier studies

- *post-delivery* contracts with money-back-warranties: see Mann and Wissink 1988, Mann and Wissink 1990, and Shiou 1996.

- Advance booking with a prebooking market and a spot market: Xie and Shugan 2001. (Here, all customers buy in advance.)

- Weatherford and Pfeifer accurate estimate of the final demand.

- Consumers' self-selection: Mahajan and van Ryzin 2001 analyze hotel customers' strategic behavior in the booking process.

- Zhao and Zheng 2001 argue that late discounts disrupt the credibility of the booking strategies.

- Bodily and Pfeifer 1992: the refundability option is a device to control the final showup probabilities.

The Model: Refundable and Non-Refundable Bookings

p^N	price of a non-refundable ticket,
p^R	price of a refundable ticket,
$c > 0$	constant unit cost,
$i = H, L$,	two groups of consumers *High* and *Low*,
σ_H, σ_L	showing up probability for type H and L consumers,
n	population size,
α_H, α_L	proportions of consumers of type H and L resp. $\alpha_H + \alpha_L = 1$. ($\alpha_i n$ are of type i),

x ($0 \le x \le 1$) Hotelling type model, consumers are indexed by according to their declining willingness to pay for this service. $x = 0$ highest willingness to pay, $x = 1$ lowest.

The utility function is given by

$$U_i(x) = \begin{cases} \sigma_i\left(1 - x - p^R\right) & \text{if buys a refundable ticket} \\ \sigma_i\left(1 - x\right) - p^N & \text{if buys a non-refundable ticket} \\ 0 & \text{if does not buy this good/service} \end{cases} \quad (1)$$

In order to avoid an immediate exclusion of all type L consumers. . .

Assumption 1. *There are some type L consumer with willingness to pay a price exceeding their marginal cost. Formally, $\sigma_L - c > 0$.*

To simplify the expressions, we define the constants

$$\psi_1 \stackrel{\text{def}}{=} \alpha_H\sigma_H + \alpha_L\sigma_L, \quad (2)$$

$$\psi_2 \stackrel{\text{def}}{=} \alpha_H\sigma_L + \alpha_L\sigma_H, \text{ and} \quad (3)$$

$$\psi_3 \stackrel{\text{def}}{=} \frac{\sigma_H\sigma_L}{\alpha_H\sigma_L + \alpha_L\sigma_H}. \quad (4)$$

Where, $\sigma_L < \psi_i < \sigma_H$ for $i = 1, 2, 3$ and $\psi_3 < \psi_1$.[1]

[1] ψ_1 is the average showing up probability in the entire population.

Refundable tickets

From (1), x_i is implicitly defined by $\sigma_i(1 - x_i - p^R) = 0$, for $i = H, L$.

Hence,

$$x_H = x_L = 1 - p^R. \tag{5}$$

No overbooking $\Longrightarrow$ production $n\left(\alpha_H x_H + \alpha_L x_L\right) = n\left(1 - p^R\right)$ units.

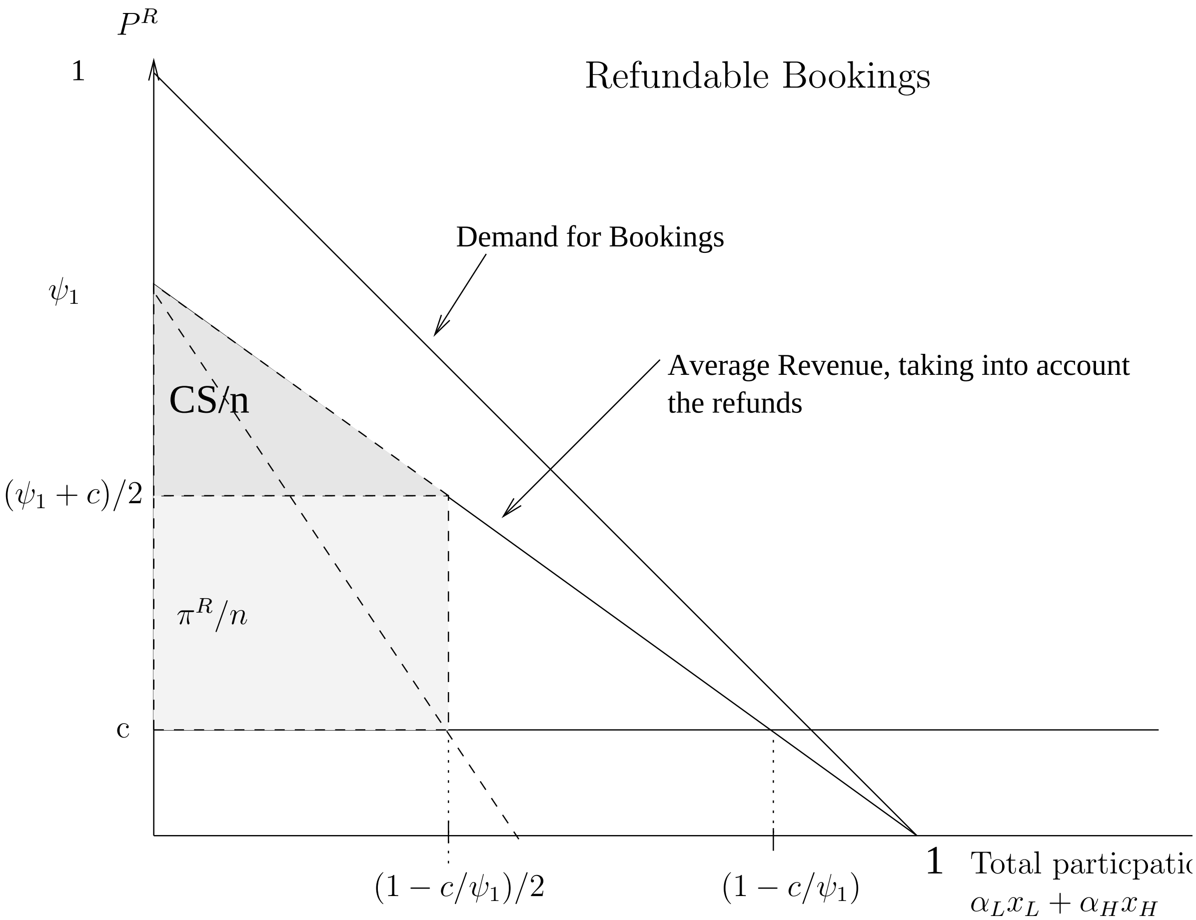

Figure 1: The fully refundable case

We verify next the intuitive solution

$$
\begin{aligned}
AR - c &= \frac{1}{2}(\psi_1 - c) = \frac{\psi_1}{2}(1 - c/\psi_1), \\
\pi^R &= \frac{n\psi_1}{4}\left(1 - \frac{c}{\psi_1}\right)^2 \\
CS^R &= 0.5\pi^R.
\end{aligned}
$$

The seller chooses a refundable ticket price p^R to solve

$$\begin{aligned} \max_{p^R} \frac{\pi^R}{n} &= (\alpha_H x_H \sigma_H + \alpha_L x_L \sigma_L) p^R \\ &\quad -(\alpha_H x_H + \alpha_L x_L) c \\ &= \psi_1 \left(1 - p^R\right) p^R - \left(1 - p^R\right) c, \end{aligned} \quad (6)$$

The profit-maximizing price and profit levels are

$$p^R = \frac{\psi_1 + c}{2\psi_1}, \text{ and} \quad (7)$$

$$\pi^R = \frac{n\psi_1}{4}\left(1 - \frac{c}{\psi_1}\right)^2 = n\psi_1 \left(x_H^R\right)^2. \quad (8)$$

The participation rates are

$$x_H^R = x_L^R = \frac{1}{2}\left(1 - \frac{c}{\psi_1}\right). \quad (9)$$

Non-refundable tickets

The utility function (1) implies that:

$$\sigma_i \left(1 - x_i^N\right) - p^N = 0$$

solves x_i^N, for $i = H, L$ (if $x_i^N > 0$). Hence,

$$x_H^N = 1 - \frac{p^N}{\sigma_H} \quad \text{and} \quad x_L^N = \max\left\{1 - \frac{p^N}{\sigma_L}, 0\right\}. \quad (10)$$

All market served? . . . or only type H customers?

