

# “RELEVANT” MARKETS: A PRODUCT DIFFERENTIATION APPROACH\*

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## Abstract

I propose a simple method of identifying a firm’s relevant market in antitrust cases in general and merger analysis in particular. Starting from a demand system associated with markets for differentiated goods, this paper computes the threshold prices associated with the “**S**mall but **S**ignificant and **N**ontransitory **I**ncrease in **P**rice” method to identify a firm’s relevant market. This procedure greatly simplifies the characterization of relevant markets as it relies only on the estimated coefficients of demand functions and the observed market prices.

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# 1. Introduction

Almost every decision based on antitrust law regarding the conduct of a specific firm relies heavily on a proper definition and characterization of the firm's "relevant" market. In particular, merger control relies heavily on a decision on which markets may be affected by reducing the number of firms in an industry. If a firm's relevant market is large, it is less likely to be able to exercise monopoly power in the form of setting high prices above unit cost. In contrast, if the relevant market is narrow, the firm should be closely monitored, and a merger with other firm in the same industry should not be allowed.

Economic theory generally does not provide policy makers with precise tools for how to measure a firm's relevant market. The present paper attempts to fill this gap by adapting the widely used "**S**mall but **S**ignificant and **N**ontransitory **I**ncrease in **P**rice" test to a system of demand functions for differentiated goods (differentiated markets). The basic idea here is that the determination of the relevant market is synonymous to determining how low the degree of market differentiation should be in order to be able to decide which markets or brands should be included in a firm's relevant market. Thus, I argue that the use of the differentiated brands model is essential for determining relevant markets. Surprisingly, as far as I know, there are no formal theories to analyze which markets should be considered to be relevant to a specific firm operating in an industry producing or selling differentiated products or services.

The SSNIP test, which is described in Section 1.1 of the 1992 Horizontal Merger Guidelines issued by U.S. Department of Justice and the Federal Trade Commission, states as follows:

"Specifically, the Agency will begin with each product (narrowly defined) produced or sold by each merging firm and ask what would happen if a hypothetical monopolist of that product imposed at least a "small but significant and nontransitory" increase in price, but the terms of sale of all other products remained constant. If, in response to the price increase, the reduction in sales of the product would be large enough that a hypothetical monopolist would not find it profitable to impose such an increase in

price, then the Agency will add to the product group the product that is the next-best substitute for the merging firm's product.”

In essence, this test asks whether a hypothetical monopoly firm would find it profitable to impose a small but significant and non-transitory increase in price (say, a 5% increase) in the firm's most narrowly-defined market. If the answer is affirmative, then the relevant market has been identified. If the answer is negative, this must be because other markets (which could be other products, services, or regions) exert competitive pressure and hence should also be included in the relevant market. In this case, the test must be repeated after the next most substitutable market (again, a product, service, or a region) is added, and so on until the relevant market is identified.

The above quote from Section 1.1 of the merger guidelines reveals two problems with the SSNIP test. First, the guidelines do not specify exactly which prices should be raised by the hypothetical monopolist. That is, when markets are added, should only the price in the firm's own market be raised by 5%, or should the prices in all added markets be raised as well? The most common interpretation is to keep raising only one price, which is the price of the firm in question and leave all other prices at the observed level, even when additional markets are added. I, therefore, adopt this interpretation. The reader is referred to Daljord, Sørsgard, and Thomassen (2007) who contrast these two interpretations of how to implement the SSNIP test, and Whinston (2007). The second problem associated with the above quote comes from “...but the terms of sale of all other products remained constant.” Clearly, this is impossible to implement because in markets for differentiated brands all sales levels are influenced by prices in all markets. The present paper cures this problem by allowing sales in all markets to adjust to a change in the price set by the firm in question. Hence, the product differentiation approach of the present paper proves to be essential for a proper implementation of the SSNIP test.

To my best knowledge, the present paper is the first to derive the SSNIP test in a differentiated markets environment. Obviously it is not the first to formalize the SSNIP test. A commonly-used method which implements the SSNIP test defines a “critical loss” of sales level associated with a small price increase. By comparing this critical level to the actual loss of sales, one can obtain

an intuitive interpretation to this “hypothetical monopoly” test, which may appeal to regulators and practitioners who rely on empirical estimation. In this respect, the present paper also seeks to theoretically derive simple formulas which rely on estimated demand coefficients and could be easily used for market definition purposes. Katz and Shapiro (2003) modify the “critical loss” formulation by introducing an “aggregate diversion ratio,” which is the percentage of the total loss of product sales when the price rises that are captured by all of the other products in the candidate market. Daljord, Sørsgard, and Thomassen (2007) modify the last approach by analyzing price increases in all markets which are candidates to be classified as relevant. Moresi, Salop, and Woodbury (2008) further extend this test to multiproduct firms. Finally, Dobbs (2003) demonstrates that ignoring cost structure may generate a bias, in that market power will often be found when in fact none is present.

