Advance Booking and Refunds Under Capacity Constraints

Staffan Ringbom*
Helsinki Center for Economic Research, Swedish School of Economics

and

Oz Shy†‡
University of Haifa and Helsinki Center for Economics Research,
Swedish School of Economics

11th April 2004

Abstract

We investigate how the presence of capacity constraints affects the advance-booking strategy of a service provider. In particular, we identify the capacity levels under which fully refundable bookings are more (less) profitable than non-refundable bookings. As in the unconstrained capacity models, we find that if the capacity is sufficiently high (low) a full-refund booking strategy is more (less) profitable than a non-refundable booking strategy. For very restrictive capacity levels, the firm will never utilize a fully refundable booking. Otherwise, a dual booking strategy that segments the market in the refundability dimension is profit maximizing.

Keywords: Advance booking, capacity constraints, reservations, refund, non-refundable tickets

JEL Classification Numbers: M2, L1

* *Corresponding author:* HECER, Department of Economics, Swedish School of Economics, P.O. Box 479, 00101 Helsinki, Finland. E-mail: staffan.ringbom@shh.fi

† Department of Economics, University of Haifa, 31905 Haifa, Israel. E-mail: ozshy@econ.haifa.ac.il

‡ We thank participants of the Economics Research Seminar in Helsinki in September 2003 for valuable comments.

1 Introduction

Consumers tend to value advance reservation systems since they guarantee service at the time of delivery. Obviously, if service providers possess an unlimited capacity, service guarantees would not be needed. In such cases advance booking would not be valued by consumers, and hence would not be utilized. Surprisingly, the economics literature on advance booking is largely based on unlimited capacity

The purpose of our paper is to introduce capacity constraints into an advance booking model in order to investigate how capacity constraints affect the advance-booking strategy of a service provider. In particular, we identify the capacity levels under which fully-refundable bookings are more (less) profitable than non-refundable bookings. In addition we provide a complete welfare analysis showing how consumer welfare and social welfare are affected by the various booking strategies under different capacity levels.

Theories of Industrial Organization tend to associate the time of purchase with the time of delivery of goods and services. In practice, there are many markets for services and goods where buyers and sellers maintain contacts long before the service or the good are scheduled to be delivered. This pre-delivery contact is usually called *advance booking*, or simply a *reservation*. Since advance booking and reservation systems are observed in almost all privately-provided services, both buyers and sellers may find them beneficial. In the case of capacity constraints, advance booking helps the seller in planning how much capacity to produce (such as aircraft size, and frequency of flights, in the case of the airline industry). However, advance booking which is not supported by a prepayment may also have an adverse effect on sellers since customers may not show up to buy the service or the good, thereby leaving the seller with underutilized capacity.

For buyers, advance booking is needed to reduce the cost of obtaining complementary goods and services. For example, if one makes travel reservations, the traveler must take time off from work and purchase other items and services for the trip. Thus, travelers will not be able to take time off from work unless they can be assured service. For this reason, car rental companies give their potential customers a *de facto* free option to rent a car.

Advance bookings are not uniform across sectors. Perhaps, the key difference lies on the issue of prepayments or refundability. More precisely, a refundable booking insures consumers against their own no shows, at the expense of service providers. Therefore, refundable (prepaid) bookings must result in higher prices than non-refundable bookings (or prepaid ones). Like any other adverse selection problem, refundable bookings tend to attract consumers who are likely to cancel or not show up for the service, and deter consumers who are less likely to cancel and are therefore more price sensitive.

Advance booking mechanisms received a limited attention in the economic literature. Gale and Holmes (1992, 1993) compare monopoly pricing with social optimum. In their analysis, the price mechanism is used to shift the demand from the peak period to the off-peak period. Gale (1993) analyzes consumers who learn their preferences after they are offered an advance purchase option. On this line, Miravete (1996) and more recently Courty and Li (2000) further investigate how consumers who learn their valuation over time can be screened via the introduction of refunds in their advance booking mechanism. Courty (2003) investigates resell and rationing strategies of a monopoly that can sell early to uninformed consumers or late to informed consumers. Dana (1998) also investigates market segmentation under advance booking of price-taking firms. There also exist a literature dealing with supply-rationing under demand uncertainty in revenue management which is broadly related to this paper (see McGill and van Ryzin 1999 for a survey).

The paper is organized as follows. Section 2 constructs the basic advance booking model. We define the mechanism through which the refundability option becomes a major consideration for consumers. Section 3 characterizes the profit maximizing booking strategies under capacity constraints. Our main results are brought in Section 4 which analyzes how varying the capacity constraint affects the choice of the profit maximizing booking strategy. Welfare consequences are also derived. Section 5 concludes.

2 The Model

Consider a single seller of a certain service or a good to be provided at a certain known date. The seller is able to offer two types of advance booking: A non-refundable ticket for a price of p^N, or a fully-refundable ticket at a price p^R. The production of this good/service exhibits constant returns to scale, with a marginal cost denoted by $c > 0$. Let K denote the capacity constraint of the service provider. In other words, K is the maximum number of consumers who can be provided with service. We assume that the capacity is exogenously given and therefore cannot be changed.