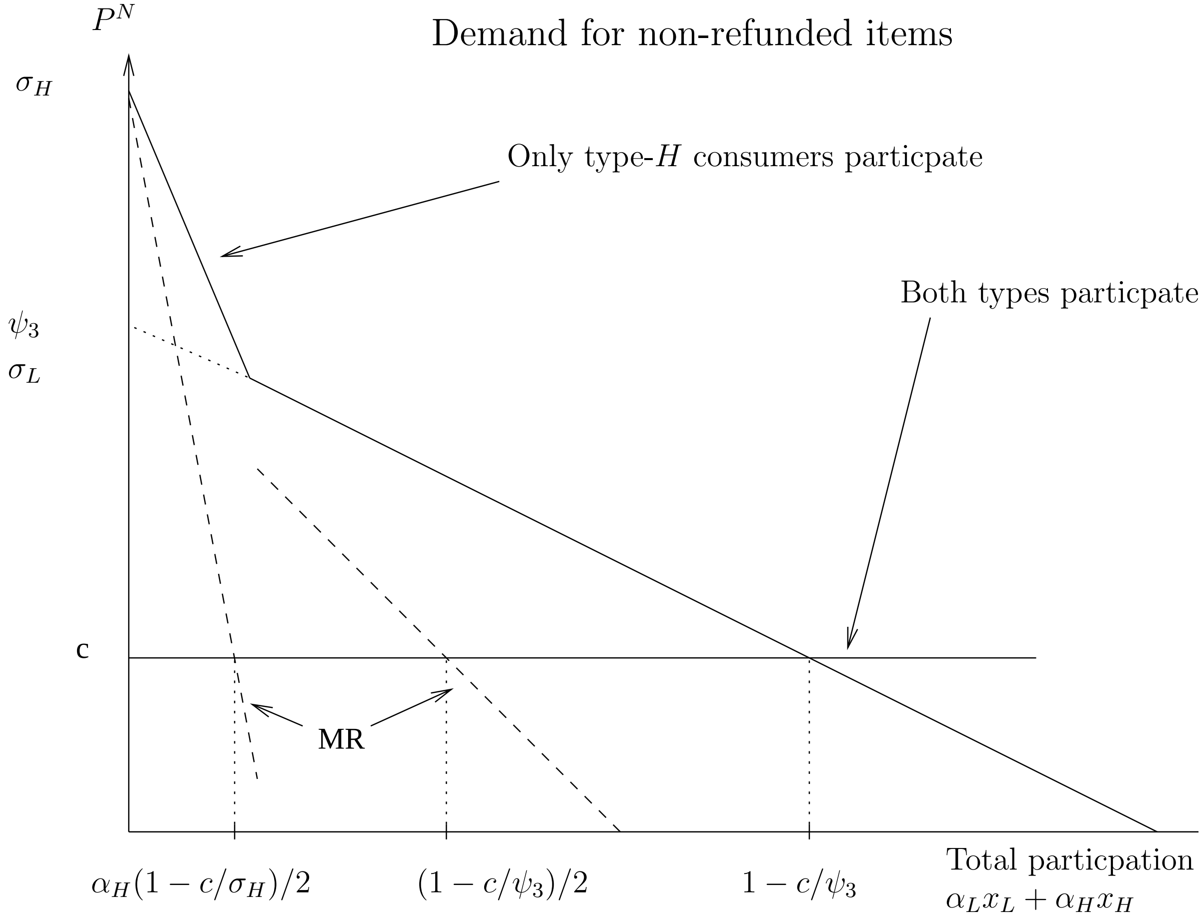

Figure 2: Non-refundable strategy - basic setup

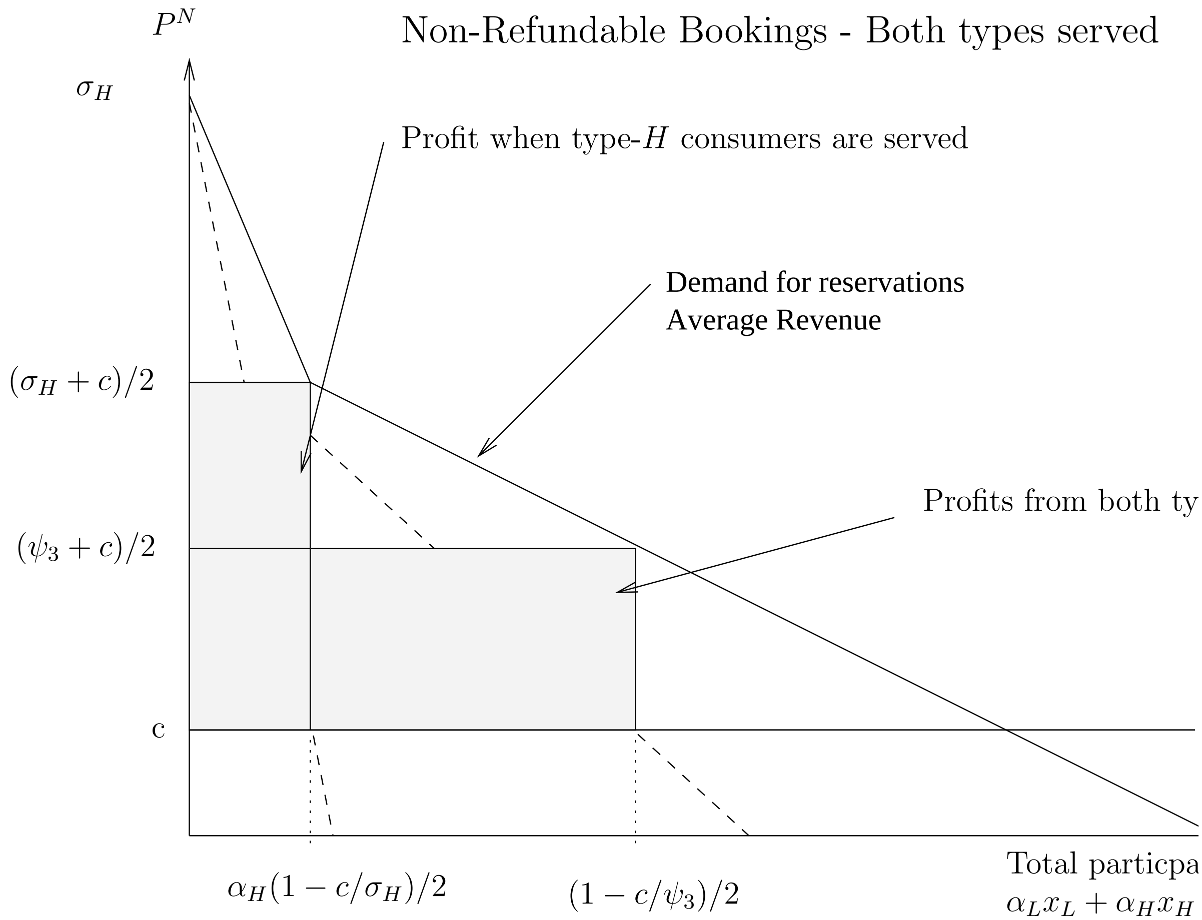

Figure 3: Non-refundable strategy — both types served

We conclude from the figure that,

$$
\begin{aligned}
p^N &= \frac{\psi_3 + c}{2} = \frac{\psi_3}{2}(1 + c/\psi_3), \\
p^N - c &= \frac{\psi_3}{2}(1 - c/\psi_3), \\
\alpha_L x_L^N + \alpha_H x_H^N &= \frac{1}{2}(1 - c/\psi_3), \text{ and} \\
\pi^N &= \frac{n\psi_3}{4}(1 - c/\psi_3)^2
\end{aligned}
$$