Section 2 provides a three-good example which demonstrates the usefulness of characterizing the relevant market using demand functions for differentiated brands. Section 3 develops the general model and computes simple formulas for determining the relevant market of a brand producing firm. Section 4 concludes with a short discussion.

## 2. An Example with Three Markets

Suppose the demand functions for orange juice (O), grape juice (G), and tomato juice (T) are given by

$$q_O = 90 - 2p_O + p_G + p_T, \quad q_G = 100 - 2p_G + p_O + p_T, \quad \text{and} \quad q_T = 120 - 2p_T + p_O + p_G.$$

where  $p_O$ ,  $p_G$ , and  $p_T$  denote the price of orange juice, grape juice, and tomato juice, respectively.<sup>1</sup> The costs of producing *one unit* of orange juice, grape juice, and tomato juice, are the same and given by  $c_O = c_G = c_T = 20$ .

Assume that the ORANGADA company is the sole producer and seller of orange juice, and that the following market prices are observed:  $p_O = 60$ ,  $p_G = p_T = 40$ . The problem faced by

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<sup>1</sup>It should be emphasized that we are dealing with actually *observed* prices as the baseline. Whether or not these prices are an equilibrium outcome of some market structure is irrelevant at least for a short-run examination.

the competition authority is to apply the SSNIP test to determine which of the following markets should be considered to be the relevant market for ORANGADA company:

- (i) Orange juice market only,
- (ii) Orange and grape juice markets combined,
- (iii) Orange, grape, and tomato juice markets combined, or
- (iv) A broader market (more juices should be included).

Under the *observed* prices  $(p_O, p_G, p_T) = (60, 40, 40)$ , the profit levels of the ORANGADA company and the grape and tomato juice producers are

$$\pi_O(60, 40, 40) = (60 - 20)q_O = (60 - 20)(90 - 2 \cdot 60 + 40 + 40) = 2000,$$

$$\pi_G(60, 40, 40) = (40 - 20)q_G = (40 - 20)(100 - 2 \cdot 40 + 60 + 40) = 2400,$$

$$\pi_T(60, 40, 40) = (40 - 20)q_T = (40 - 20)(120 - 2 \cdot 40 + 60 + 40) = 2800.$$

Using the SSNIP test, we first check whether the market for orange juice alone is the relevant market for the ORANGADA company by raising the *observed* price  $p_O$  by 5% from  $p_O = 60$  to  $p'_O = 63$ . Then,

$$\pi_O(63, 40, 40) = (63 - 20)q_O = (63 - 20)(90 - 2 \cdot 63 + 40 + 40) = 1892 < 2000,$$

which means that a price increase is not profitable. Thus, under the SSNIP criterion, the market for orange juice alone should *not* be regarded as the relevant market for the ORANGADA company.

Next, we ask whether the combined markets for orange and grape juice should be considered as the relevant market. Setting again  $p'_O = 63$  makes the combined profit in both markets<sup>2</sup>

$$\pi_O(63, 40, 40) + \pi_G(63, 40, 40) = 1892 + (40 - 20)(100 - 2 \cdot 40 + 63 + 40) = 4352 < 2000 + 2400.$$

Thus, the combined markets for orange and grape juice should *not* be considered as the relevant market for the ORANGADA company.

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<sup>2</sup>As discussed in the introduction, for the purpose of applying the SSNIP test to the ORANGADA company, only the price of orange juice should be raised whereas all other prices should be kept at the observed levels, regardless of which potentially-relevant markets are investigated.

Next, should the markets for the three juices combined be considered as the relevant market?

To answer this question we sum up all three profits

$$\begin{aligned}\pi_O(63, 40, 40) + \pi_G(63, 40, 40) + \pi_T(63, 40, 40) &= 4352 + (40 - 20)(120 - 2 \cdot 40 + 63 + 40) = 7212 \\ &> 7200 = \pi_O(60, 40, 40) + \pi_G(60, 40, 40) + \pi_T(60, 40, 40).\end{aligned}$$

Yes, according to the SSNIP test, the relevant market for the ORANGADA company is the market for all three juices combined.

### 3. The General Model

The section extends the above three market example to  $N \geq 2$  markets. Our goal is to derive general formulas that facilitate the determination of the relevant market of a firm using observed market prices and estimated coefficients demand functions for differentiated goods (markets).