Each consumer buys one unit of this service. There are two groups of consumers, indexed by $i = H, L$, who differ according to their probability of showing up to collect this good or service. Type H consumers have a "high" showing up probability of σ_H (i.e., will cancel the reservation with probability $(1 - \sigma_H)$); whereas type L have a "low" showing up probability given by σ_L. We assume that $0 < \sigma_L < \sigma_H \leq 1$. There is a total of n customers in the population. $\alpha_H n$ are of type H, and $\alpha_L n = (1 - \alpha_H)n$ are of type L, where $0 < \alpha_H < 1$. The fractions α_H and α_L and the showing up probabilities σ_H and σ_L are exogenous parameters, and they are not affected by the booking strategy chosen by the seller.

Within each group, consumers are indexed by x ($0 \leq x \leq 1$) according to their declining willingness to pay for this service. A consumer indexed by $x = 0$ has the highest willingness to pay. Consumers indexed by $x = 1$ have the lowest willingness to pay. Formally, the utility of a type i consumer indexed by x, $i = H, L$, is given by

$$U_i(x) = \begin{cases} \sigma_i \left(1 - x - p^R\right) & \text{if buys a refundable ticket} \\ \sigma_i (1 - x) - p^N & \text{if buys a non-refundable ticket} \\ 0 & \text{if does not buy this good/service.} \end{cases} \tag{1}$$

The utility functions (1) imply that a consumer indexed by x derives a basic utility of $1 - x$ if he shows up (with probability σ_i). If the ticket is refundable he pays only if he does not cancel, thus with probability σ_i. In contrast, if the ticket is nonrefundable, the ticket is paid regardless of whether the consumer shows up.

Without loss of generality we have normalized the price and quantity in our utility functions (1) such that consumers' maximum willingness to pay for the ticket is $(1 - x)$ if the ticket is refundable, and $\sigma_i (1 - x)$ if the ticket is non-refundable. The following assumption is needed in order to avoid an immediate exclusion of all type L consumers.

Assumption 1. There are some type L consumer with willingness to pay a price exceeding the service's marginal cost. Formally, $\sigma_L > c$.

For the sake of reducing the amount of writing, we define the following constants.

$$\psi_1 \stackrel{\text{def}}{=} \alpha_H \sigma_H + \alpha_L \sigma_L = \alpha_H \sigma_H + (1 - \alpha_H) \sigma_L, \tag{2}$$

$$\psi_2 \stackrel{\text{def}}{=} \frac{\sigma_H \sigma_L}{\alpha_H \sigma_L + \alpha_L \sigma_H}. \tag{3}$$

The constant ψ_1 is the aggregate showing up probability for the entire population. Obviously, $\sigma_L < \psi_i < \sigma_H$ for $i = 1, 2$. We will frequently use the following lemma.

Lemma 1. $\psi_1 > \psi_2$.

Proof. $(\psi_1 - \psi_2)(\alpha_H \sigma_L + \alpha_L \sigma_H) = \alpha_H \alpha_L (\sigma_H - \sigma_L)^2 > 0$. □

3 Equilibrium Prices, Profits, and Welfare

When the capacity constraint is binding, the aggregate participation index will always equal K/n. That is,[1]

$$\frac{K}{n} = \alpha_H x_H + \alpha_L x_L. \tag{4}$$

3.1 Fully refundable tickets

Suppose that the seller's advance booking strategy is to offer only refundable tickets for the price of p^R. Let x_H denote the index of a type H consumer, who is indifferent between making and not making a reservation. x_L is similarly defined. That is, all consumers indexed by $x \le x_i$ make reservations, whereas all consumers $x > x_i$ do not book the service. From (1), x_i is implicitly defined by $\sigma_i(1 - x_i - p^R) = 0$, for $i = H, L$. Hence,

$$x_H = x_L = 1 - p^R. \tag{5}$$

In order to keep our qualitative statments independent of linear tranformation of the the underlying population size and capacity, we are ruling out overbooking here. Thus, the seller must produce $K = n(\alpha_H x_H + \alpha_L x_L)$ units. Therefore, by (5) the participation index for refundable tickets are $K/n = \alpha_H x_H^R + \alpha_L x_L^R = x_H^R = x_L^R = 1 - p^R$. Hence,

$$p^R = 1 - \frac{K}{n}. \tag{6}$$

However, since tickets are fully-refundable, the revenue is affected by the showing up probabilities. As ψ_1 is the average showing up probability in the population and the participations in both groups are the same, the total revenue is the average revenue $\psi_1 p^R$ times the produced capacity K. Therefore, the profit function is

$$\pi^R = (\psi_1 p^R - c)K = \psi_1 \left(1 - \frac{K}{n}\right) K - cK. \tag{7}$$

We define aggregate consumer surplus as the sum of utilities. Firstly, observe that

[1] It can be shown that capacity is binding if $K/n < (1 - c/\psi_2)/2$. Therefore, there is a tradeoff between the level of a binding capacity K and the marginal cost of providing the service c. When c is high the binding capacity K is lower.

$$\alpha_i \int_0^{x_i} \sigma_i(1-x-p)dx = \frac{\alpha_i \sigma_i x_i^2}{2} \quad \text{and} \quad \alpha_i \int_0^{x_i} \left(\sigma_i(1-x)-p\right)dx = \frac{\alpha_i \sigma_i x_i^2}{2}, \tag{8}$$

for the refundable and non-refundable cases, respectively (for $i = H, L$). The integration has eliminated the price since from (1), $1 - x_i - p = 0$ in the refundable case, and $\sigma_i(1 - x_i) - p = 0$ in the non-refundable case. Consequently, the consumer surplus is always n times a linear combination of the expressions in (8) which becomes

$$CS = \frac{n}{2}\left(\alpha_H \sigma_H x_H^2 + \alpha_L \sigma_L x_L^2\right). \tag{9}$$