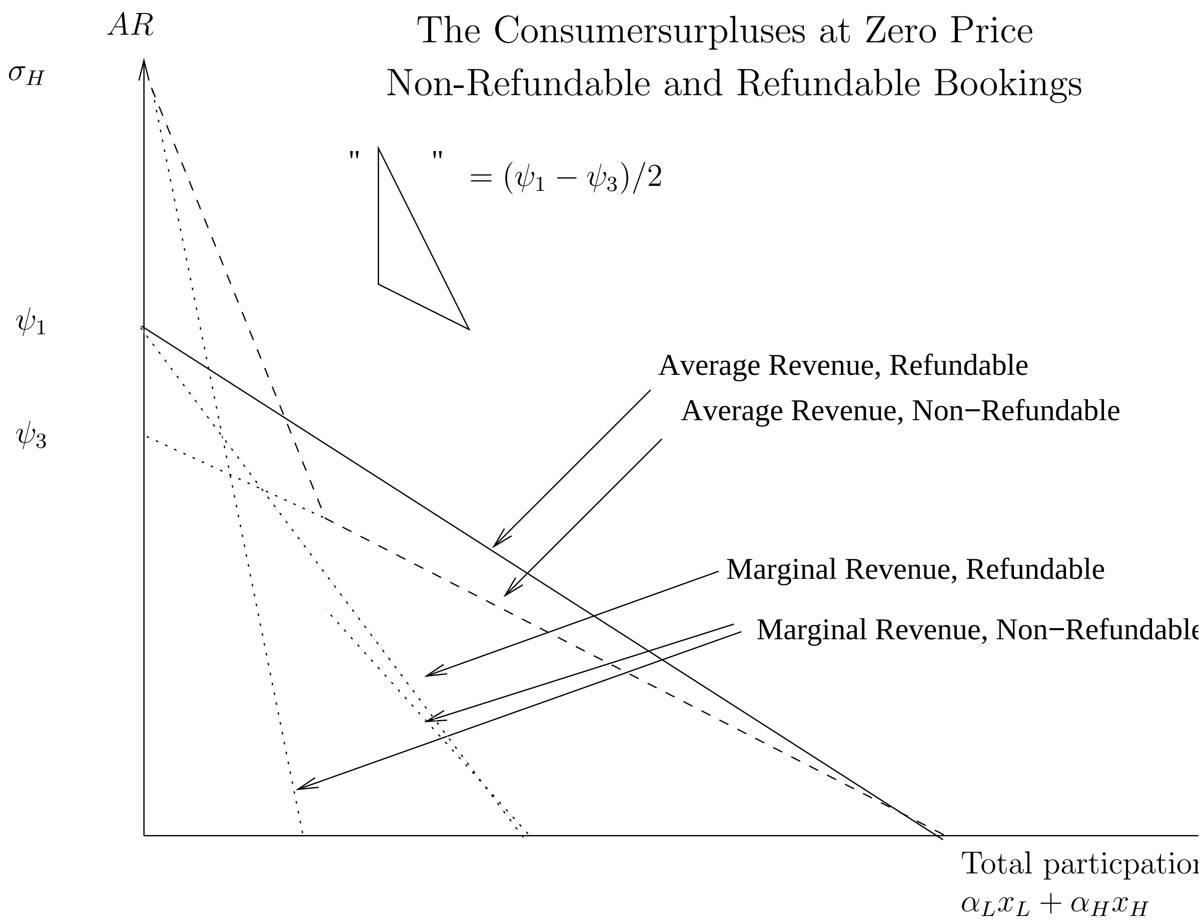

Figure 4: For a zero price the consumer surpluses are equal regardless of the refundability feature.

As the areas under the AR-functions are equal, the upper left triangle under the non-refundable strategy is $\psi_1/2$ - $\psi_3/2$.

Observe also that AR is always lower under the non-refundable strategy when both types are served compared with the AR under the refundable strategy.

We observe that ...

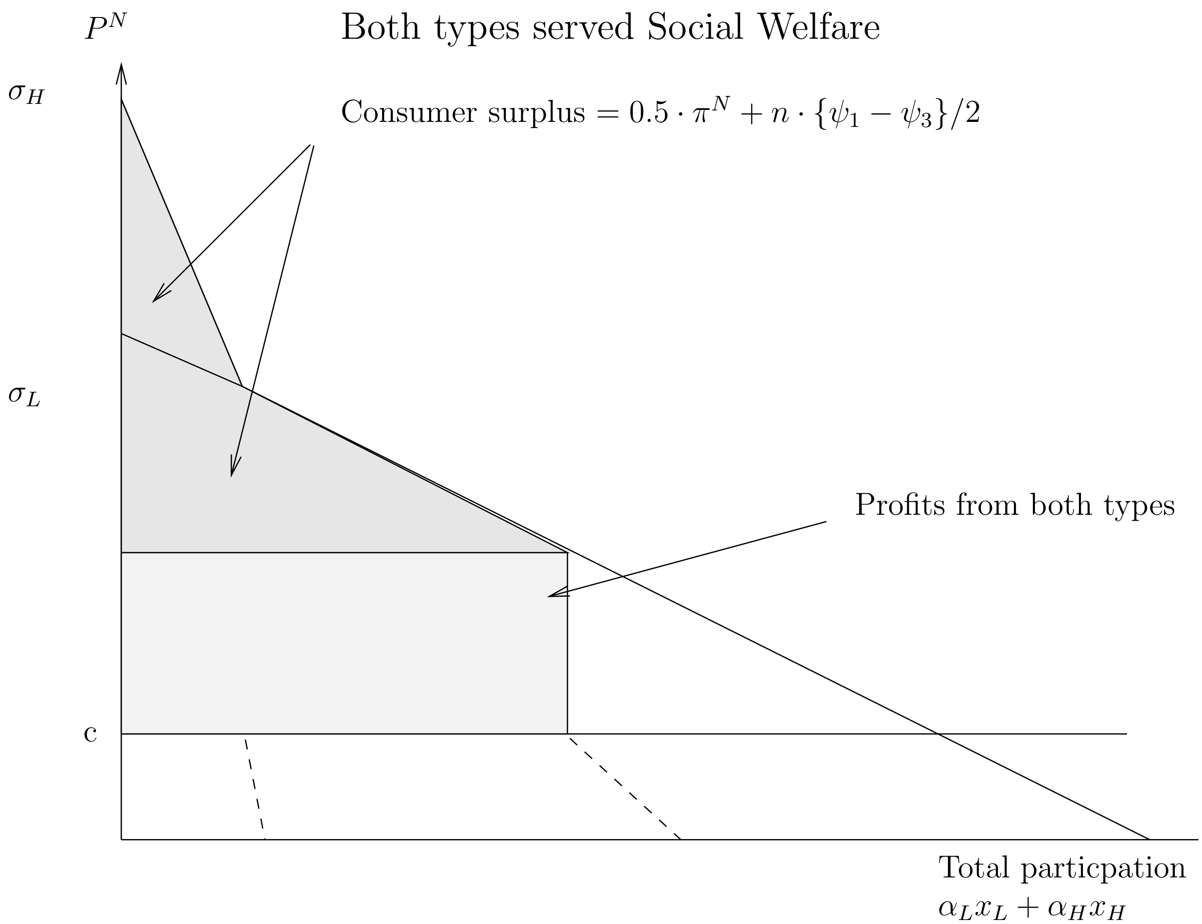

Figure 5: Social Welfare under the non-refundable strategy.

$$SW^{N_{\text{Both}}} = 1.5\pi^{N_{\text{Both}}} + \frac{n}{2}\{\psi_1 - \psi_3\}$$