Each market  $i$  ( $i = 1, \dots, N$ ) is served by a single firm with a unit production  $c_i \geq 0$ . Our goal is to characterize the relevant market for the firm operating in market 1. Therefore, we index markets according to potential competition facing firm 1. Thus, the good sold in market 2 is the most substitutable to the good sold market 1, market 3 is the next most substitutable, and market  $N$  is the least substitutable to the good sold in market 1.<sup>3</sup> The system of demand functions to be estimated by the econometrician is assumed to take the linear form given by

$$\begin{aligned}q_1(p_1, \dots, p_N) &= z_1 + a_{11}p_1 + a_{12}p_2 + \dots + a_{1N}p_N, \\ q_2(p_1, \dots, p_N) &= z_2 + a_{21}p_1 + a_{22}p_2 + \dots + a_{2N}p_N, \\ &\vdots \\ q_N(p_1, \dots, p_N) &= z_N + a_{N1}p_1 + a_{N2}p_2 + \dots + a_{NN}p_N.\end{aligned}\tag{1}$$

The econometrician should estimate the coefficients  $z_i$  and  $a_{ij}$ . It should be verified that the “diagonal” coefficients satisfy  $a_{ii} < 0$  which means that the demand in market  $i$  is downward sloping with respect to  $p_i$ , for  $i = 1, \dots, N$ .

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<sup>3</sup>This paper does not specify any formal criterion for how to determine the degree of substitutability among markets (according to which markets are indexed). Here, I assume that there is no dispute that market 2 is the closest to market 1, and so forth. See the concluding section, Section 4, for more discussions.

Using the SSNIP criterion, Market 1 is the relevant market for firm 1 if the following condition is satisfied when evaluated at the *observed* market prices,

$$\frac{\partial \pi_1}{\partial p_1} = \frac{\partial [(p_1 - c_1) q_1(p_1, \dots, p_N)]}{\partial p_1} > 0,$$

hence, if

$$p_1 < P_{\{1\}} \stackrel{\text{def}}{=} \frac{z_1 - a_{11} c_1 + \sum_{i=2}^N (a_{1i} p_i)}{-2a_{11}}. \quad (2)$$

Next, suppose that condition (2) is not fulfilled which implies that market 1 alone should not be regarded as the relevant market for firm 1.<sup>4</sup> In this case, according to the SSNIP test, the combined markets 1 and 2 should be considered as the relevant market for firm 1 if the following condition is satisfied at the *observed* market prices,

$$\frac{\partial(\pi_1 + \pi_2)}{\partial p_1} = \frac{\partial [(p_1 - c_1) q_1(p_1, \dots, p_N)]}{\partial p_1} + \frac{\partial [(p_2 - c_2) q_2(p_1, \dots, p_N)]}{\partial p_1} > 0,$$

hence, if

$$p_1 < P_{\{1,2\}} \stackrel{\text{def}}{=} \frac{z_1 - a_{11} c_1 + \sum_{i=2}^N (a_{1i} p_i) + a_{21}(p_2 - c_2)}{-2a_{11}} = P_{\{1\}} + \frac{a_{21}(p_2 - c_2)}{-2a_{11}}. \quad (3)$$

Next, suppose that condition (3) is not fulfilled which implies that the combined markets 1 and 2 should not be regarded as the relevant markets for firm 1. Thus, according to the SSNIP test, the combined markets 1, 2 and 3 should be considered to be the relevant markets for firm 1 if

$$\frac{\partial(\pi_1 + \pi_2 + \pi_3)}{\partial p_1} = \sum_{i=1}^3 \frac{\partial [(p_i - c_i) q_i(p_1, \dots, p_N)]}{\partial p_1} > 0,$$

hence, if

$$\begin{aligned} p_1 < P_{\{1,2,3\}} \stackrel{\text{def}}{=} \frac{z_1 - a_{11} c_1 + \sum_{i=2}^N (a_{1i} p_i) + a_{21}(p_2 - c_2) + a_{31}(p_3 - c_3)}{-2a_{11}} \\ = P_{\{1,2\}} + \frac{a_{31}(p_3 - c_3)}{-2a_{11}}. \quad (4) \end{aligned}$$

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<sup>4</sup>Note that the familiar “Cellophane case” may occur here if  $p_1$  already exceeds the monopoly price in market 1. In this case, condition (2) would indicate that market 1 is not the relevant market although this need not be the case. Therefore, the SSNIP test should be applied to firms facing some degree of price competition rather than to firms that already set their monopoly price.