Substituting $x_H^R = x_L^R = K/n$ into (9) yields

$$CS^R = \frac{n\psi_1}{2}\left(\frac{K}{n}\right)^2. \tag{10}$$

Finally, the social welfare is defined as the sum of the firm's profit (7) and the consumer surplus (10). Hence,

$$SW^R = CS^R + \pi^R = \frac{n\psi_1}{2}\left(\frac{K}{n}\right)^2 + \psi_1\left(1 - \frac{K}{n}\right)K - cK. \tag{11}$$

3.2 Non-refundable tickets

We now analyze the case where advance booking implies a commitment to pay a ticket price of p^N regardless of whether the customer cancels the reservation or not. In practice, consumers prepay for the service and do not obtain any refund for no shows or cancellations. The utility function (1) implies that a consumer x_i^N who is indifferent between buying and not buying must satisfy $\sigma_i\left(1 - x_i^N\right) - p^N = 0$, for $i = H, L$ (if $x_i > 0$). Therefore, the participation rates will be higher for customers with higher showing up probabilities. Obviously, there will exist two profit maximizing candidates. That is, either the service is targeted toward both consumer categories or toward type-H only. Interestingly, there might exist some capacity levels where it might be profitable to have slack capacity rather than lowering the price in order to up the slack with low paying customers of type-L.

The utility function (1) implies that the particpation indexes are implicitly solved from $\sigma_H(1 - x_H) - p^N = 0$, and $\sigma_L(1 - x_L) - p^N = 0$, and therefore are given by

$$x_H^N = 1 - \frac{p^N}{\sigma_H} \quad \text{and} \quad x_L^N = 1 - \frac{p^N}{\sigma_L}. \tag{12}$$

As in the refundable case, the price is lowered until the demand matches the capacity constraint (4). Hence, when both consumer types participate we have

$$\frac{K}{n} = \alpha_H \left(1 - \frac{p^N}{\sigma_H}\right) + \alpha_L \left(1 - \frac{p^N}{\sigma_L}\right). \tag{13}$$

Solving (13) for p^N yields

$$p^N = \psi_2 \left(1 - \frac{K}{n}\right). \tag{14}$$

Under this price, the expected profit of the seller is given by

$$\pi^{N_{\text{both}}} = (p^N - c)K = \psi_2 \left(1 - \frac{K}{n}\right) K - cK. \tag{15}$$

The participation indexes for the groups L and H are obtained by substituting (14) into (12) to obtain

$$x_i^N \;=\; 1 - \frac{p^N}{\sigma_i} = 1 - \frac{\psi_2}{\sigma_i}\left(1 - \frac{K}{n}\right) \quad i = H, L. \tag{16}$$

Substituting the above participation indexes into the general expression for the consumer surplus (9) we obtain

$$CS^{N_{\text{both}}} = \frac{n}{2} \cdot \left[\psi_1 - \psi_2 + \psi_2 \left(\frac{K}{n}\right)^2\right]. \tag{17}$$

Summing the profit (15) and the consumer surplus (17) yields

$$SW^{N_{\text{both}}} = CS^{N_{\text{both}}} + \pi^{N_{\text{both}}} = \frac{n}{2} \cdot \left[\psi_1 - \psi_2 + 2\psi_2 \frac{K}{n} - \psi_2 \left(\frac{K}{n}\right)^2\right] - cK. \tag{18}$$

So far, we have not ruled out the possibility, that the capacity restriction is binding if the tickets are sold to the entire population, but non-binding under a price where only customers of type-H are served. Due to the fact that the demand is kinked, the profit function is also kinked. The behavior of the profit function with respect to the available capacity is illustrated in Figure 1. We observe from Figure 1 that the capacity constraint is binding for very small capacity levels, even though only the customers of type-H are served. The capacity is binding until it matches the monopoly production quantity for the H-type,

$$\overline{K} \stackrel{\text{def}}{=} \frac{n\alpha_H}{2}\left(1 - \frac{c}{\sigma_H}\right). \tag{19}$$

For low capacities where $K < \overline{K}$, the market price is solved from (13), that is $p^{N_H} = \sigma_H(1 - K/(n\alpha_H))$. Thus, the corresponding profit is $\pi^{N_H} = [\sigma_H(1 - K/(n\alpha_H)) - c] \cdot K$.