Non-refundable booking: Only type-H are served

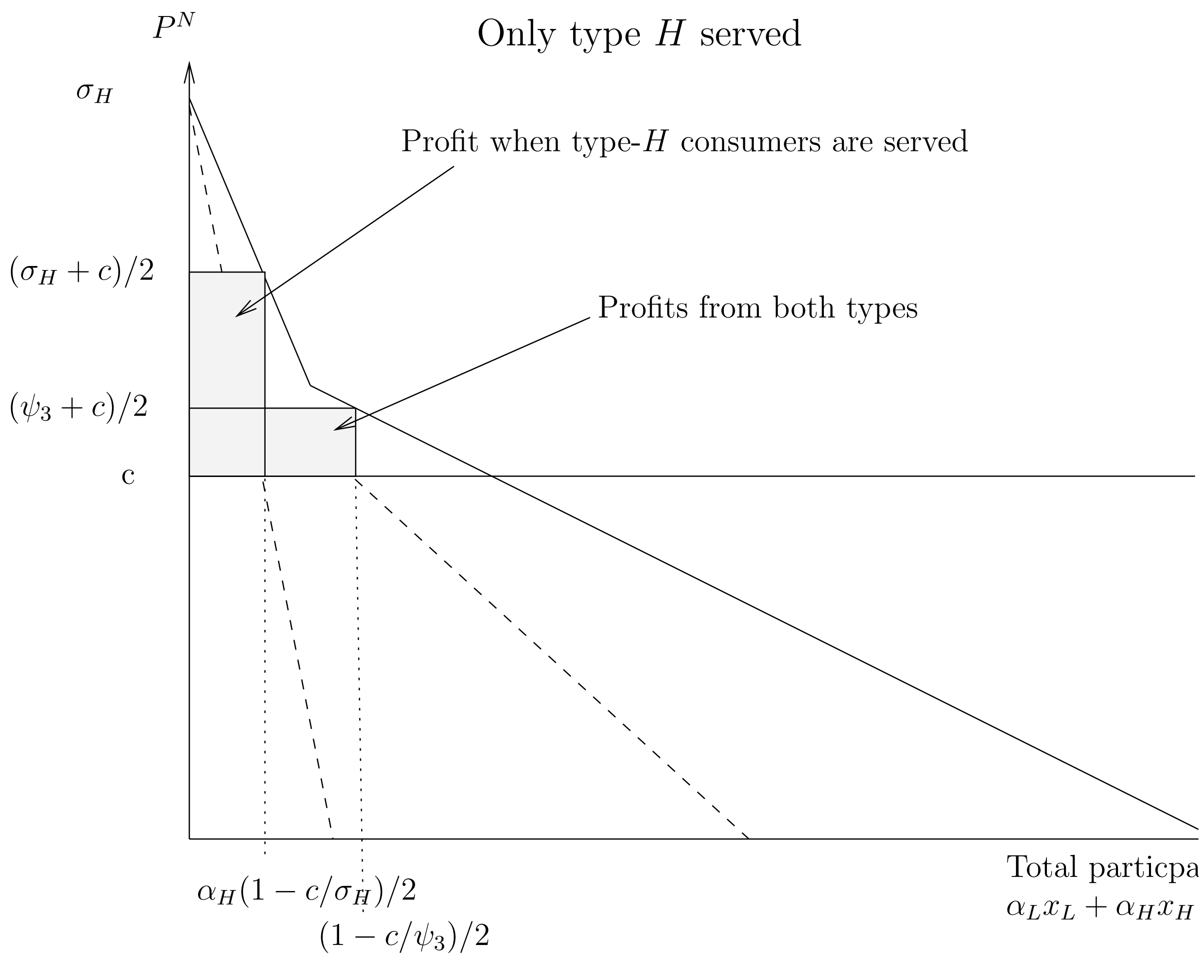

Figure 6: Non-refundable strategy type-L excluded

$$\max_{p^{N_H}} \pi^{N_H} = n\alpha_H \left(p^{N_H} - c\right)\left(1 - \frac{p^{N_H}}{\sigma_H}\right), \quad (11)$$

$$CS^{N_H} = 0.5\pi^{N_H}. \quad (12)$$

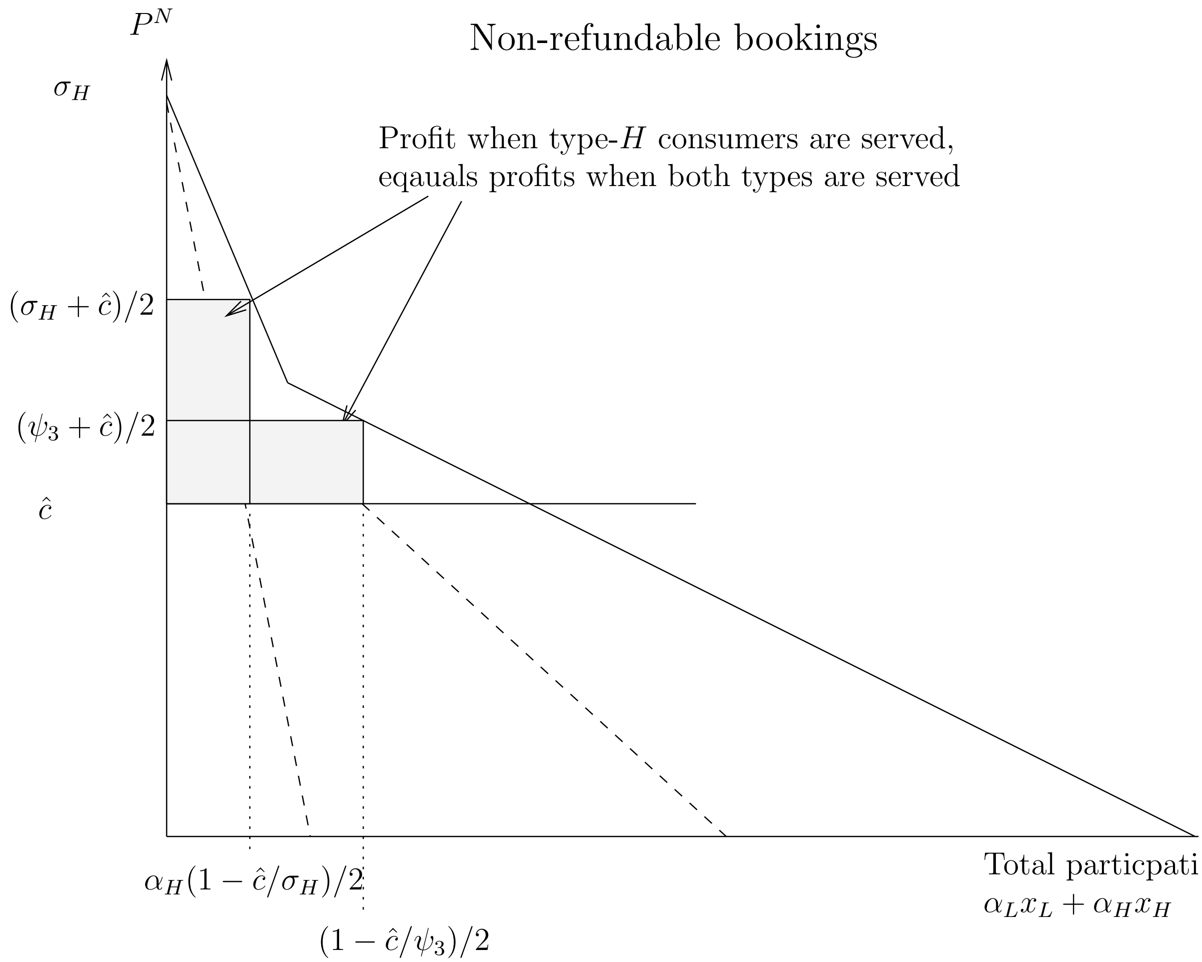

Figure 7: Non-refundable strategy — the threshold marginal cost is $c = \hat{c}$.

$$\hat{c} \quad \stackrel{\text{def}}{=} \quad \arg_{0<c<\sigma_L} \left\{ \pi^{N_{\text{both}}} = \pi^{N_H} \right\} \qquad (13)$$

Proposition 1. *Let $\hat{c}$ be given in (13). Under the non-refundable booking strategy, if the unit production cost is sufficiently low ($c \leq \hat{c}$) the service provider lowers the price so that all consumers are booked. Conversely, only type H consumers are booked if the unit production cost is sufficiently high ($c > \hat{c}$). Formally, $\pi^{N_{\text{both}}} \geq \pi^{N_H}$ if and only if $c \leq \hat{c}$.*

Dual-booking strategy: selling refundable and non-refundable tickets

The seller chooses $p^{N'}$ and $p^{R'}$ to solve

$$\max_{\substack{p^{N'},p^{R'} \text{ subject to:} \\ p^{N'}/p^{R'} \in [\sigma_L, \sigma_H]}} \frac{\pi^D}{n} =$$

$$\alpha_H \left(p^{N'} - c\right) x_H + \alpha_L \left(\sigma_L p^{R'} - c\right) x_L =$$

$$\alpha_H \left(p^{N'} - c\right) \left(1 - \frac{p^{N'}}{\sigma_H}\right) + \alpha_L \left(\sigma_L p^{R'} - c\right) \left(1 - p^{R'}\right) \tag{14}$$

where x_H was substituted from (10), and x_L from (5). The profit maximizing prices are

$$p^{R'} = \frac{1}{2}\left(1 + \frac{c}{\sigma_L}\right) \text{ and} \tag{15}$$

$$p^{N'} = \frac{\sigma_H}{2}\left(1 + \frac{c}{\sigma_H}\right). \tag{16}$$

These prices segment the market, since $\sigma_L p^{R'} \leq p^{N'} = (\sigma_H + c)/2 \leq \sigma_H p^{R'}$. The participation rates are

$$x_H^D = \frac{1}{2}\left(1 - \frac{c}{\sigma_H}\right), \text{ and} \tag{17}$$

$$x_L^D = \frac{1}{2}\left(1 - \frac{c}{\sigma_L}\right). \tag{18}$$

Hence, the aggregate participation is

$$\bar{x}^D = \alpha_L \frac{\sigma_L - c}{2\sigma_L} + \alpha_H \frac{\sigma_H - c}{2\sigma_H} = \frac{1}{2}\left(1 - \frac{c}{\psi_3}\right). \tag{19}$$

The firm's profit and consumer surplus become

$$\pi^D = \frac{n}{4} \cdot \left\{\psi_1 - \psi_3 + \psi_3 \left(1 - \frac{c}{\psi_3}\right)^2\right\}, \tag{20}$$

$$CS^D = 0.5\pi^D. \tag{21}$$

Therefore, $SW^D = 1.5\pi^D$.