Next, suppose that condition (4) is not satisfied, which means that the combined markets 1, 2 and 3 should not be regarded as the relevant markets for firm 1 under the SSNIP test. Therefore, we now investigate whether the combined markets 1, 2, 3 and 4 should be considered to be the relevant markets for firm 1. This would be the case if

$$\frac{\partial(\pi_1 + \pi_2 + \pi_3 + \pi_4)}{\partial p_1} = \sum_{i=1}^4 \frac{\partial[(p_i - c_i) q_i(p_1, \dots, p_N)]}{\partial p_1} > 0,$$

hence, if

$$p_1 < P_{\{1,2,3,4\}} \stackrel{\text{def}}{=} \frac{z_1 - a_{11} c_1 + \sum_{i=2}^N (a_{1i} p_i) + a_{21}(p_2 - c_2) + a_{31}(p_3 - c_3) + a_{41}(p_4 - c_4)}{-2a_{11}} = P_{\{1,2,3\}} + \frac{a_{41}(p_4 - c_4)}{-2a_{11}}. \quad (5)$$

The “relevant” conditions (2), (3), (4), and (5) reveal a general pattern under which we can state the main result.

**Result 1.** *Let  $(p_1, p_2, \dots, p_N)$  be the observed prices in  $i = 1, \dots, N$  markets characterized by the estimated system of demand functions (1) with  $a_{ii} < 0$ . Let  $(c_1, c_2, \dots, c_N)$  denote the unit production costs of the firms operating in these markets. Then, the SSNIP test implies that the relevant market for firm 1 is the set of markets  $\{1, 2, \dots, M\}$ , where  $M$  is the smallest integer  $1 \leq M \leq N$ , satisfying*

$$p_1 < P_{\{1,2,\dots,M\}} \stackrel{\text{def}}{=} \frac{z_1 - a_{11} c_1 + \sum_{i=2}^N (a_{1i} p_i) + \sum_{i=2}^M [a_{1i}(p_i - c_i)]}{-2a_{11}} \quad (6)$$

Finally, observe that condition (6) can be derived recursively by iterating on

$$P_{\{1,2,\dots,M\}} = P_{\{1,2,\dots,M-1\}} + \frac{a_{M1}(p_M - c_M)}{-2a_{11}} \quad (7)$$

where  $p_{\{1\}}$  is defined in (2). Note that (7) holds as long as the relevant market for firm 1 is broader than its own market (formally, for all cases in which  $M \geq 2$ ).



## 4. Discussions

The simple framework constructed in this paper derives a formula (6) under which a relevant market of a firm could be easily identified. All that is required is to properly estimate the system of demand functions associated with differentiated markets, and to know the observed market price in each market. Once this information is fed into a computer, the computer can iterate according to (7) to identify all markets that do not compete very much with the firm in question. These markets are then classified as relevant markets.

The algorithm for identifying the relevant market constructed in this paper requires the user to index markets according to declining degree of substitutability to the good sold in market 1. This paper does not suggest any algorithm for how to determine and rank the degree of substitutability among markets. Clearly, there may several ways of doing that. However, without any further knowledge of the particular industry in question, one simple way to index the markets is to use the empirically-estimated market demand functions (1) to index markets according to  $a_{12} \geq a_{13} \geq \dots \geq a_{1N}$ . This ranking would imply that the quantity demanded in market 1 is more sensitive to a price increase in market 2 than to a price increase in market 3, and so on; so a price increase in market  $N$  would have the smallest effect on quantity demanded in market 1. However, note that this measure of substitutability is far from perfect simply because, for example, if the demand intercept  $z_2$  is much smaller relative to  $z_3, z_4$ , the competition authority may not find it useful to consider the good sold in market 2 to be the most substitutable to good 1 despite having  $a_{12} \geq a_{13} \geq \dots \geq a_{1N}$ .

Finally, given the problem at hand, the analysis would ideally extend to multi-product firms, rather than assuming that each market is served by a single firm. Indeed it is products, not firms, for which relevant markets are defined. All that has to be done is to replace the term “market” with the term “product,” to obtain a measure of an industry’s product definition.

## References

- Daljord, Ø, L. Sjørgard, and Ø. Thomassen. 2007. "The SSNIP Test and Market Definition with the Aggregate Diversion Ratio." *Journal of Competition Law & Economics* 4: 263–270.
- Dobbs, I. 2003. "Demand, Cost Elasticities, and Pricing Benchmarks in the Hypothetical Monopoly Test: The Consequences of a simple SSNIP." *Applied Economics Letters* 10: 545–548.
- Katz, M. and C. Shapiro. 2003. "Critical Loss: Let's Tell the Whole Story." *Antitrust Magazine*, 17:2, Spring.
- Moresi, S., S. Salop, and J. Woodbury. 2008. "Implementing the Hypothetical Monopolist SSNIP Test with Multi-Product Firms." SSRN Working Paper.
- Whinston, M. 2007. "Antitrust Policy Toward Horizontal Mergers." In: M. Armstrong and R. Porter (eds.) *Handbook of Industrial Organization*, Volume 3, Ch.36: 2370–2440.