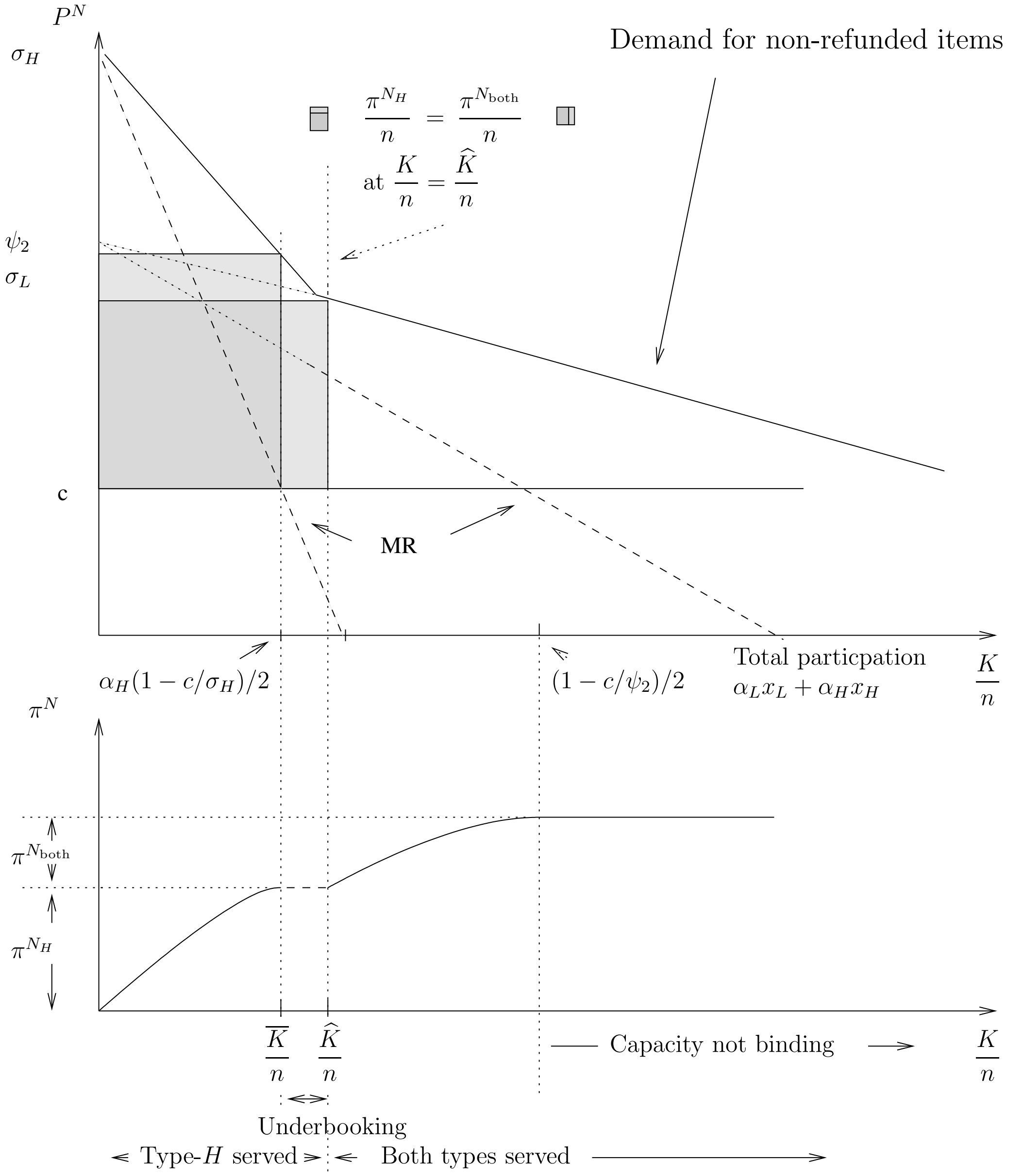

Figure 1: The profit under a non-refundable booking strategy with capacity constraints

The capacity levels $\overline{K} < K < \widehat{K}$, define a region where profit is maximized when the service provider books exactly $\overline{K}$ customers. As shown in Figure 1, booking at full capacity in this region does not enhance profits. The threshold value, $\widehat{K}$, is

$$\widehat{K} \stackrel{\text{def}}{=} \arg\min_{K>0} \left\{ \left[\sigma_H \left(1 - \frac{\overline{K}}{n\alpha_H} \right) - c \right] \cdot \overline{K} = \left[\psi_2 \left(1 - \frac{K}{n} \right) - c \right] \cdot K \right\} \tag{20}$$

To summarize our analysis of non-refundable bookings for restricted capacity when type-L consumers are excluded, $K < \widehat{K}$, the profit function and the consumer surplus are given by

$$\pi^{N_H} = \left[\sigma_H \left(1 - \frac{\min\{K, \overline{K}\}}{n\alpha_H} \right) - c \right] \cdot \min\{K, \overline{K}\}, \text{ and} \tag{21}$$

$$CS^{N_H} = n \frac{\sigma_H}{2\alpha_H} \left(\frac{\min\{K, \overline{K}\}}{n} \right)^2. \tag{22}$$

3.3 Dual price strategy

We now enlarge the seller's action set by allowing the sale of, both, refundable and non-refundable tickets. Thus, the seller sets a non-refundable booking price of $p^{N'}$ to target consumers with a high probability of showing up, and a fully-refundable price of $p^{R'}$ to target consumers with a low probability of showing up. Hence, the seller chooses $p^{N'}$ and $p^{R'}$ to solve

$$\begin{aligned} \frac{\pi^D}{n} &= \alpha_H \left(p^{N'} - c \right) x_H + \alpha_L \left(\sigma_L p^{R'} - c \right) x_L = \\ &= \alpha_H \left(p^{N'} - c \right) \left(1 - \frac{p^{N'}}{\sigma_H} \right) + \alpha_L \left(\sigma_L p^{R'} - c \right) \left(1 - p^{R'} \right), \end{aligned} \tag{23}$$