Optimal booking strategy - Refundable versus non-refundable bookings

Both consumer types participate

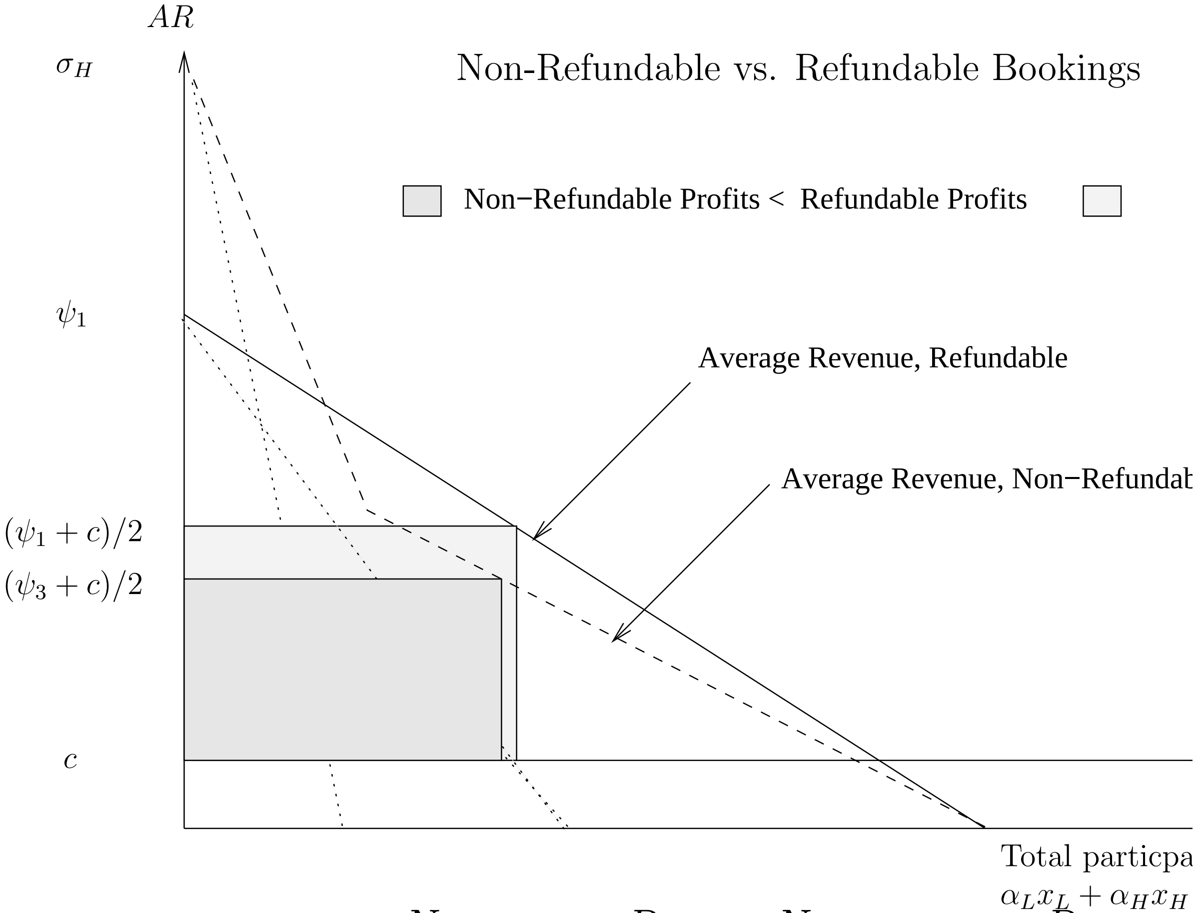

Figure 8: $\pi^{N_{\text{Both}}} < \pi^R$, $CS^{N_{\text{Both}}} > CS^R$, and $\alpha_L x_L^{N_{\text{Both}}} + \alpha_H x_H^{N_{\text{Both}}} < \alpha_L x_L^R + \alpha_H x_H^R$

Proposition 2. *When both consumer types participate*

a. Selling refundable tickets yields a higher profit than selling non-refundable tickets. Formally, $\pi^R > \pi^{N_{\mathrm{both}}}$.

b. Aggregate consumer surplus and social welfare is lower under the refundable booking strategy compared with the non-refundable booking strategy. Formally, $CS^R < CS^{N_{\mathrm{both}}}$ and $SW^R < SW^{N_{\mathrm{both}}}$.

c. In the limit, $\pi^R = \pi^{N_{\mathrm{both}}}$ and $CS^R = CS^{N_{\mathrm{both}}}$, when $\sigma_L = \sigma_H$.

$$\begin{aligned} x_H^N &= \frac{1}{2}\left(1-\frac{c}{\sigma_H}+1-\frac{\sigma_L}{\psi_2}\right) \\ &> \frac{1}{2}\left(1-\frac{c}{\sigma_H}\right) > \frac{1}{2}\left(1-\frac{c}{\psi_1}\right) = x_H^R = \\ x_L^R &> \frac{1}{2}\left(1-\frac{c}{\sigma_L}\right) \\ &> \frac{1}{2}\left(1-\frac{c}{\sigma_L}+1-\frac{\sigma_H}{\psi_2}\right) = x_L^N. \qquad (22) \end{aligned}$$

The population average of x_L^N and x_H^N is

$$\bar{x}_H^N \stackrel{\text{def}}{=} \alpha_H x_H^N + \alpha_L x_L^N = \frac{1}{2}\left(1-\frac{c}{\psi_3}\right) \leq \frac{1}{2}\left(1-\frac{c}{\psi_1}\right) = x^R. \qquad (23)$$

Proposition 3. *The participation of consumers with a high probability of cancellation is higher under the refundable booking strategy than under the non-refundable booking strategy. Also, the participation of consumers with a low probability of cancellation is lower under the refundable booking strategy than under the non-refundable booking strategy. The aggregate participation is higher under the refundable booking strategy than under the non-refundable booking strategy.*

Proposition 4. *Consumer surplus is lower under the refundable strategy although the aggregate consumer participation is higher under the refundable strategy.*

Proposition 4 is remarkable, a strategy which ends up in more reservations generates a reduction in the consumer surplus!

Only type H participates

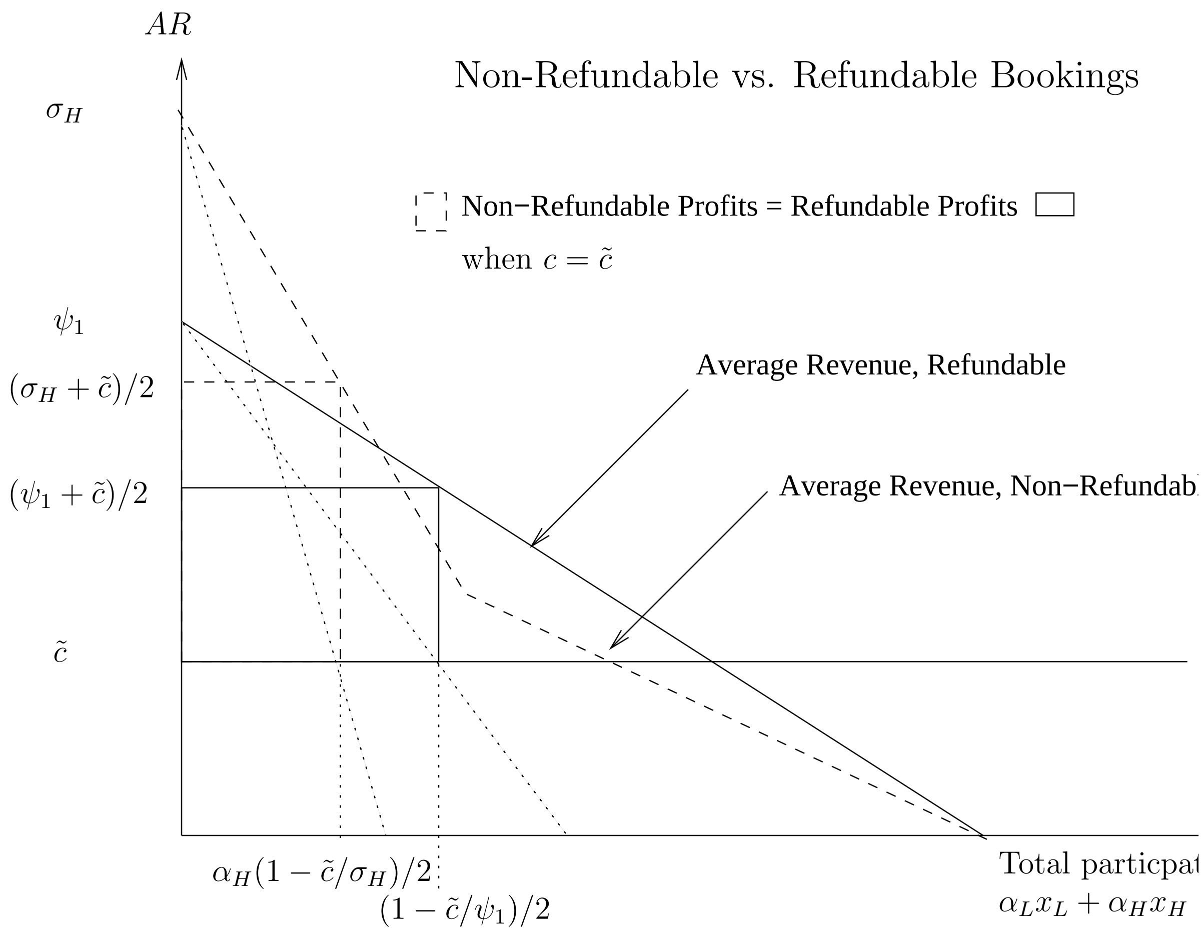

Figure 9: $\pi^{N_{\mathrm{H}}} < \pi^R$ when $c < \tilde{c}$ and $\pi^{N_{\mathrm{H}}} > \pi^R$ when $c > \tilde{c}$.

$$\tilde{c} \stackrel{\text{def}}{=} \arg_{\hat{c}<c<\sigma_L} \left\{\pi^R = \pi^{N_H}\right\} \qquad (24)$$

Proposition 5. *The refundable booking strategy is more profitable than the non-refundable strategy when only type H are served, if an only if the marginal cost is sufficiently low.*
Formally, $\pi^R \geq \pi^{N_H}$ if and only if $c \leq \tilde{c}$.

Propositions 2 and 5, imply that an optimal single price strategy has the following features.

Proposition 6. *If the seller adopts an optimal single price strategy, then if it adopts*

a. a non-refundable booking strategy, it serves only type H consumers.

b. a refundable booking strategy, it serves both type of customers.

Dual booking strategy vs. single booking strategies

Proposition 7. *Let* $\bar{x}^N$, $\bar{x}^{N_H}$, $\bar{x}^D$, *and* $x_L^R = x_H^R$, *be the equilibrium participation rates under non-refundable booking without exclusion, the non-refundable when type* L *are excluded, market segmentation, and fully-refundable booking strategy, respectively, given by (9), (19) and (23). Then,*

$$\bar{x}^{N_{Both}} = \bar{x}^D < x_L^R = x_H^R \quad \text{and} \quad \alpha_H \bar{x}^{N_H} < \bar{x}^D.$$

Proposition 7 hints that the refundable booking strategy may be associated with some inefficiency since it implies some over-participation of consumers with a high cancellation rate. The participation rates given in (17), (18) and (22) yield the following proposition.

Proposition 8. *Suppose that it is more profitable to offer refundable bookings than non-refundable bookings, meaning that $c < \tilde{c}$ by Proposition 5. Then,*

a. *Participation of consumers with low showing up probability is higher under the refundable strategy than under the dual price strategy. Formally, $x_L^R > x_L^D$; and*

b. *Participation of consumers with high showing up probability is lower under the refundable strategy compared with the dual price strategy. Formally, $x_H^R < x_H^D$.*

Proposition 9. *The dual price booking strategy (yielding market segmentation), generates a higher profit, a higher consumer surplus, and consequently a higher social welfare when compared with the single price strategies. Formally,*
$\pi^D > \max\left\{\pi^R, \pi^{N_H}\right\}$ *and*
$CS^D > \max\left\{CS^R, CS^{N_H}\right\}$*, and therefore,*
$SW^D > \max\left\{SW^R, SW^{N_H}\right\}$*.*

Clearly, the fact that market segmentation may lead to a welfare improvement is not novel (See Varian 1985). Here we demonstrate that the welfare improvement can be achieved via the refundability option which leads to a self-selection of the type of booking according to the probability of showing up.

Capacity Constraint

When the capacity constraint is binding the aggregate participation index in the population will always equal K/n. That is

$$\frac{K}{n} = \alpha_H x_H + \alpha_L x_L. \qquad (25)$$

It follows from (19) and (23) that the capacity is binding under all full participation strategies, only if $K/n < (1 - c/\psi_3)/2$.

The basic intuition of the introduction of a capacity constraint is given in Figures 10 – 13.

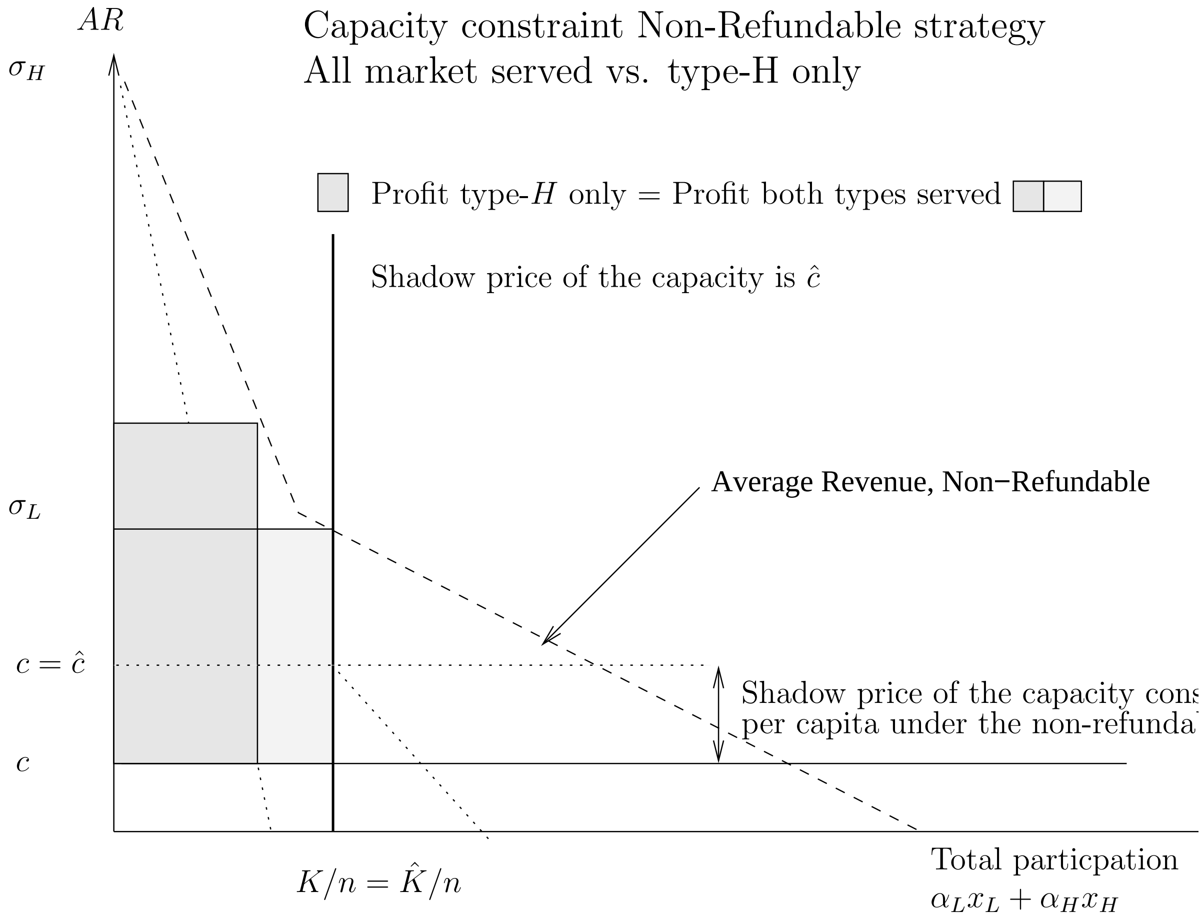

Figure 10: $\pi^{N_{\text{Both}}} = \pi^{N_H}$ for a "small capacity K", $K = \widehat{K}$. The alternative cost of the capacity constraint K is under the non-refundable strategy $\psi_3(1 - 2K/n) - c = \hat{c} - c$.