subject to

$$\begin{aligned} \frac{K}{n} &= \alpha_H \left(1 - \frac{p^{N'}}{\sigma_H} \right) + \alpha_L \left(1 - p^{R'} \right) \\ \frac{p^{N'}}{p^{R'}} &\in [\sigma_L, \sigma_H], \end{aligned}$$

where x_H was substituted from (12), and x_L from (5). The profit function is a negative definite quadratic form in $p^{R'}$ and $p^{N'}$, and the constraint is linear. The profit maximization problem is next rewritten as the Lagrangian

$$\begin{aligned} L\left(\lambda, p^{N'}, p^{R'}\right) &= \alpha_H (p^{N'} - c) \left(1 - \frac{p^{N'}}{\sigma_H} \right) + \alpha_L \left(\sigma_L p^{R'} - c \right) \left(1 - p^{R'} \right) \\ &\quad + \lambda \left(\frac{K}{n} - \alpha_H \left(1 - \frac{p^{N'}}{\sigma_H} \right) - \alpha_L \left(1 - p^{R'} \right) \right). \end{aligned} \tag{24}$$

Since the objective function is concave, and the constraint is linear, the necessary optimality conditions for the Lagrangian (24), are also sufficient, as long as both types participate. The F.O.C:s are:

$$\begin{aligned}\frac{\sigma_H}{\alpha_H}\cdot\frac{\partial L}{\partial p^{N'}} &= \sigma_H - 2p^{N'} + c + \lambda = 0, &(25)\\ \frac{1}{\alpha_L}\cdot\frac{\partial L}{\partial p^{R'}} &= \sigma_L - 2\sigma_L p^{R'} + c + \lambda = 0, \text{ and} &(26)\\ \frac{\partial L}{\partial \lambda} &= \frac{K}{n} - \alpha_H\left(\sigma_H - p^{N'}\right)/\sigma_H - \alpha_L(1 - p^{R'}) = 0. &(27)\end{aligned}$$

From (25) and (26) we verify that the solution implies that both types participate: $\sigma_L p^{R'} = (\sigma_L + c + \lambda)/2 \leq p^{N'} = (\sigma_H + c + \lambda)/2 \leq \sigma_H p^{R'}$. Next, we substitute these optimal prices into (27) and solve for $c + \lambda$. $(c + \lambda) = \psi_2 (1 - 2K/n) > 0$. Therefore, the profit-maximizing prices are:

$$\begin{aligned}p^{R'} &= \frac{1}{2\sigma_L}\left[\sigma_L + \psi_2\left(1 - 2\frac{K}{n}\right)\right], \text{ and}\\ p^{N'} &= \frac{1}{2}\left[\sigma_H + \psi_2\left(1 - 2\frac{K}{n}\right)\right]. &(28)\end{aligned}$$

Substitution of these prices, into the expressions for the corresponding participation indexes (5) and (12) imply

$$\begin{aligned}x_L^D &= 1 - p^{R'} = \frac{1}{2\sigma_L}\left[\sigma_L - \psi_2\left(1 - 2\frac{K}{n}\right)\right], \text{ and}\\ x_H^D &= 1 - \frac{p^{N'}}{\sigma_H} = \frac{1}{2\sigma_H}\left[\sigma_H - \psi_2\left(1 - 2\frac{K}{n}\right)\right], &(29)\end{aligned}$$

with the corresponding profit-maximum

$$\pi^D = n\frac{\psi_1 - \psi_2}{4} + \psi_2\left(1 - \frac{K}{n}\right)K - cK. \quad (30)$$

Substituting of the participation rates (29) into (9) yields

$$CS^D = n\cdot\left[\frac{\psi_1 - \psi_2}{8} + \frac{\psi_2}{2}\left(\frac{K}{n}\right)^2\right]. \quad (31)$$

Finally, the social welfare is given by

$$SW^D = CS^D + \pi^D = \frac{n}{2}\cdot\left[\frac{3}{4}(\psi_1 - \psi_2) + 2\psi_2\frac{K}{n} - \psi_2\left(\frac{K}{n}\right)^2\right] - cK. \quad (32)$$

4 Profit-maximizing Booking Strategy

We now turn to the main results of the paper by showing how capacity constraints affect the profit as well as the welfare maximizing booking strategies. Define the threshold capacity level $\widetilde{K}$ by

$$\widetilde{K} \stackrel{\text{def}}{=} \arg\min_{K>0} \left\{\pi^R = \pi^{N_H}\right\}, \quad \text{and} \quad \underline{K} = \frac{n\alpha_H}{2}\left(1 - \frac{\sigma_L}{\sigma_H}\right) \tag{33}$$

where π^R and π^{N_H} are given by (7) and (21). If $K < (>)\widetilde{K}$ then it it more (less) profitable to serve the customers of type-H only under a non-refundable strategy than it is to use the refundable strategy. It follows from Lemma 1 that $\widetilde{K} < \widehat{K}$. If $K < \underline{K}$, then the entire capacity can be sold to the consumers of type-H with a marginal revenue above σ_L. Therefore, for heavily restricted capacity levels satisfying $K < \underline{K}$ the problem is reduced to a standard monopoly of selling to type-H only. Thus, in what follows we confine our analysis to $K > \underline{K}$. For the remainder of the paper we make use of the following terminology, see also Figure 2.

Definition 1. Let $\widehat{K}, \widetilde{K}$ and $\underline{K}$ be given in (20) and (33). We say that the service provider possesses

a. a *large* capacity if $K > \widehat{K}$,

b. a *medium* capacity if $\widetilde{K} < K < \widehat{K}$, and

c. a *small* capacity if $\underline{K} < K < \widetilde{K}$.