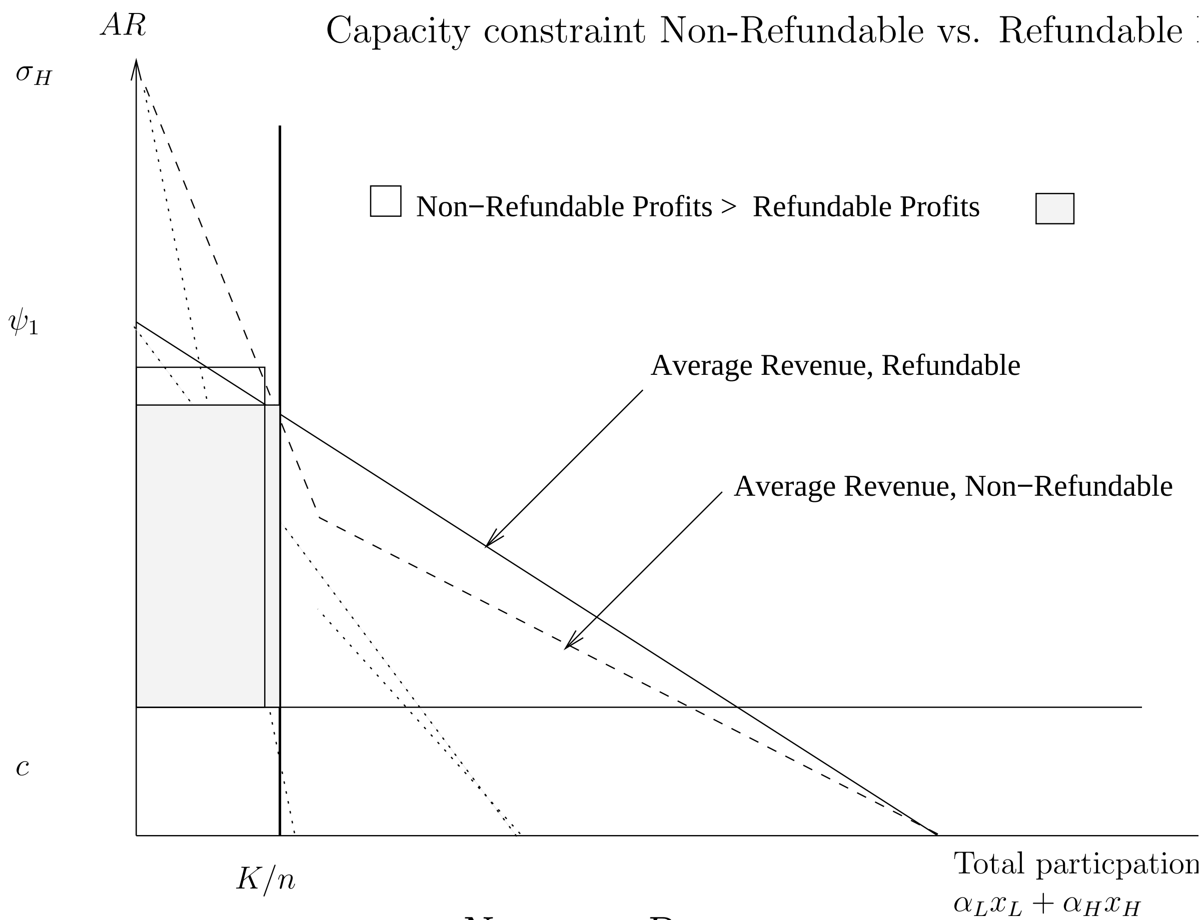

Figure 11: $\pi^{N_{\mathrm{H}}} > \pi^R$ for a "small capacity K" (or high c), $K < \tilde{K} < \widehat{K}$. Tickets are sold at premium price exclusively to the type-Hcustomers.

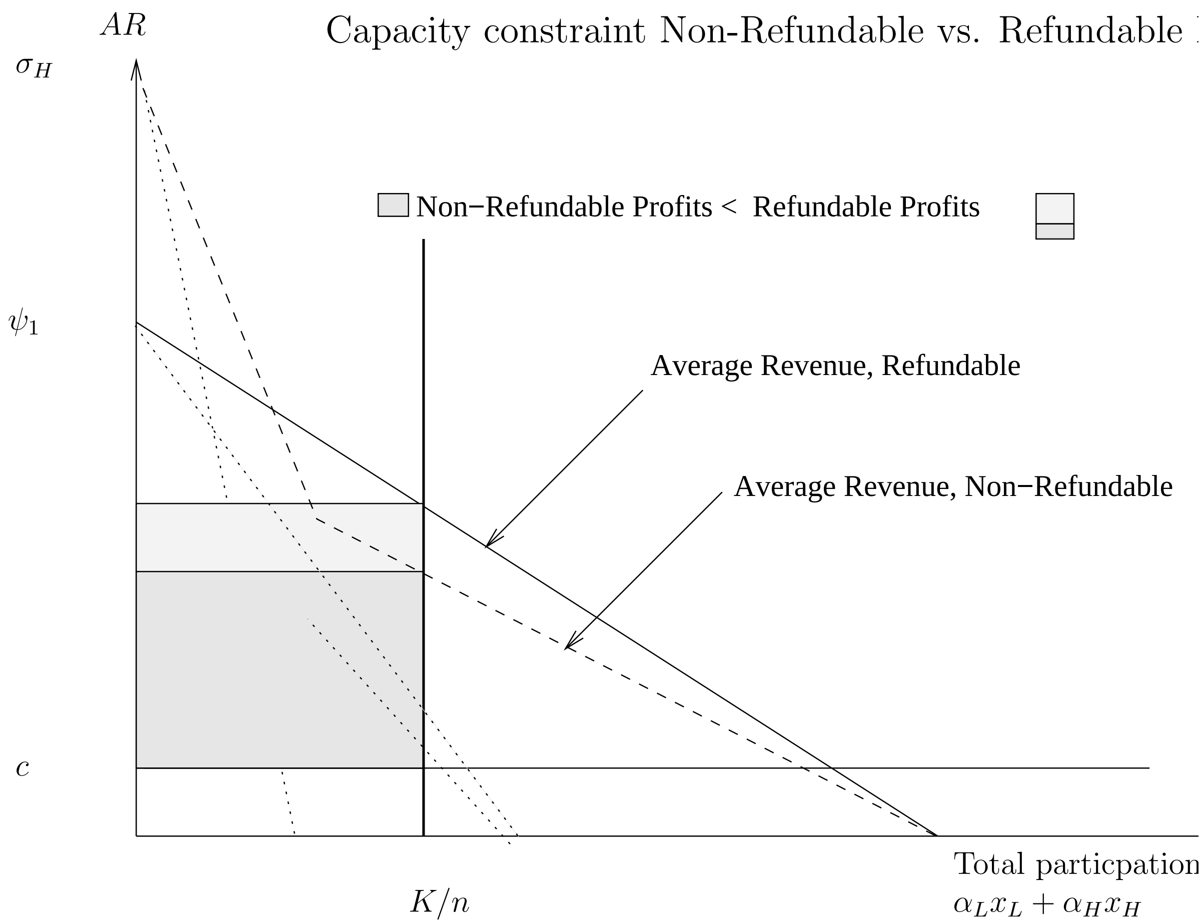

Figure 12: $\pi^{N_{\text{Both}}} < \pi^R$ and $SW^{N_{\text{Both}}} > SW^R$, when $c < \tilde{c}$ and K "not very restrictive". The alternative cost of the capacity constraint K, under the non-refundable strategy: $\psi_3(1-2K/n)-c \in [0, \hat{c}-c]$.

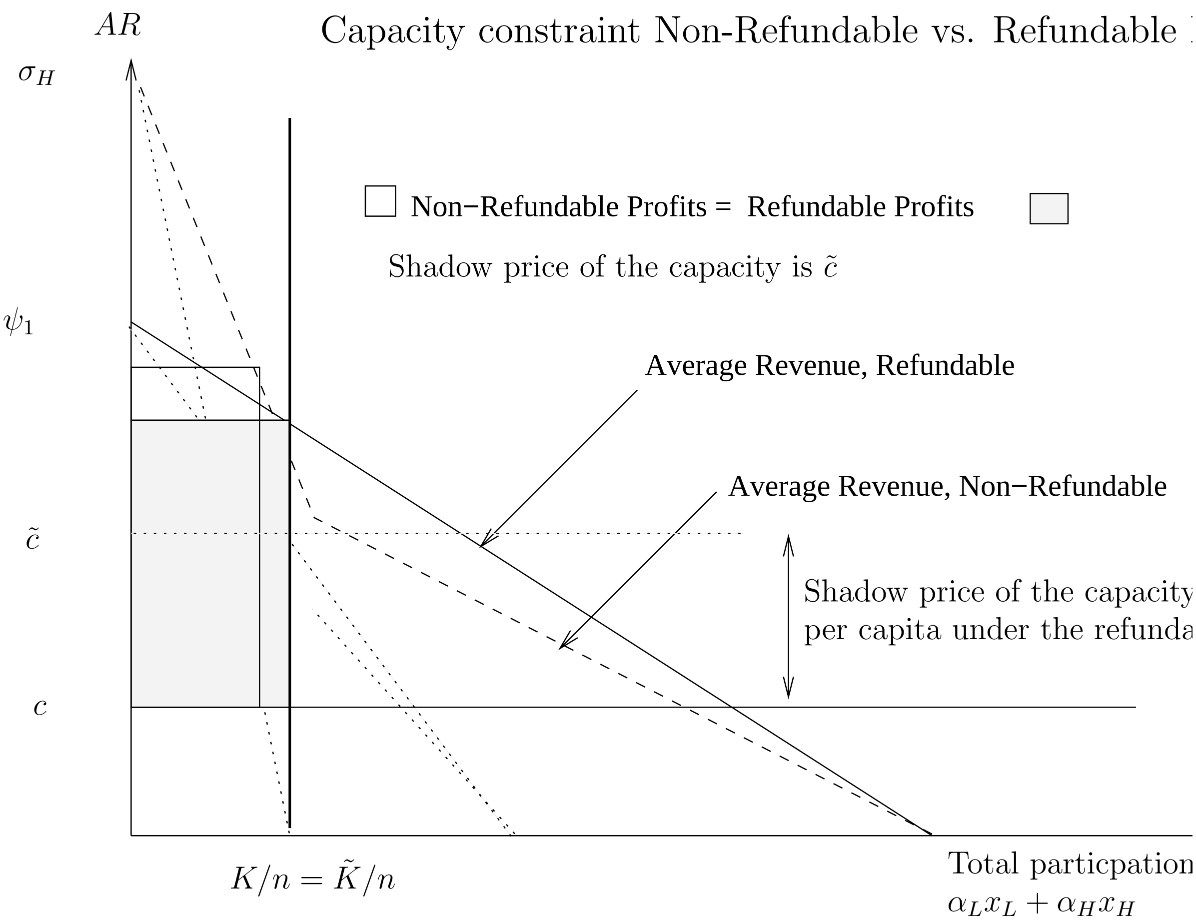

Figure 13: $\pi^{N_H} = \pi^R$ when $c < \tilde{c}$ and the capacity is "rather restrictive" $K = \tilde{K} < \widehat{K}$. The alternative cost of the capacity constraint K under the refundable strategy is $\psi_1(1 - 2K/n) - c = \tilde{c} - c$.

Refundable tickets under capacity constraint — algebra

Recall from (5) and (25) that the participation index for refundable tickets was $K/n = \alpha_H x_H^R + \alpha_L x_L^R = x_H^R = x_L^R = 1 - p^R$. Therefore,

$$p^R = 1 - \frac{K}{n}, \tag{26}$$

$$\pi^R = (\psi_1 p^R - c)K = \psi_1 \left(1 - \frac{K}{n}\right) K - cK, \tag{27}$$

$$CS^R = \frac{n\psi_1}{2} \left(\frac{K}{n}\right)^2, \tag{28}$$

$$SW^R = \frac{n}{2} \cdot \left\{ 2\psi_1 \frac{K}{n} - \psi_1 \left(\frac{K}{n}\right)^2 \right\} - cK, \text{ and} \tag{29}$$

$$\frac{\partial \pi^R}{\partial K} = \psi_1 \left(1 - \frac{2K}{n}\right) - c. \tag{30}$$

Non-refundable tickets under capacity constraint — algebra

Both types served:

$$\frac{K}{n} = \alpha_H \left(1 - \frac{p^N}{\sigma_H}\right) + \alpha_L \left(1 - \frac{p^N}{\sigma_L}\right). \qquad (31)$$

Solving (31) for p^N yields

$$p^N = \frac{\sigma_H \sigma_L}{\psi_2} \left(1 - \frac{K}{n}\right) = \psi_3 \left(1 - \frac{K}{n}\right). \qquad (32)$$

We observe after some algebra that

$$\pi^{N_{\text{both}}} = (p^N - c)K = \psi_3 \left(1 - \frac{K}{n}\right) K - cK, \qquad (33)$$

$$CS^{N_{\text{both}}} = \frac{n}{2} \cdot \left\{ \psi_1 - \psi_3 + \psi_3 \left(\frac{K}{n}\right)^2 \right\}, \text{ and} \qquad (34)$$

$$\frac{\partial \pi^{N_{\text{Both}}}}{\partial K} = \psi_3 \left(1 - \frac{2K}{n}\right) K - c. \qquad (35)$$

Dual price strategy under capacity constraint

The Lagrangian associated with the profit maximization problem is

$$
\begin{aligned}
L &= \alpha_H(p^{N'}-c)\left(1-\frac{p^{N'}}{\sigma_H}\right) \\
&+ \alpha_L\left(\sigma_L p^{R'}-c\right)\left(1-p^{R'}\right) \\
&+ \lambda\left(\frac{K}{n}-\alpha_H\left(1-\frac{p^{N'}}{\sigma_H}\right)-\alpha_L\left(1-p^{R'}\right)\right)
\end{aligned} \tag{36}
$$

The profit-maximizing prices are:

$$
\begin{aligned}
p^{R'} &= \frac{1}{2\sigma_L}\left\{\sigma_L+\psi_3\left(1-2\frac{K}{n}\right)\right\}, \text{ and} \\
p^{N'} &= \frac{1}{2}\left\{\sigma_H+\psi_3\left(1-2\frac{K}{n}\right)\right\}.
\end{aligned} \tag{37}
$$

The corresponding participation indexes are

$$
\begin{aligned}
x_L^D &= 1 - p^{R'} = \frac{1}{2}\left\{1 - \frac{\psi_3}{\sigma_L}\left(1 - 2\frac{K}{n}\right)\right\}, \text{ and} \\
x_H^D &= 1 - \frac{p^{N'}}{\sigma_H} = \frac{1}{2}\left\{1 - \frac{\psi_3}{\sigma_H}\left(1 - 2\frac{K}{n}\right)\right\}. \quad (38)
\end{aligned}
$$

We obtain the following profit-maximum

$$
\begin{aligned}
\pi^D &= \frac{n}{4}\cdot\left\{\psi_1 - \psi_3\left(1 - 2\frac{K}{n}\right)^2\right\} - cK \\
&= n\frac{\psi_1 - \psi_3}{4} + \psi_3\left(1 - \frac{K}{n}\right)K - cK. \quad (39)
\end{aligned}
$$

The consumer surplus is

$$
CS^D = n\frac{\psi_1 - \psi_3}{8} + \frac{\psi_3}{2}\left(\frac{K}{n}\right)^2, \quad (40)
$$

and finally the shadow price of K

$$
\frac{\partial \pi^D}{\partial K} = \psi_3\left(1 - \frac{2K}{n}\right)K - c. \quad (41)
$$

Profit-maximizing booking strategy under capacity constraint

When $c < \hat{c}$ we define a threshold capacities $\widehat{K}$ and $\widetilde{K}$ as:

$$\widehat{K} \stackrel{\text{def}}{=} \arg_{K>0} \left\{ \psi_3(1 - K/n)K - cK = \pi^{N_H} \right\}, \quad (42)$$

$$\widetilde{K} \stackrel{\text{def}}{=} \arg_{K>0} \left\{ \psi_1(1 - K/n)K - cK = \pi^{N_H} \right\}. \quad (43)$$

Because, $\psi_1 > \psi_3$, we can conclude that $\widetilde{K} < \widehat{K}$.

We summarize the implications of introducing any exogenously given fixed capacity constraint in the following two propositions

Proposition 10. *Suppose that both consumer groups participate under all booking strategies, i.e.* $K > \widehat{K}$, *then rankings of profits, consumer surpluses and social welfares are*

a. $\pi^D > \pi^R > \pi^{N_{\text{both}}}$,

b. $CS^R < CS^D < CS^{N_{\text{both}}}$, *and*

c. $SW^R < SW^D < SW^{N_{\text{both}}}$.

We observe that **a more restricted capacity K has similar effects to those of a higher marginal cost c.**

Discussion

- Refundable bookings to increase the surplus extracted from consumers. If the customer group preferring a refundability option is large, and the marginal cost is not too high, a fully refundable strategy will outperform a non-refundable strategy.

- Selling both refundable and non-refundable tickets leads to a separation of consumers with high showing up probability from consumers with a low showing up probability, and this this strategy is desired from a social welfare consideration, compared with a single booking strategy. This result is robust with respect to any non-trivial capacity constraint.

- Open questions: several industries do not utilize multiple refundability options on their advanced booking systems.

- Extension of the model. Allow for
  - overbooking,
  - competition.

Appendix

Consumer surplus

We define aggregate consumer surplus as the sum of utilities. Firstly, observe that

$$\begin{aligned}\alpha_i \int_0^{x_i} \sigma_i(1-x-p)dx &= \frac{\alpha_i \sigma_i x_i^2}{2} \quad \text{and} \\ \alpha_i \int_0^{x_i} \left(\sigma_i(1-x)-p\right)dx &= \frac{\alpha_i \sigma_i x_i^2}{2}, \qquad (44)\end{aligned}$$

for the refundable and non-refundable cases

, respectively (for $i = H, L$). The integration has eliminated the price since from (1), $1 - x_i - p = 0$ in the refundable case, and $\sigma_i(1-x_i) - p = 0$ in the non-refundable case. Consequently, the consumer surplus is always n times a linear combination of the expressions in (44) which becomes

$$CS = \frac{n}{2}\left(\alpha_H \sigma_H x_H^2 + \alpha_L \sigma_L x_L^2\right). \qquad (45)$$

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