The next four propositions, proved in Appendix A, demonstrate how capacity constraints affect the profit maximizing booking strategies and consumer welfare.

Proposition 1. *Suppose that capacity is large, i.e. $K > \widehat{K}$, so that both consumer types participate under all booking strategies. The rankings of profits, consumer surplus and social welfare are as follows:*

a. $\pi^D > \pi^R > \pi^{N_{\text{both}}}$,

b. $CS^{N_{\text{both}}} > CS^D > CS^R$, *and*

c. $SW^{N_{\text{both}}} > SW^D > SW^R$.

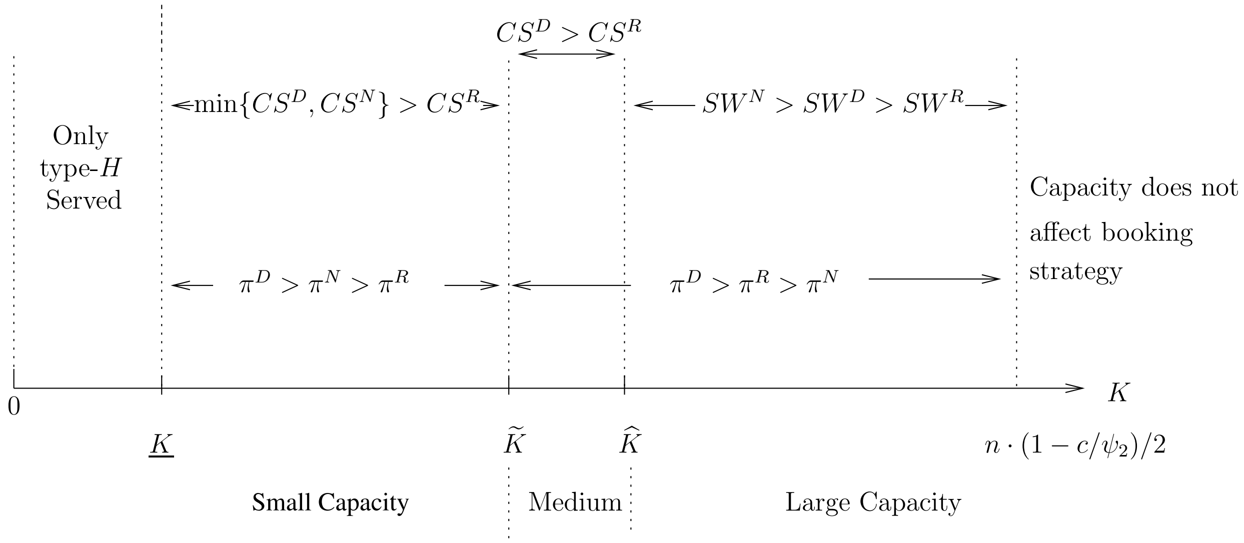

Figure 2: Capacity constraints and optimal booking strategies

The rankings listed in Proposition 1 are the same as if the capacity would be unlimited. When capacity is unlimited a refundable booking strategy is more profitable than a non-refundable booking strategy. In addition, consumers are worse off when offered refundable bookings. More importantly, social welfare is lowest under the fully refundable strategy, and highest under the non-refundable strategy. This is because under the fully refundable booking strategy "too many" customers of type-L or "too few" customers of type-H are served.

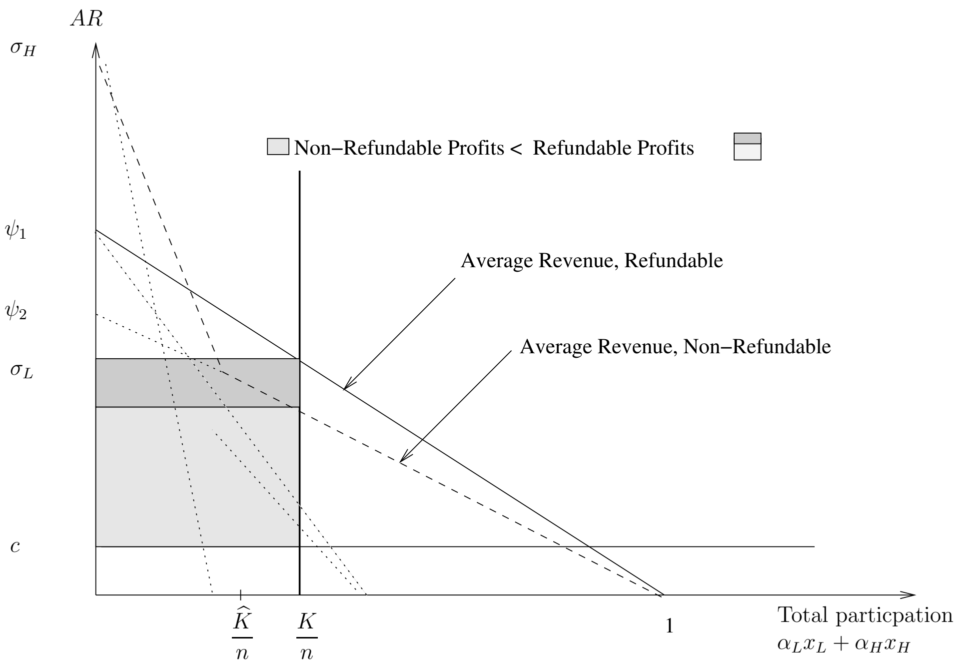

Figure 3: Profit comparison under large capacity.

The shaded areas in Figure 3 illustrates the profit comparisons made in Proposition 1. Recall that ψ_1 is the average participation rate from the entire consumer population. Under the refundable strategy, the intercept of the average revenue curve is at ψ_1 instead of at 1, as it is scaled down by the expected refunds. Therefore, the average revenue function under full refund forms a straight line from ψ_1 down to maximal participation at 1. Under the non-refundable strategy we have a kinked market demand curve. Moreover, when the ticket price is zero a refunded and a non-refunded ticket are equivalent from consumers' point of view. Therefore, the areas under the average revenue curves should be equal under both strategies. Since the demand curve under the non-refundable strategy is kinked, the average revenue under the non-refundable strategy must be below the average revenue under the refundable strategy when both consumer types are served.

The next proposition postulates our results for the range of medium capacity.

Proposition 2. *Suppose that capacity is at a medium level, i.e.* $\widetilde{K} < K < \widehat{K}$*. Then the rankings of profits and consumer surplus are*

a. $\pi^D > \pi^R > \pi^{N_H}$ *and*

b. $CS^D > CS^R$

Under these equilibria, type-L customers are not served under the non-refundable strategy. The ranking of the consumer surplus under the non-refundable strategy CS^{N_H} *is ambiguous.*

Comparing Proposition 2 with Proposition 1 reveals that the profit-ranking remains the same despite the fact that under a medium capacity level type-L are not served under non-refundable booking. However, the ranking of consumer surpluses differ. The consumer surplus under the non-refundable strategy is heavily reduced as a result of market exclusion and a higher price. Therefore, the non-refundable strategy yields a lower consumer surplus and social welfare compared with large capacity levels.

The rankings at small capacity levels are analyzed in the following proposition.

Proposition 3. *Suppose that capacity is small, i.e.* $\underline{K} < K < \widetilde{K}$*. Then the rankings of profits and consumer surplus are*

a. $\pi^D > \pi^{N_H} > \pi^R$*, and*

b. $\min\left\{CS^D, CS^{N_H}\right\} > CS^R$.

Proposition 3 demonstrates that when capacity is heavily restricted, the refund booking strategy is the least profitable and desirable from a social point of view. This is, it is more profitable to serve type-H consumers who do not require the refundability option. Selling only refundable tickets would attract consumer of type-L, who would contribute to more no-shows, which otherwise would have been better allocated to consumers of type-H. Figure 4 illustrates the profit comparisons made in Propostion 3.

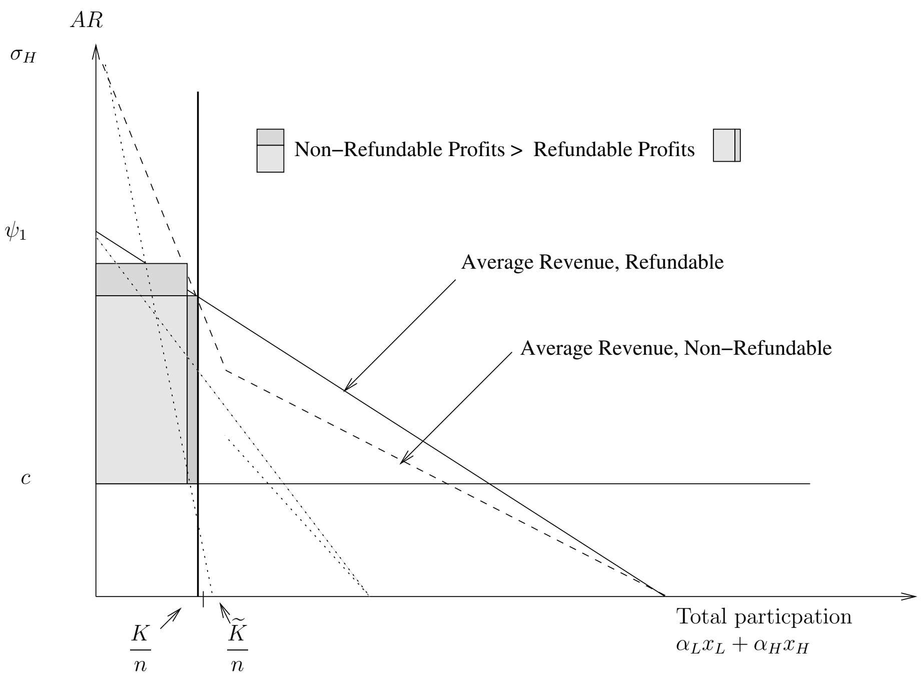

Figure 4: Profit comparison under low capacity

Next we further specify the ranking of the consumer surplus.

Proposition 4. *Suppose that the capacity is in the low range* $\underline{K} < K < \overline{K}$*, where and* $\overline{K}$ *and* $\underline{K}$ *are given in (19) and (33). Then the ranking of the consumer surplus is* $CS^{N_H} > CS^D > CS^R$.

If follows from Propositions 1 and 4, that the consumer surplus is highest under the non-refundable strategy, when capacity, K, is larger than $\widehat{K}$ or lower than $\overline{K}$, that is when the capacity is fully utilized. In the capacity range $\overline{K} < K < \widehat{K}$, the ranking of the non-refundable strategy is ambiguous.

Finally, our last proposition answers the question when it is profitable for the service provider to book below the available capacity.

Proposition 5. *Suppose that the service provider is restricted to a single booking strategy. Then if* $\overline{K} < K < \widetilde{K}$*, the profit maximizing booking level is below the capacity.*

The possibility of profitable underbooking is demonstrated in the following numerical example. With the parameter values $\sigma_H = 0.9$, $\sigma_L = 0.6$, $\alpha_H = 0.3$ and $c = 0.4$, the threshold capacity levels are $\overline{K} \approx 0.0833 < \widetilde{K} \approx 0.09196$.

5 Discussion

The novelty of our approach is that we integrate capacity constaints into screening models of advance booking. We believe that this integration is essential because the advance booking problem is closely related to capacity constraints. The optimal booking strategy varies with the amount of available capacity. If capacity is large, it will always be profitable, under a single booking strategy, to expand the customer base by selling refundable tickets. Refundable tickets attract more consumers with low probability of showing up. However, having more no shows becomes increasingly costly when capacity is scarce. At more restrictive capacity levels the profit maximizing service provider, will exclude participation of customers with low showing up probability by selling more expensive tickets targeted primarily for the customers with higher showing up probability. The outcome from a consumer point of view is different. A dual booking strategy always yields a higher consumer surplus than a single booking strategy with refunds. The consumer surplus is maximized under the non-refundable strategy, if the entire capacity is booked. Since, it isn't always profitable to utilize the entire capacity under the non-refundable strategy, we cannot conclude that the consumer surplus is universally highest under the non-refundable strategy.

Our research confirms the observation that refundability options are more frequently observed during recession periods than under peak-demand periods. This suggest that service providers adjust their booking strategies under business cycles.

Appendix

A Proofs of Propositions 1 – 4

We prove Propostion 1 in order.

Part *a*. Recall the profits π^R (7), $\pi^{N_{\text{both}}}$ (15), and π^D (30) from the main text. As $\psi_1 > \psi_2$, $\pi^R - \pi^{N_{\text{both}}} = (\psi_1 - \psi_2)(1 - K/n)K > 0$; $4n^{-1}\left(\pi^D - \pi^R\right) = (\psi_1 - \psi_2) + 4\psi_2 K/n - 4\psi_2 (K/n)^2 - 4\psi_1 K/n - 4\psi_1 (K/n)^2 = (\psi_1 - \psi_2)(1 - 2K/n)^2 > 0$; and $\pi^D - \pi^{N_{\text{both}}} = n \cdot (\psi_1 - \psi_2)/4 > 0$.

Part *b*. The difference between the consumer surpluses CS^N in (17) and CS^D in (31) is $CS^{N_{\text{both}}} - CS^D = n \cdot \dfrac{3}{8}(\psi_1 - \psi_2) > 0$. The difference between the consumer surpluses CS^D in (31) and CS^R in (10) is $n \cdot (CS^D - CS^R)/2 = (\psi_1 - \psi_2) + \psi_2 (K/n)^2$ - $\psi_1 (K/n)^2$ $= (\psi_1 - \psi_2)\left(1 - (K/n)^2\right) > 0$.

Part *c*. By combining the differences in Parts *a* and *b* above we observe that $SW^{N_{\text{both}}} - SW^D = CS^{N_{\text{both}}} - CS^D + \pi^{N_{\text{both}}} - \pi^D = n \cdot (\psi_1 - \psi_2)/8 > 0$, and that $SW^D - SW^R = \left(CS^D - CS^R\right) + \left(\pi^D - \pi^R\right) > 0$.

Proposition 2 : Here the welfare ranking might switch due to the fact that $CS^{N_H} = 0.5\pi^{N_H} = \dfrac{n\sigma_H}{2\alpha_H}\left(\dfrac{\min\{K, \overline{K}\}}{n}\right)^2$ and $CS^R = \dfrac{n\psi_1}{2}\left(\dfrac{K}{n}\right)^2$. Under medium capacity the ranking between CS^{N_H} and $CS^R$is without further restrictions unclear.

Proposition 3. If $\overline{K} < K < \widetilde{K}$ we have $CS^{N_H} = 0.5\pi^{N_H} > 0.5\pi^R > CS^R$. If $\underline{K} < K < \overline{K}$, the demand curve is steeper under the non-refundable strategy, and therefore $CS^{N_H} > CS^R$.

Finally, Proposition 4 states that $CS^{N_H} > CS^D$ when $K < \overline{K}$. Proof: We first combine equations (22) and (31) as follows

$$\frac{CS^{N_H}}{n} = \frac{\sigma_H}{2\alpha_H}\left(\frac{K}{n}\right)^2 = \frac{\psi_1 - \psi_2}{8} + \frac{\psi_2}{2}\left(\frac{K}{n}\right)^2 = \frac{CS^D}{n},$$

and solve for K/n.

$$\frac{K}{n} = \frac{\alpha_H}{2}\left(\frac{\sigma_H - \sigma_L}{\sigma_H}\right),$$

which is exactly $\underline{K}/n$ in (33). As $\sigma_H/\alpha_H > \psi_2$ and $\psi_1 - \psi_2 > 0$, we conclude that $\overline{K} > K > \underline{K} \iff CS^{N_H} > CS^D$.

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